Q1 If Input Prices Are W = 4, And R = 1, And Q = 4K0.5L0.5, What Is The Least Cost Input Combination

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When analyzing production processes and aiming to optimize costs, understanding the least-cost input combination is essential. This involves determining the optimal quantities of inputs—typically capital (K) and labor (L)—that minimize total production costs while producing a specified level of output. In this article, we will explore how to identify the least-cost combination of inputs given the input prices and the production function: Q = 4K^0.5L^0.5, with input prices W = 4 for labor and R = 1 for capital. We will dissect the problem step-by-step, employ mathematical and economic principles, and provide practical insights into cost minimization strategies.

Understanding the Production Function

The production function provided is:

$$ Q = 4K^{0.5}L^{0.5} $$

This is a Cobb-Douglas production function, characterized by diminishing marginal returns and constant returns to scale when the exponents sum to one. Here, the exponents are 0.5 for both capital (K) and labor (L), summing to 1, indicating constant returns to scale.

Key features of this production function include:
- Symmetric input contributions
- Marginal products decreasing as input quantities increase
- Ease of analysis due to its mathematical properties

Input Prices and Cost Function

Given input prices:
- W (price of labor, L) = 4
- R (price of capital, K) = 1

The total cost (C) of producing a certain output level Q is:

$$ C = W \times L + R \times K = 4L + 1 \times K $$

Our goal is to minimize C subject to the production constraint $Q = 4K^{0.5}L^{0.5}$.

Step-by-Step Approach to Find Least Cost Input Combination

The problem can be formulated as an optimization problem:

Minimize:

$$C = 4L + K$$

Subject to:

$$Q = 4K^{0.5}L^{0.5}$$

Step 1: Express the production constraint

Rearranged to express one input in terms of the other:

$$Q = 4K^{0.5}L^{0.5}$$

Divide both sides by 4:

$$\frac{Q}{4} = K^{0.5}L^{0.5}$$

Square both sides:

$$\left(\frac{Q}{4}\right)^2 = K \times L$$

So,

$$K \times L = \left(\frac{Q}{4}\right)^2$$

This is a key relationship linking the inputs for a given output Q.

Step 2: Formulate the cost minimization problem

Our goal is:

Minimize:

$$C = 4L + K$$

subject to:

$$ K \times L = \left(\frac{Q}{4}\right)^2 $$

This is a constrained optimization problem, best approached using Lagrangian multipliers.

Step 3: Set up the Lagrangian

Define the Lagrangian function:

$$ \mathcal{L} = 4L + K + \lambda \left(\left(\frac{Q}{4}\right)^2 - K L \right) $$

where λ is the Lagrange multiplier.

Step 4: Take partial derivatives and set to zero

Compute partial derivatives:

- With respect to K:

$$ \frac{\partial \mathcal{L}}{\partial K} = 1 - \lambda L = 0 \Rightarrow 1 = \lambda L $$

- With respect to L:

$$ \frac{\partial \mathcal{L}}{\partial L} = 4 - \lambda K = 0 \Rightarrow 4 = \lambda K $$

- With respect to λ:

$$ \left(\frac{Q}{4}\right)^2 - K L = 0 $$

Step 5: Solve the system of equations

From the first two equations:

$$ 1 = \lambda L \Rightarrow \lambda = \frac{1}{L} $$

$$ 4 = \lambda K \Rightarrow 4 = \frac{1}{L} K \Rightarrow K = 4L $$

Substitute $K = 4L$ into the production constraint:

$$ K L = \left(\frac{Q}{4}\right)^2 $$

$$ (4L) \times L = \left(\frac{Q}{4}\right)^2 $$

$$ 4L^2 = \left(\frac{Q}{4}\right)^2 $$

$$ L^2 = \frac{1}{4} \left(\frac{Q}{4}\right)^2 $$

\[L^2 = \frac{Q^2}{4 \times 16} = \frac{Q^2}{64} \]

\[L = \frac{Q}{8} \] (taking the positive root, as input quantities are positive)

Now, find \(K \):

\[K = 4L = 4 \times \frac{Q}{8} = \frac{Q}{2} \]

Final Least Cost Input Combination

\[
\boxed{
\begin{aligned}
L^ &= \frac{Q}{8} \\
K^ &= \frac{Q}{2}
\end{aligned}
}
\]

These are the input quantities that minimize total cost for producing output \(Q\).

Calculating the Least Cost for a Specific Output Level

Suppose we want to produce \(Q = 4\) units. Plugging into the formulas:

- Labor:

\[L^ = \frac{4}{8} = 0.5 \]

- Capital:

\[K^ = \frac{4}{2} = 2 \]

Total minimum cost:

\[C^ = 4L^ + R K^ = 4 \times 0.5 + 1 \times 2 = 2 + 2 = 4 \]

This means producing 4 units of output costs at least 4 currency units with the optimal input combination.

Implications and Insights

Understanding the least-cost input combination allows firms to optimize resource allocation, reduce expenses, and improve profitability. Several key insights emerge from this analysis:

- Input Proportions: The optimal ratio of capital to labor is directly proportional to the output level \(Q\). Specifically, \(K^ = Q/2\) and \(L^ = Q/8\). This indicates that as production increases, both inputs need to increase proportionally to maintain cost efficiency.

- Cost Efficiency: The formula for minimum cost at a given \(Q\) is:

\[C_{min} = 4L^ + R K^ = 4 \times \frac{Q}{8} + 1 \times \frac{Q}{2} = \frac{Q}{2} + \frac{Q}{2} = Q \]

This shows that the minimum cost of producing \(Q\) units equals \(Q\), which is a linear relationship. In other words, the cost per unit remains constant at 1, aligning with the constant returns to scale of the production function.

- Economies of Scale: Since the production function exhibits constant returns to scale, doubling the output doubles the minimum cost, maintaining proportionality.

Practical Applications and Considerations

In real-world scenarios, firms can leverage this analysis to:

- Budget for production costs based on desired output levels, ensuring resource allocation aligns with optimal input combinations.
- Adjust input proportions dynamically as production targets change.
- Negotiate input prices or explore technological improvements to reduce costs further.
- Plan capital and labor investments strategically, considering the proportional relationships derived.

Conclusion

Determining the least-cost input combination is a foundational aspect of production and cost management in economics. For the production function \(Q = 4K^{0.5}L^{0.5}\) with input prices \(W=4\) and \(R=1\), the optimal inputs are:

- \(L^ = \frac{Q}{8}\)
- \(K^ = \frac{Q}{2}\)

This analysis highlights the importance of mathematical optimization techniques, such as Lagrangian multipliers, in solving real-world economic problems. By understanding and applying these principles, businesses can achieve cost efficiency, improve competitiveness, and make informed strategic decisions.

Keywords: least cost input combination, production function, Cobb-Douglas, cost minimization, input optimization, input prices, economic analysis, production costs, resource allocation

Frequently Asked Questions

What are the input prices provided in the problem?

The input prices are W = 4 for labor and R = 1 for capital.

What is the given production function?

The production function is Q = 4K^{0.5}L^{0.5}.

How do you determine the least cost input combination for a given output?

By minimizing total cost, which is WL + RK, subject to the production function constraint Q = 4K^{0.5}L^{0.5}.

What is the first step in finding the least cost combination?

Express one input in terms of the other using the production function, then set up the cost function and use optimization techniques to minimize it.

What is the formula for the total cost in this problem?

Total cost C = 4L + 1K.

What is the least cost input combination when Q = 4K^{0.5}L^{0.5} and input prices are W=4, R=1?

The least cost input combination occurs when K = L = (Q/4)^2. For example, if Q=10, then K=L= (10/4)^2 = (2.5)^2 = 6.25 units.

Additional Resources

Q1: If Input Prices Are W = 4, And R = 1, And Q = 4K^{0.5}L^{0.5}, What Is The Least Cost Input Combination?

Introduction

In the realm of microeconomics, understanding how firms minimize costs while producing a given level of output is fundamental. This question explores the optimal input combination—specifically, the least cost combination of capital (K) and labor (L)—given input prices and a production function. The problem is set with input prices W = 4 (for labor) and R = 1 (for capital), and a Cobb-Douglas production function: Q = 4K^{0.5}L^{0.5}. The goal is to determine the combination of K and L that minimizes total cost while producing a specified output level.

This analysis not only illustrates core principles of cost minimization but also demonstrates the application of mathematical techniques like setting up the cost function, using Lagrangian multipliers, and understanding the properties of Cobb-Douglas functions. The insights derived here are crucial for managers, economists, and policymakers aiming to optimize resource allocation in production.

Section 1: Understanding the Components of the Problem

1.1 Production Function: Cobb-Douglas Form

The production function:

$$ Q = 4K^{0.5}L^{0.5} $$

is a classic Cobb-Douglas form, characterized by constant returns to scale when the sum of the exponents equals 1 (here, 0.5 + 0.5 = 1). This property implies that if inputs are scaled proportionally, output scales proportionally, which simplifies analysis.

The function indicates that output depends symmetrically on capital and labor, each contributing equally to production, scaled by a factor of 4. The exponents imply diminishing marginal returns to each input individually but constant returns to scale overall.

1.2 Input Prices

- W (wage rate for labor): 4
- R (rental rate for capital): 1

These prices directly influence the firm's choice of input combination, as the firm aims to minimize total cost:

$$ \text{Total Cost} = W \times L + R \times K $$

Given the prices, labor is relatively expensive compared to capital, which suggests the firm may prefer to substitute toward the cheaper input, all else equal, to minimize costs for a given output.

1.3 Output Level (Q)

The problem as stated does not specify a particular output level Q to be produced. To proceed with a meaningful analysis, assume a target output level Q_0. For illustration, let's choose $Q_0 = 16$. The methodology applies generally, and the specific value can be adjusted as needed.

Section 2: Formulating the Cost Minimization Problem

2.1 The Objective Function: Total Cost

The total cost function based on input prices:

$$ C = 4L + 1 \times K $$

Our goal is to find the combination of K and L that minimizes C subject to the production constraint:

$$ 4K^{0.5}L^{0.5} = Q_0 $$

or, rearranged:

$$ K^{0.5} L^{0.5} = \frac{Q_0}{4} $$

which simplifies to:

$$ \sqrt{K} \times \sqrt{L} = \frac{Q_0}{4} $$

or equivalently,

$$ \sqrt{K L} = \frac{Q_0}{4} $$

Section 3: Solving the Cost Minimization Problem

3.1 Using the Lagrangian Method

To find the minimum cost, set up the Lagrangian:

$$ \mathcal{L} = 4L + R K + \lambda \left(Q - 4K^{0.5}L^{0.5} \right) $$

Given $R=1$:

$$ \mathcal{L} = 4L + K + \lambda \left(Q - 4K^{0.5}L^{0.5} \right) $$

The first-order conditions (FOCs):

1. Derivative with respect to K:

$$ \frac{\partial \mathcal{L}}{\partial K} = 1 - \lambda \times 4 \times 0.5 \times K^{-0.5} L^{0.5} = 0 $$

2. Derivative with respect to L:

$$ \frac{\partial \mathcal{L}}{\partial L} = 4 - \lambda \times 4 \times 0.5 \times K^{0.5} L^{-0.5} = 0 $$

3. Production constraint:

$$ Q - 4K^{0.5}L^{0.5} = 0 $$

3.2 Simplifying the FOCs

From the derivatives:

$$ 1 = \lambda \times 2 \times K^{-0.5} L^{0.5} \Rightarrow \lambda = \frac{1}{2 K^{-0.5} L^{0.5}} $$

Similarly,

$$ 4 = \lambda \times 2 \times K^{0.5} L^{-0.5} \Rightarrow \lambda = \frac{4}{2 K^{0.5} L^{-0.5}} = \frac{2}{K^{0.5} L^{-0.5}} $$

Setting these equal:

$$ \frac{1}{2 K^{-0.5} L^{0.5}} = \frac{2}{K^{0.5} L^{-0.5}} $$

Rewrite each:

$$ \frac{1}{2} \times K^{0.5} L^{-0.5} = 2 \times K^{0.5} L^{-0.5} $$

But note that:

$$ K^{-0.5} = \frac{1}{K^{0.5}} $$

So the left side:

$$ \frac{1}{2} \times \frac{K^{0.5}}{L^{0.5}} $$

The right side:

$$ 2 \times \frac{K^{0.5}}{L^{0.5}} $$

Therefore:

$$ \frac{1}{2} \times \frac{K^{0.5}}{L^{0.5}} = 2 \times \frac{K^{0.5}}{L^{0.5}} $$

Dividing both sides by $\frac{K^{0.5}}{L^{0.5}}$:

$$ \frac{1}{2} = 2 $$

which is impossible unless the term cancels out, indicating a need to revisit the algebra.

Section 4: Deriving the Optimal Input Ratio

4.1 Alternative Approach: Equal Marginal Products per Cost

Given the symmetry in the production function, a more straightforward method involves equating the marginal product per dollar spent on each input:

$$ \frac{\text{MP}_L}{W} = \frac{\text{MP}_K}{R} $$

Where:

- $\text{MP}_L = \frac{\partial Q}{\partial L}$

- $\text{MP}_K = \frac{\partial Q}{\partial K}$

Calculate:

$$ \frac{\partial Q}{\partial L} = 4 \times 0.5 \times K^{0.5} L^{-0.5} = 2 K^{0.5} L^{-0.5} $$

$$ \frac{\partial Q}{\partial K} = 4 \times 0.5 \times K^{-0.5} L^{0.5} = 2 L^{0.5} K^{-0.5} $$

The cost-minimizing condition:

$$ \frac{2 K^{0.5} L^{-0.5}}{W} = \frac{2 L^{0.5} K^{-0.5}}{R} $$

Plugging in $W=4$, $R=1$:

$$ \frac{2 K^{0.5} L^{-0.5}}{4} = 2 L^{0.5} K^{-0.5} $$

Simplify:

$$ \frac{K^{0.5} L^{-0.5}}{2} = 2 L^{0.5} K^{-0.5} $$

Expressed differently:

\[\frac{K^{0.5}}{2 L^{0.5}} = 2 \frac{L^{0.5}}{K^{0.5}} \]

Cross-multiplied:

\[\frac{K^{0.5}}{2 L^{0.5}} \times K^{0.5} = 2 L^{0.5} \]

But better to manipulate directly:

\[\frac{K^{0.5}}{L^{0.5}} = 4 \frac{L^{0.5}}{K^{0.5}} \]

Cross-multiplied:

\[(K^{0.5})^2 = 4 (L^{0.5})^2 \]

which simplifies to:

\[K = 4 L

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