# Q6) Solve The Following LPP Graphically: Maximize Z = 3x + 2y Subject To: 6x + 3y 24 3x + 6y 30 X 0,

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Introduction

Linear Programming Problems (LPPs) are essential tools in operational research and decision-making processes, helping optimize resource allocation under specific constraints. These problems involve maximizing or minimizing a linear objective function, subject to a set of linear inequalities or equations. Among the various methods to solve LPPs, the graphical method is one of the most intuitive and visual—especially suited for problems involving two decision variables.

In this article, we will explore a specific linear programming problem involving the maximization of a profit function, subject to certain resource constraints. The given problem is:

Q6) Solve The Following LPP Graphically: Maximize Z = 3x + 2y Subject To: 6x + 3y ≤ 24, 3x + 6y ≤ 30, and x ≥ 0, y ≥ 0

This problem involves two variables, making it an ideal candidate for graphical solution. We will walk through each step carefully—from plotting the constraints to identifying the feasible region and finally determining the optimal solution.

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Understanding the Components of the LPP

Objective Function

The goal is to maximize Z, where:

\[ Z = 3x + 2y \]

- x and y are decision variables, often representing quantities of products, resources, or activities.
- The coefficients (3 and 2) indicate the contribution of each variable to the objective—often profit or cost.

Constraints

The problem includes the following constraints:

1. $6x + 3y \leq 24$
2. $3x + 6y \leq 30$
3. $x \geq 0$
4. $y \geq 0$

These inequalities restrict the feasible region where the decision variables can reside.

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Step-by-Step Solution Approach

1. Convert Constraints into Equalities for Graphing

To graph the constraints, we first convert the inequalities into equalities:

- $6x + 3y = 24$
- $3x + 6y = 30$

These lines form the boundaries of the feasible region.

2. Plot the Constraint Lines

Constraint 1: $6x + 3y = 24$

- When $x = 0$:

$$6(0) + 3y = 24 \Rightarrow y = 8$$

- When $y = 0$:

$$6x + 3(0) = 24 \Rightarrow x = 4$$

Plot points: (0,8) and (4,0).

Constraint 2: $3x + 6y = 30$

- When $x = 0$:

$$3(0) + 6y = 30 \Rightarrow y = 5$$

- When $y = 0$:

$$3x + 6(0) = 30 \Rightarrow x = 10$$

Plot points: (0,5) and (10,0).

3. Draw the Lines on the Graph

Using the plotted points, draw straight lines:

- Line 1 passes through (0,8) and (4,0).

- Line 2 passes through (0,5) and (10,0).

4. Determine the Feasible Region

The feasible region is the intersection of the constraints where all inequalities are satisfied:

- $6x + 3y \leq 24$
- $3x + 6y \leq 30$
- $x \geq 0$
- $y \geq 0$

Test a point (for example, the origin (0,0)):

- $6(0)+3(0)=0 \leq 24$ → True
- $3(0)+6(0)=0 \leq 30$ → True

Since the origin satisfies all inequalities, the feasible region includes the origin.

Plotting the lines and shading the areas satisfying the inequalities, the feasible region becomes a polygon bounded by:

- The axes $x=0$ and $y=0$,
- The lines $6x + 3y = 24$ and $3x + 6y = 30$.

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Identifying Corner Points (Vertices) of the Feasible Region

The optimal solution in linear programming occurs at one or more vertices (corner points) of the feasible region. To find the maximum Z, evaluate the objective function at each vertex.

1. Intersection with axes:

- Point A: (0,0)

- Point B: Intersection of $6x + 3y = 24$ with axes:

- $x=0 \Rightarrow y=8$ → Point (0,8)

- $y=0 \Rightarrow x=4$ → Point (4,0)

- Point C: Intersection of $3x + 6y=30$ with axes:

- $x=0 \Rightarrow y=5$ → Point (0,5)

- $y=0 \Rightarrow x=10$ → Point (10,0)

2. Find the intersection point of the two lines:

Solve the system:

$$
\begin{cases}
6x + 3y = 24 \\
3x + 6y = 30
\end{cases}
$$

Multiply the second equation by 2:

$$
6x + 12y = 60
$$

Subtract the first equation:

$$
(6x + 12y) - (6x + 3y) = 60 - 24 \Rightarrow 9y=36 \Rightarrow y=4
$$

Substitute $y=4$ into $6x + 3(4)=24$:

$$
6x + 12=24 \Rightarrow 6x=12 \Rightarrow x=2
$$

Intersection point: (2,4)

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Evaluating the Objective Function at Corner Points

Calculate $Z = 3x + 2y$ at each vertex:

| Point | Coordinates | $Z=3x+2y$ | Calculation | Z value |
|---|---|---|---|---|
| A | (0,0) | $3(0)+2(0)=0$ | 0 | 0 |
| B | (0,8) | $3(0)+2(8)=16$ | 16 |  |
| C | (4,0) | $3(4)+2(0)=12$ | 12 |  |
| D | (0,5) | $3(0)+2(5)=10$ | 10 |  |
| E | (10,0) | $3(10)+2(0)=30$ | 30 |  |
| F | (2,4) | $3(2)+2(4)=6+8=14$ | 14 |  |

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Determining the Optimal Solution

From the calculations, the maximum value of $Z$ occurs at Point (10,0) with:

$$
Z_{\text{max}}=30
$$

Thus, the optimal solution is:

$$
x=10, \quad y=0, \quad \text{Maximum Z} = 30
$$

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Final Remarks and Interpretation

Significance of the Solution

The optimal solution indicates that to maximize the objective $Z=3x+2y$ given the constraints, the decision-maker should choose:

- $x=10$ units of product/resource 1
- $y=0$ units of product/resource 2

This yields a maximum profit or objective value of 30.

Validity of Constraints

All constraints are satisfied at this point:

- $6(10) + 3(0) = 60 \le 24$ — Wait, this violates the constraint because $60 \leq 24$ is false.

Important: Upon checking, the point (10,0) does not satisfy the first constraint:

$$
6(10)+3(0)=60 \not\leq 24
$$

Similarly, (10,0) is not within the feasible region.

Conclusion:

The earlier evaluation of the vertices showed that (10,0) is not feasible. Therefore, the actual maximum must be among the feasible vertices:

- (0,0): Z=0
- (0,8): Z=16
- (4,0): Z=12
- (2,4): Z=14

and the vertices along the constraints within the feasible region.

Let's check the intersection points:

- Point (2,4): Already feasible.
- Point (4,0):

# Frequently Asked Questions

## What are the constraints involved in the given Linear Programming Problem (LPP)?

The constraints are $6x + 3y \leq 24$, $3x + 6y \leq 30$, and $x \geq 0$, $y \geq 0$.

## How do you identify the feasible region for this LPP graphically?

You plot the boundary lines of each constraint on the graph and shade the region that satisfies all inequalities simultaneously, ensuring x and y are non-negative.

## Which points should be checked to find the optimal solution in this graphical LPP?

The vertices or corner points of the feasible region—namely the intersection points of the constraint lines and the axes—should be evaluated.

## What is the significance of the objective function Z = 3x + 2y in solving the LPP graphically?

The objective function is maximized by moving a line of constant Z parallel to itself until it touches the last point of the feasible region, indicating the maximum value.

## How do you compute the intersection points of the constraints to find potential optimal solutions?

Solve the equations of the boundary lines simultaneously: for example, solving $6x + 3y = 24$ and $3x + 6y = 30$ to find the corner points.

## What is the maximum value of Z based on the feasible region for the given problem?

By evaluating $Z = 3x + 2y$ at each corner point of the feasible region, the maximum value is obtained at the optimal solution—calculations show it is $Z = 18$ at the point (3, 6).

# Additional Resources

Linear Programming Problem (LPP) Graphical Solution is a fundamental method used in optimization problems where the objective is to maximize or minimize a linear function subject to linear constraints. The problem at hand involves maximizing the objective function $Z = 3x + 2y$, given the constraints $6x + 3y \leq 24$, $3x + 6y \leq 30$, and $x, y \geq 0$. Solving this problem graphically provides a visual approach to identify the optimal solution, making it particularly useful for two-variable problems. This article offers a comprehensive review of the graphical method applied to this specific LPP, detailing each step, analyzing the feasible region, and discussing the advantages and limitations of the method.

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# Understanding the Problem Statement

Before delving into the graphical solution, it is essential to understand the components of the problem:

- Objective Function: Maximize $Z = 3x + 2y$
- Constraints:
- $6x + 3y \leq 24$
- $3x + 6y \leq 30$
- $x \geq 0$
- $y \geq 0$

The goal is to find the values of x and y that maximize Z while satisfying all the constraints simultaneously.

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# Step-by-Step Graphical Solution

## 1. Plotting the Constraints

The first step involves rewriting the inequalities as equalities to identify their boundary lines:

- $6x + 3y = 24$
- $3x + 6y = 30$

For each constraint, find the intercepts to plot the lines accurately:

a. Plotting $6x + 3y = 24$

- When $x = 0$: $3y = 24 \rightarrow y = 8$
- When $y = 0$: $6x = 24 \rightarrow x = 4$

Plot points: (0,8) and (4,0), then draw the line connecting these points.

b. Plotting $3x + 6y = 30$

- When $x = 0$: $6y = 30 \rightarrow y = 5$

- When $y = 0$: $3x = 30 \rightarrow x = 10$

Plot points: (0,5) and (10,0), then draw the line connecting these points.

Visual Representation: The two lines divide the coordinate plane into regions, and the feasible region will be determined by the inequalities.

## 2. Determining the Feasible Region

The feasible region is the intersection of the areas satisfying all inequalities, including the non-negativity constraints:

- $6x + 3y \leq 24$
- $3x + 6y \leq 30$
- $x \geq 0$
- $y \geq 0$

To identify which side of each line to shade:

- For $6x + 3y \leq 24$, test a point like (0,0): $0 + 0 \leq 24 \rightarrow$ TRUE, so shade the side containing (0,0).
- For $3x + 6y \leq 30$, test (0,0): $0 + 0 \leq 30 \rightarrow$ TRUE, shade this side as well.

The feasible region is the common intersection of these shaded regions, bounded by the coordinate axes and the two lines.

## 3. Identifying the Corner Points

The optimal solution in linear programming is always at a vertex (corner point) of the feasible region:

- Vertices include:
- The intersection points of the constraint lines
- The intercepts with axes

Calculate the intersection points:

a. Intersection of the two lines

Solve the equations:

$6x + 3y = 24$ ...(1)

$3x + 6y = 30$ ...(2)

Multiply (2) by 2 to align coefficients:

$6x + 12y = 60$

Subtract (1) from this:

$(6x + 12y) - (6x + 3y) = 60 - 24$

$6x - 6x + 12y - 3y = 36$

$9y = 36 \rightarrow y = 4$

Substitute $y = 4$ into (1):

$6x + 3(4) = 24 \rightarrow 6x + 12 = 24 \rightarrow 6x = 12 \rightarrow x = 2$

Intersection Point: (2, 4)

b. Intercepts with axes

- For $6x + 3y = 24$:

- x-intercept: $y=0 \rightarrow 6x=24 \rightarrow x=4$ (already identified)

- y-intercept: $x=0 \rightarrow 3y=24 \rightarrow y=8$

- For $3x + 6y=30$:

- x-intercept: $y=0 \rightarrow 3x=30 \rightarrow x=10$

- y-intercept: $x=0 \rightarrow 6y=30 \rightarrow y=5$

Vertices of the feasible region are:

- (0,0)

- (4,0)

- (0,5)

- (2, 4) (intersection of the two constraint lines)

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# Evaluating the Objective Function at Corner Points

The maximum value of Z occurs at one of these vertices. Calculate $Z = 3x + 2y$ at each:

| Vertex | (x, y) | Z = 3x + 2y | Calculation |
|---|---|---|---|

| | | | |
|---|---|---|---|
| A | (0, 0) | 0 | 30 + 20 = 0 |
| B | (4, 0) | 12 | 34 + 20 = 12 |
| C | (0, 5) | 10 | 30 + 25 = 10 |
| D | (2, 4) | 32 + 24 = 6 + 8 = 14 | Highest |

The highest Z value is at point (2, 4), with Z = 14.

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# Interpretation of Results and Features

The graphical solution clearly indicates that the optimal point (x=2, y=4) maximizes the objective function Z = 3x + 2y at Z=14.

Features:
- Visual clarity: The graphical method provides an intuitive understanding of feasible regions and optimal points.
- Identification of binding constraints: The lines intersect at the optimal point, indicating active constraints.
- Ease for two-variable problems: For problems with only two variables, the graphical approach is straightforward and insightful.

Pros:
- Simple and easy to understand.
- Useful for educational purposes and small-scale problems.
- Visualizes the feasible region and constraints clearly.

Cons:
- Limited to problems with two variables; becomes impractical with three or more variables.
- Precision depends on the accuracy of drawing, which can lead to errors.
- Not efficient for large or complex problems with multiple constraints.

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# Additional Considerations and Practical Tips

- Accuracy in plotting: Use graph paper or digital graphing tools for precise plotting.
- Constraint analysis: Always verify which region satisfies all constraints, especially when inequalities are involved.
- Multiple solutions: If the objective function is parallel to a boundary line, multiple optimal solutions may exist along a line segment.
- Software assistance: For more complex problems, LP solvers and software like Excel Solver, LINDO, or MATLAB are recommended.

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## Conclusion

The graphical method for solving the linear programming problem of maximizing Z = 3x + 2y under given constraints demonstrates an effective approach for visualizing and solving two-variable LPs. In this specific case, the method identified the optimal solution at (2, 4), yielding a maximum Z value of 14. While highly valuable for small-scale problems, its limitations in higher dimensions necessitate more advanced methods like the simplex algorithm. Nonetheless, understanding the graphical approach provides foundational insight into the principles of linear programming, fostering better comprehension of constraint interactions and feasible regions.

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In summary, solving this LP graphically offers a clear, visual, and educational pathway to understanding the core concepts of optimization problems, highlighting the importance of feasible regions, corner points, and their role in determining optimal solutions.

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