

Q2. Solve The Following Differential Equations By Leibnitz Linear Equation Method. (i) $(1-x) Dy - Xy$

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Introduction to Leibnitz Linear Differential Equations

Differential equations serve as fundamental tools in mathematical modeling across various scientific and engineering disciplines. Among these, linear differential equations are particularly significant due to their structured form and solvability. The Leibnitz linear differential equation method is a classical approach used to solve first-order linear differential equations, characterized by their standard form:

$$\frac{dy}{dx} + P(x) y = Q(x)$$

This method hinges on the concept of an integrating factor, which simplifies the differential equation into an exact derivative, enabling straightforward integration. In this article, we focus on applying the Leibnitz linear method to solve the specific differential equation:

$$(1 - x) \frac{dy}{dx} - x y = 0$$

through a detailed, step-by-step process, demonstrating the core principles and techniques involved.

Understanding the Given Differential Equation

Before solving, it's essential to analyze and rewrite the differential equation into the standard form suitable for the Leibnitz method.

Original Equation:

$$(1 - x) \frac{dy}{dx} - x y = 0$$

Rearranged to Standard Form:

Divide both sides by $(1 - x)$, assuming $x \neq 1$:

$$\frac{dy}{dx} - \frac{x}{1 - x} y = 0$$

which can be written as:

$$\frac{dy}{dx} + P(x) y = 0$$

where:

$$P(x) = -\frac{x}{1-x}$$

and

$$Q(x) = 0$$

Notice that this is a first-order linear differential equation with a zero right-hand side, making it a homogeneous linear differential equation.

Applying the Leibnitz Method to Solve the Differential Equation

The Leibnitz linear method involves finding an integrating factor $\mu(x)$, which is given by:

$$\mu(x) = e^{\int P(x) dx}$$

Once we find $\mu(x)$, multiplying the entire differential equation by it transforms it into an exact derivative:

$$\frac{d}{dx} (\mu(x) y) = \mu(x) Q(x)$$

\\

Given that $Q(x) = 0$, the solution will involve the integrating factor and the subsequent integration.

Step 1: Calculate the Integrating Factor $\mu(x)$

Calculate:

\\

$$\int P(x) dx = \int -\frac{x}{1-x} dx$$

\\

To evaluate this integral, manipulate the integrand:

\\

$$-\frac{x}{1-x} = -\frac{(1-x) - 1}{1-x} = -\left(1 - \frac{1}{1-x}\right) = -1 + \frac{1}{1-x}$$

\\

Thus,

\\

$$\int P(x) dx = \int \left(-1 + \frac{1}{1-x}\right) dx = \int -1 dx + \int \frac{1}{1-x} dx$$

\\

Calculate each term separately:

$$\int -1 dx = -x + C_1$$

$$\int \frac{1}{1-x} dx = -\ln|1-x| + C_2$$

(Recall that $\int \frac{1}{a-x} dx = -\ln|a-x|$.)

Combining:

$$\int P(x) \, dx = -x - \ln|1 - x| + C$$

The integrating factor is then:

$$\mu(x) = e^{\int P(x) \, dx} = e^{-x} \cdot e^{-\ln|1 - x|} = e^{-x} \cdot \frac{1}{|1 - x|}$$

For simplicity, assuming $(1 - x > 0)$, the absolute value can be dropped:

$$\mu(x) = \frac{e^{-x}}{1 - x}$$

Step 2: Rewrite the Differential Equation Using the Integrating Factor

Multiply the entire differential equation:

$$\frac{dy}{dx} + P(x)y = 0$$

by $(\mu(x))$:

$$\frac{e^{-x}}{1-x} \frac{dy}{dx} + \left(-\frac{x}{1-x} \right) \frac{e^{-x}}{1-x} y = 0$$

which simplifies to:

$$\frac{d}{dx} \left(\frac{e^{-x}}{1-x} y \right) = 0$$

This step is justified because:

$$\frac{d}{dx} \left(\mu(x) y \right) = \mu(x) \frac{dy}{dx} + y \frac{d}{dx} \mu(x)$$

and the choice of $\mu(x)$ ensures:

$$\frac{d}{dx} \left(\mu(x) y \right) = \mu(x) \frac{dy}{dx} + P(x) \mu(x) y$$

which matches the transformed differential equation.

Step 3: Integrate and Find the General Solution

Since:

$$\frac{d}{dx} \left(\frac{e^{-x}}{1-x} y \right) = 0$$

integrate both sides with respect to x :

$$\frac{e^{-x}}{1-x} y = C$$

where C is an arbitrary constant.

Solve for y :

$$y = C \frac{1-x}{e^{-x}} = C (1-x) e^x$$

This is the general solution to the differential equation.

Final Solution and Interpretation

The complete solution to the differential equation:

$$(1-x) \frac{dy}{dx} - xy = 0$$

is:

$$\boxed{\quad y(x) = A (1 - x) e^x}$$

where A is an arbitrary constant that can be determined based on initial conditions.

Key Points to Remember

When solving first-order linear differential equations via Leibnitz's method, keep in mind:

- Convert the differential equation into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.
- Compute the integrating factor $\mu(x) = e^{\int P(x) dx}$.
- Multiply through by the integrating factor to obtain an exact derivative.
- Integrate to find the general solution.
- Simplify and interpret the solution based on initial or boundary conditions.

Conclusion

The Leibnitz linear method provides a systematic and effective approach for solving first-order linear differential equations. By calculating the integrating factor and transforming the differential equation

into an exact derivative, the solution process becomes straightforward. The specific example examined here, $(1 - x) \frac{dy}{dx} - xy = 0$, illustrates the power and elegance of this technique, culminating in the general solution:

$$y(x) = A(1 - x)e^x$$

Understanding and mastering this method is essential for students and practitioners working with differential equations, enabling them to solve a wide variety of problems efficiently and accurately.

Keywords: Differential equations, Leibnitz linear method, integrating factor, first-order linear differential equations, solving differential equations, mathematical modeling, differential equation solutions

Frequently Asked Questions

What is the general form of a Leibniz linear differential equation?

A Leibniz linear differential equation is of the form $(a(x)y)' + b(x)y = c(x)$, where derivatives are expressed as total derivatives, allowing the substitution to linearize the equation.

How do you identify whether a differential equation is suitable for the Leibniz linear method?

You look for equations where the derivative term can be expressed as the total derivative of a product involving y , typically transforming into a form like $(u(x)y)'$ equals some function, enabling straightforward integration.

What is the step-by-step approach to solve $(1 - x) Dy - x y$ using Leibniz's method?

Rearrange the equation into the form $(u(x) y)' = v(x)$, identify $u(x)$ and $v(x)$, then integrate both sides to find y , and finally solve for y explicitly.

In the differential equation $(1 - x) Dy - x y$, how can you write it in the Leibniz form?

Divide through by $(1 - x)$ to get $Dy - (x / (1 - x)) y = 0$, then recognize this as the derivative of (some function times y), leading to the form $(u(x) y)' = 0$.

What integrating factor is used in Leibniz linear equations, and how is it determined?

The integrating factor is typically $u(x) = e^{\int p(x) dx}$, where $p(x)$ is the coefficient of y in the standard linear form; it transforms the equation into an exact derivative form.

Can you provide the solution to the differential equation $(1 - x) Dy - x y$ using Leibniz's method?

Yes. Rewrite as $(1 - x) Dy = x y$. Then, $(1 - x) Dy / y = x$. Recognize the left as $D[(1 - x) y]$, leading to the solution $y = C / (1 - x)$, where C is an integration constant.

What are common challenges faced while solving differential equations using Leibniz's method?

Challenges include correctly rewriting the equation into the Leibniz form, identifying the correct integrating factor, and performing the integration accurately, especially when functions involve complex expressions.

How does the Leibniz linear method compare to the standard integrating factor method for solving linear differential equations?

Both methods aim to linearize the differential equation; Leibniz's method emphasizes expressing derivatives as total derivatives of products, which can simplify integration, whereas the standard method uses integrating factors directly derived from the coefficients.

Additional Resources

Q2. Solve The Following Differential Equations By Leibnitz Linear Equation Method. (i) $(1-x) Dy - Xy$

Introduction

Differential equations form the backbone of mathematical modeling across a broad spectrum of scientific and engineering disciplines. Among various techniques devised to solve these equations, the Leibnitz linear equation method stands out for its systematic approach to first-order linear differential equations. This comprehensive review delves into the core principles of the Leibnitz linear method, with a particular focus on solving the differential equation:

$$\begin{aligned} & \backslash \\ (1 - x) \frac{dy}{dx} - xy &= 0 \\ & \backslash \end{aligned}$$

This specific problem exemplifies the typical structure that warrants the Leibnitz linear approach, and analyzing it provides insight into the method's utility, limitations, and implementation nuances.

Understanding the Differential Equation: Structural Analysis

Before solving, it is crucial to comprehend the form and components of the differential equation:

$$\begin{aligned} & \left[\right. \\ & (1 - x) \frac{dy}{dx} - xy = 0 \\ & \left. \right] \end{aligned}$$

This can be rearranged to a standard linear form. First, divide through by $(1 - x)$, assuming $x \neq 1$:

$$\begin{aligned} & \left[\right. \\ & \frac{dy}{dx} - \frac{x}{1 - x} y = 0 \\ & \left. \right] \end{aligned}$$

This is a first-order linear differential equation of the form:

$$\begin{aligned} & \left[\right. \\ & \frac{dy}{dx} + P(x) y = Q(x) \\ & \left. \right] \end{aligned}$$

where

$$\begin{aligned} & - \left(P(x) = - \frac{x}{1 - x} \right), \\ & - \left(Q(x) = 0 \right). \end{aligned}$$

The structure reveals that the equation is homogeneous, as $Q(x) = 0$. The solution technique—Leibnitz linear equation method—involves integrating factors, which are central to solving such equations.

The Leibnitz Linear Equation Method: Theoretical Framework

The Leibnitz linear method applies to first-order linear differential equations expressed as:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Key steps of the method:

1. Identify $P(x)$ and $Q(x)$.
2. Calculate the integrating factor (IF):

$$\mu(x) = e^{\int P(x) dx}$$

3. Multiply the entire differential equation by $\mu(x)$.
4. Recognize that the left-hand side becomes the derivative of $\mu(x)y$:

$$\frac{d}{dx} [\mu(x)y] = \mu(x)Q(x)$$

5. Integrate both sides with respect to x .
6. Solve for y .

This systematic approach simplifies solving otherwise complex differential equations, especially when $Q(x) \neq 0$.

Applying the Method to the Given Equation

Step 1: Write the Equation in Standard Form

As shown earlier,

$$\left[\frac{dy}{dx} - \frac{x}{1-x} y = 0 \right]$$

Here,

$$\begin{aligned} - \quad & P(x) = - \frac{x}{1-x}, \\ - \quad & Q(x) = 0. \end{aligned}$$

Step 2: Find the Integrating Factor $\mu(x)$

Compute:

$$\left[\mu(x) = e^{\int P(x) dx} = e^{\int - \frac{x}{1-x} dx} \right]$$

Focus on the integral:

$$\left[I = \int - \frac{x}{1-x} dx \right]$$

To evaluate I , rewrite the integrand:

$$\left[- \frac{x}{1-x} = - \frac{(1 - (1-x))}{1-x} = - \frac{1 - (1-x)}{1-x} = - \left(\frac{1}{1-x} - 1 \right) \right]$$

Simplify:

$$\begin{aligned} & - \left(\frac{1}{1-x} - 1 \right) = - \frac{1}{1-x} + 1 \end{aligned}$$

Thus,

$$I = \int \left(- \frac{1}{1-x} + 1 \right) dx = \int - \frac{1}{1-x} dx + \int 1 dx$$

Calculate each separately:

$$- \left(\int - \frac{1}{1-x} dx \right):$$

Let $(u = 1 - x)$, then $(du = -dx)$. So,

$$\begin{aligned} \int - \frac{1}{u} dx &= \int \frac{1}{u} du = \ln |u| + C = \ln |1-x| + C \end{aligned}$$

Remember, because of the substitution, the integral becomes:

$$\int - \frac{1}{1-x} dx = \ln |1-x| + C$$

$$- \left(\int 1 dx = x + C \right).$$

Putting it all together:

$$I = \ln |1 - x| + x + C$$

Now, the integrating factor:

$$\mu(x) = e^I = e^{\ln |1 - x| + x} = e^{\ln |1 - x|} \cdot e^x = |1 - x| \cdot e^x$$

For simplicity, consider $(1 - x > 0)$, i.e., $(x < 1)$, so:

$$\boxed{\mu(x) = (1 - x) e^x}$$

Step 3: Multiply the Equation by the Integrating Factor

Multiply both sides of the differential equation by $(\mu(x))$:

$$(1 - x) e^x \frac{dy}{dx} - \frac{x}{1 - x} (1 - x) e^x y = 0$$

Simplify:

$$(1 - x) e^x \frac{dy}{dx} - x e^x y = 0$$

Recognize that the left side is the derivative of $(\mu(x) y)$:

$$\frac{d}{dx} [(1-x)e^x y] = 0$$

Step 4: Integrate Both Sides

Since the derivative of $(1-x)e^x y$ with respect to x is zero, the expression must be a constant:

$$(1-x)e^x y = C$$

where C is an arbitrary constant of integration.

Step 5: Solve for y

Finally, express y :

$$y = \frac{C}{(1-x)e^x}$$

which is the general solution to the original differential equation.

Summary of the Solution

$$\boxed{\text{General solution: } y(x) = \frac{C}{(1-x)e^x}}$$

where C is an arbitrary constant.

Special Cases and Domain Considerations

- The solution is valid for $x \neq 1$ to avoid division by zero.
- The sign of $(1-x)$ affects the absolute value in the integrating factor, but since it appears in the denominator, the domain of the solution excludes $x = 1$.
- The exponential term (e^x) is always positive; thus, the sign and behavior of the solution are primarily influenced by the constant C and the domain constraints.

Critical Analysis of the Leibnitz Linear Method

This method simplifies the process of solving first-order linear differential equations, especially those with variable coefficients. Its systematic approach—finding an integrating factor and recognizing the resulting total derivative—provides a straightforward pathway to solutions.

Advantages include:

- Clear procedural steps.
- Applicability to various forms of linear equations.
- Facilitates analytical solutions where other methods may struggle.

Limitations:

- Requires the integral of $P(x)$ to be manageable.
- Not suitable for nonlinear or higher-order differential equations.
- Assumes $P(x)$ and $Q(x)$ are continuous over the domain.

Conclusion

The application of the Leibnitz linear equation method to the differential equation $(1 - x) Dy - xy = 0$ demonstrates the elegance and efficiency of integrating factor techniques. By transforming the original equation into a recognizable form, computing the integrating factor, and recognizing the total derivative, we obtain the general solution seamlessly.

This method remains a fundamental tool in the mathematician's toolkit when tackling first-order linear differential equations, underpinning many modeling efforts across scientific disciplines. Its systematic approach ensures clarity, precision, and adaptability, making it indispensable for both theoretical investigation and practical problem-solving.

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