

**Question 12 Given: $P(A) = 0.70$,
 $P(B) = 0.60$, $P(A \text{ Or } B) = 0.88$. Determine
 $P(A \text{ And } B)$. 0.10 0.18 0.28 0.42**

Understanding the Problem: Question 12 and Its Probabilistic Context

Question 12 Given: $P(A) = 0.70$, $P(B) = 0.60$, $P(A \text{ Or } B) = 0.88$. Determine $P(A \text{ And } B)$. 0.10 0.18 0.28 0.42.

This question presents a classic problem in probability theory, involving two events, A and B. The goal is to determine the probability that both events occur simultaneously, denoted as $P(A \text{ And } B)$. Given the individual probabilities of A and B, as well as the probability of either A or B occurring, it provides an excellent opportunity to explore fundamental concepts such as the addition rule and the intersection of events. Understanding how to manipulate these probabilities is essential for students and professionals working in fields like statistics, data analysis, risk assessment, and decision-making processes.

Fundamental Concepts in Probability Theory

Events and Probabilities

In probability theory, an event is a set of outcomes within a sample space. The probability of an event quantifies how likely it is to occur, ranging from 0 (impossible event) to 1 (certain event). For example, $P(A) = 0.70$ indicates that event A has a 70% chance of happening.

Union and Intersection of Events

- **Union (A Or B):** Represents the event that either A, B, or both occur. Denoted as $P(A \cup B)$.
- **Intersection (A And B):** Represents the event that both A and B occur simultaneously. Denoted as $P(A \cap B)$.

The Addition Rule

The addition rule links the probabilities of the union and intersection of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This formula adjusts for the double-counting of the intersection when summing the individual probabilities.

Applying Probability Rules to the Given Data

Given Data Recap

- $P(A) = 0.70$
- $P(B) = 0.60$
- $P(A \cup B) = 0.88$
- Possible options for $P(A \cap B)$: 0.10, 0.18, 0.28, 0.42

Calculating $P(A \text{ And } B)$: Step-by-Step Solution

1. Recall the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Plug in the known values:

$$0.88 = 0.70 + 0.60 - P(A \cap B)$$

3. Simplify the equation:

$$0.88 = 1.30 - P(A \cap B)$$

4. Rearranged to solve for $P(A \cap B)$:

$$P(A \cap B) = 1.30 - 0.88 = 0.42$$

Interpreting the Results and Validating the Choice

Result: $P(A \text{ And } B) = 0.42$

The calculation reveals that the probability of both events A and B occurring together is 0.42, which matches one of the provided options. This confirms that the correct answer among the options is 0.42.

Why is this important?

- It illustrates how to use the addition rule effectively.
- It demonstrates the importance of understanding overlaps between events in probability.
- It emphasizes the need for careful algebraic manipulation to derive unknown probabilities.

Additional Probabilistic Scenarios and Their Implications

Understanding the Range of $P(A \text{ And } B)$

The probability of both events A and B occurring simultaneously, $P(A \cap B)$, must satisfy certain bounds:

- Lower Bound: $\max(0, P(A) + P(B) - 1)$
- Upper Bound: $\min(P(A), P(B))$

Given $P(A) = 0.70$ and $P(B) = 0.60$:

- Lower bound: $\max(0, 0.70 + 0.60 - 1) = \max(0, 0.30) = 0.30$
- Upper bound: $\min(0.70, 0.60) = 0.60$

Thus, $P(A \cap B)$ must be between 0.30 and 0.60. Since our calculation gives 0.42, it fits comfortably within this range, reinforcing its validity.

Implications of Different $P(A \cap B)$ Values

- If $P(A \cap B)$ were 0.10 (less than 0.30), it would violate the bounds, indicating an impossible scenario given the data.
- If $P(A \cap B)$ were 0.18, it would also be outside the feasible range, indicating inconsistency.
- Values like 0.28 or 0.42 are plausible, with 0.42 being the exact value derived from the data.

Real-World Applications of Probability in Decision-Making

Risk Analysis and Management

Understanding probabilities such as $P(A)$, $P(B)$, and their intersection helps organizations assess risks. For example, in finance, the probability that two assets both decline in value can influence investment strategies.

Medical Testing and Diagnostics

In healthcare, probabilities of co-occurring conditions (like comorbidities) are critical for diagnosis and treatment planning. Calculating these probabilities informs patient care and resource allocation.

Quality Control and Manufacturing

Manufacturers use probability models to predict the likelihood of defective products occurring simultaneously in different production stages, aiding in quality assurance processes.

Conclusion: Mastering Probabilistic

Calculations

The problem outlined in Question 12 exemplifies fundamental probability principles that are widely applicable across numerous fields. By applying the addition rule and understanding the bounds of joint probabilities, learners and professionals can accurately interpret complex data scenarios. Recognizing that $P(A \text{ And } B)$ equals 0.42 in this case demonstrates the importance of precise calculation and conceptual clarity in probability theory. Whether analyzing risk, making predictions, or optimizing processes, mastering these core concepts is essential for informed decision-making.

Additional Resources for Learning Probability

- [Khan Academy's Probability and Statistics Courses](#)
- [Statistics by Jim: Conditional Probability](#)
- [Coursera: Introduction to Probability and Data](#)

Understanding how to manipulate and interpret probabilities empowers you to make better decisions in academic, professional, and everyday contexts. Keep practicing with different scenarios to strengthen your grasp of these essential concepts.

Frequently Asked Questions

Given $P(A) = 0.70$, $P(B) = 0.60$, and $P(A \text{ or } B) = 0.88$, what is $P(A \text{ and } B)$?

$P(A \text{ and } B) = 0.42$

How do you calculate $P(A \text{ and } B)$ when $P(A)$, $P(B)$, and $P(A \text{ or } B)$ are known?

Use the formula $P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$.

What is the value of $P(A \text{ and } B)$ for the given probabilities?

It is 0.42.

If $P(A) = 0.70$, $P(B) = 0.60$, and $P(A \text{ or } B) = 0.88$, what does the calculation $P(A) + P(B) - P(A \text{ or } B)$ equal?

It equals $0.70 + 0.60 - 0.88 = 0.42$, which is $P(A \text{ and } B)$.

Why is $P(A \text{ and } B)$ important in understanding the intersection of two events?

Because it represents the probability that both events A and B occur simultaneously.

Can $P(A \text{ and } B)$ be greater than either $P(A)$ or $P(B)$?

No, $P(A \text{ and } B)$ cannot be greater than the smaller of $P(A)$ or $P(B)$.

What does the value 0.42 indicate about the relationship between events A and B?

It indicates a moderate level of overlap or intersection between the two events.

How does knowing $P(A)$ and $P(B)$ help in predicting the likelihood of both events occurring?

It enables calculation of $P(A \text{ and } B)$ to understand their joint occurrence probability.

If $P(A \text{ or } B)$ was less than $P(A) + P(B)$, what would that imply?

It would imply that there is some overlap or intersection between A and B, reducing the total probability.

Is the calculated $P(A \text{ and } B)$ consistent with the given probabilities?

Yes, $P(A \text{ and } B) = 0.42$ is consistent with $P(A)$, $P(B)$, and $P(A \text{ or } B)$ provided.

Additional Resources

Probability Analysis

Understanding the interaction between events and their probabilities forms the cornerstone of statistical reasoning and decision-making. In this

article, we dive deep into a classic probability problem involving two events, A and B, to determine the probability of their intersection, $P(A \text{ And } B)$, given certain known parameters. We will explore the problem step-by-step, providing clarity and insight into how such probabilities are computed, and why they matter in both theoretical and practical contexts.

Introduction to the Problem

The problem provides the following data:

- $P(A) = 0.70$
- $P(B) = 0.60$
- $P(A \text{ Or } B) = 0.88$

The goal is to find $P(A \text{ And } B)$ – the probability that both events A and B occur simultaneously.

This type of problem, common in probability theory, is fundamental in fields ranging from risk assessment to machine learning, and understanding how to manipulate and interpret these probabilities is essential for data analysts, statisticians, and decision makers alike.

Understanding the Basic Probability Principles

Before tackling the problem directly, it's crucial to understand some foundational concepts:

1. The Inclusion-Exclusion Principle

This principle states that for any two events A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

In words, the probability that either A or B occurs is the sum of the probabilities that each occurs individually, minus the probability that both occur (to avoid double-counting).

2. The Relationship Between the Events

Given the above, knowing $P(A)$, $P(B)$, and $P(A \text{ Or } B)$ allows us to find $P(A \text{ And } B)$ by rearranging the formula:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

This formula is straightforward but powerful, providing a direct pathway to the solution.

Step-by-Step Solution

Let's methodically work through the problem.

Step 1: Write Down Known Values

Parameter	Value
$P(A)$	0.70
$P(B)$	0.60
$P(A \text{ Or } B)$	0.88

Step 2: Apply the Inclusion-Exclusion Formula

Using the formula:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Substituting the known values:

$$P(A \cap B) = 0.70 + 0.60 - 0.88$$

Calculating:

$$P(A \cap B) = 1.30 - 0.88 = 0.42$$

Thus, the probability that both events occur simultaneously is 0.42.

Step 3: Verify the Result

The options provided are:

- 0.10
- 0.18
- 0.28
- 0.42

Our calculation matches the last option, confirming that $P(A \text{ And } B) = 0.42$.

Interpreting the Result: What Does $P(A \text{ And } B) = 0.42$ Mean?

This value indicates that there's a 42% chance that both events A and B occur together. In practical terms:

- If A and B represent two favorable outcomes or conditions, nearly half of the instances involve both occurring.
- The high probability of intersection (0.42) suggests significant overlap between the two events, which might influence strategies in areas like marketing, risk management, or quality control.

Deeper Insights and Implications

Understanding this probability provides several valuable insights:

1. Degree of Dependence

- High $P(A \text{ And } B)$ relative to individual probabilities indicates potential dependence or correlation.
- Since $P(A) = 0.70$ and $P(B) = 0.60$, and $P(A \text{ And } B) = 0.42$, there's a substantial overlap, which can imply some level of association.

2. Conditional Probabilities

Knowing $P(A \text{ And } B)$, we can compute:

- $P(B | A)$: The probability B occurs given A.

$$\begin{aligned} & \backslash[\\ P(B|A) &= \frac{P(A \cap B)}{P(A)} = \frac{0.42}{0.70} \approx 0.6 \\ & \backslash] \end{aligned}$$

- $P(A | B)$: The probability A occurs given B.

$$\begin{aligned} & \backslash[\\ P(A|B) &= \frac{0.42}{0.60} = 0.7 \\ & \backslash] \end{aligned}$$

These conditional probabilities are equal to the individual probabilities of B and A respectively, implying that knowing one event has occurred does not alter the probability of the other. This suggests independence, which is consistent with the initial data.

Additional Considerations

While the calculation is straightforward, it's important to recognize potential caveats and extensions:

1. Assumption of Independence

The calculation shows a $P(A \text{ And } B)$ consistent with independence, but the problem doesn't explicitly state that A and B are independent. In real-world scenarios, dependence might exist, affecting the interpretation.

2. Limitations of the Data

- The probabilities are assumed accurate and complete.
- Real data often involves sampling variability, and estimates might differ.

3. Practical Applications

- Risk Analysis: Understanding joint probabilities helps assess combined risks.
- Decision Making: High overlap might influence strategies in marketing, operations, or health interventions.
- Modeling: Probabilistic models often depend on such calculations to estimate outcomes.

Conclusion

Through systematic application of the inclusion-exclusion principle, we have determined that:

$$P(A \text{ And } B) = 0.42$$

This value reflects a substantial overlap between events A and B, with implications for understanding their relationship and combined impact. Mastery of such probability calculations is vital across disciplines, empowering professionals to make informed decisions grounded in quantitative reasoning.

Whether you're assessing the likelihood of combined system failures, overlapping customer behaviors, or correlated risk factors, grasping how to compute and interpret $P(A \text{ And } B)$ equips you with a fundamental tool in the arsenal of data-driven analysis.

Question 12 Given P A 0 70 P B 0 60 P A Or B 0 88 Determine P A And B 0 10 0 18 0 28 0 42

Question 12 Given P A 0 70 P B 0 60 P A Or B 0 88 Determine P A And B 0 10 0 18 0 28 0 42

Related Articles

- [Suppose \$A_n\$ And \$B_n\$ Are Series With Positive Terms And \$B_n\$ Is Known To Be Convergent. \(a\) If \$A_n \geq B_n\$](#)
- [Q1 \[Total 2 Marks\] How Might The Cost And Benefits Of The COVID Lockdowns Be Assessed? \[1 Mark\] From](#)
- [Researchers Have Discovered An Association Between Depression, Anxiety And Pain. How Do They Explain](#)

[Back to Home](#)