

Rectangle Has A Length Of 24 In. And A Width Of 14 In. It Is Dilated And The Image Has A Width Of 21

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Understanding how geometric transformations like dilation affect the dimensions of shapes is fundamental in both mathematics and real-world applications such as engineering, architecture, and design. In this article, we explore the scenario where a rectangle with a given length and width undergoes dilation, resulting in a scaled image with a different width. We will analyze the original dimensions, the nature of dilation, determine the scaling factor, and discuss how this transformation impacts the rectangle's length and overall area. By the end of this guide, you'll have a comprehensive understanding of dilation effects on rectangles, supported by detailed explanations and illustrative examples.

Understanding the Original Rectangle Dimensions

Before delving into dilation, it is crucial to clarify the initial properties of the rectangle in question.

Original Dimensions

- **Length:** 24 inches
- **Width:** 14 inches

These measurements define the size and shape of the rectangle before any transformation occurs. The length and width are perpendicular sides, with the length being the longer side in this case.

Significance of Dimensions

- These measurements determine the rectangle's area and perimeter.
- They serve as the basis for calculating scaled dimensions after dilation.
- Understanding the original shape helps visualize the effects of geometric transformations.

What Is Dilation in Geometry?

Dilation is a transformation that produces an image (or scaled version) of a figure by enlarging or reducing it, based on a specific scale factor, while maintaining the shape's proportions.

Key Features of Dilation

- **Center of Dilation:** The fixed point about which the figure is expanded or contracted.
- **Scale Factor:** A number indicating how much the figure is enlarged or reduced.
- **Preservation of Shape:** The figure remains similar; only the size changes, not the shape.

In our scenario, we are primarily concerned with how the width of the rectangle changes due to dilation, and whether the length is affected proportionally.

Analyzing the Dilation: From Original to Image

The key information provided is that the original rectangle, with width 14 inches, is dilated, and its image has a width of 21 inches. This allows us to determine the scale factor of the dilation.

Calculating the Scale Factor

1. Original Width (W_0): 14 inches
2. Image Width (W_1): 21 inches
3. Scale Factor (k): $W_1 / W_0 = 21 / 14 = 1.5$

This indicates that the rectangle was enlarged by a factor of 1.5 in the width dimension.

Implication for the Length

Since dilation preserves the shape's proportions:

- The scale factor for the length should be the same as for the width, assuming uniform dilation.
- Therefore, the new length (L_1) can be calculated as:

$$L_1 = \text{Original Length} \times \text{Scale Factor} = 24 \text{ inches} \times 1.5 = 36 \text{ inches}$$

This shows that the length of the rectangle also increases proportionally during dilation.

Results of the Dilation Transformation

Based on the calculations:

New Dimensions of the Dilated Rectangle

- **Length:** 36 inches
- **Width:** 21 inches

The rectangle has been scaled uniformly, maintaining its aspect ratio.

Area Comparison

- Original area: 24 inches $\times$ 14 inches = 336 square inches
- Dilated area: 36 inches $\times$ 21 inches = 756 square inches

The area increases by the square of the scale factor:

$$\text{Area scale factor} = (\text{Scale factor})^2 = 1.5^2 = 2.25$$

Indeed, $336 \times 2.25 = 756$, confirming the consistency of the transformation.

Applications and Significance of Dilation in Real-World Contexts

Understanding dilation is essential in various contexts:

Architectural Design

- Scaling blueprints to real-world dimensions.
- Adjusting room layouts proportionally.

Engineering and Manufacturing

- Creating scaled models of machinery or structures.
- Ensuring proportionality in component dimensions.

Mathematics Education

- Teaching concepts of similarity and scale.
- Visualizing transformations and their effects on shapes.

Additional Considerations in Dilation

While the above scenario assumes a uniform dilation, it's important to consider other aspects:

Non-Uniform Dilation (Anisotropic Scaling)

- If the scale factors differ for length and width, the figure is scaled non-uniformly.
- For example, if only the width is scaled, the length might remain unchanged or be scaled differently.

Center of Dilation

- The position of the center of dilation affects how the shape enlarges.
- If the dilation is centered at the rectangle's centroid, the shape enlarges uniformly around its center.

Practical Limitations

- Physical constraints may limit the extent of dilation.
- In digital modeling, maintaining aspect ratios is essential for accuracy.

Summary and Key Takeaways

- The original rectangle measures 24 inches in length and 14 inches in width.
- Dilation with a scale factor of 1.5 increases the width to 21 inches and the length to 36 inches.
- The transformation preserves the shape's proportions, leading to a similar, larger rectangle.

- The area of the rectangle increases by a factor of 2.25, from 336 to 756 square inches.
- Understanding dilation helps in practical applications like scaling blueprints, models, and designing proportional layouts.

Conclusion

The transformation of a rectangle with a length of 24 inches and a width of 14 inches into a larger image with a width of 21 inches exemplifies the fundamental concept of dilation in geometry. By calculating the scale factor based on the change in width, we determined that the entire shape scales uniformly by a factor of 1.5, resulting in a new length of 36 inches and an increased area. Recognizing how dilation affects shape dimensions and area is vital in both academic contexts and practical applications, reinforcing the importance of similar triangles, scale models, and proportional reasoning in everyday problem-solving. Whether in construction, design, or education, mastering the concept of dilation empowers a deeper understanding of geometric transformations and their real-world significance.

Frequently Asked Questions

What is the original width and length of the rectangle?

The original rectangle has a length of 24 inches and a width of 14 inches.

What is the scale factor of the dilation based on the width?

The scale factor based on the width is 21 divided by 14, which equals 1.5.

What is the new length of the rectangle after dilation?

Multiplying the original length by the scale factor: $24 \text{ inches} \times 1.5 = 36 \text{ inches}$.

Does the height or width change proportionally during dilation?

Yes, both the height and width change proportionally based on the scale factor of 1.5.

What is the dilated rectangle's new dimensions?

The new width is 21 inches, and the new length is 36 inches.

How do you find the scale factor when only the width of the dilated image is given?

Divide the dilated width by the original width: $21 \div 14 = 1.5$.

Is the area of the rectangle increased after dilation? How to calculate it?

Yes, the area increases by the square of the scale factor. Original area: $24 \times 14 = 336$ sq. in. Dilated area: $36 \times 21 = 756$ sq. in.

What is the importance of understanding dilation in similar figures?

Understanding dilation helps in determining how dimensions, areas, and perimeters scale, which is essential in geometry and real-world applications like scale models and maps.

Additional Resources

Understanding the Impact of Dilation on a Rectangle with a Length of 24 Inches and a Width of 14 Inches

When working with geometric figures, especially rectangles, understanding how transformations such as dilation affect their dimensions is fundamental in both academic contexts and practical applications. In this article, we will analyze the scenario: Rectangle has a length of 24 inches and a width of 14 inches. It is dilated, and the image has a width of 21 inches. This case provides an excellent opportunity to explore the concept of dilation, scale factors, and their effects on rectangles' dimensions, area, and perimeter.

What Is Dilation in Geometry?

Before delving into the specifics of the given problem, it's essential to understand the concept of dilation in geometry.

Dilation is a transformation that enlarges or reduces a figure by a scale factor relative to a fixed point called the center of dilation. Key aspects include:

- Scale factor (k): Determines how much the figure is enlarged or reduced.
- Center of dilation: The fixed point about which the figure is dilated.
- Effects on dimensions: Lengths of sides are multiplied by the scale factor.
- Effects on area: Areas are multiplied by the square of the scale factor.

Initial Dimensions of the Rectangle

Let's establish the original dimensions:

- Length (L_1): 24 inches
- Width (W_1): 14 inches

From these, we can calculate the original area and perimeter:

- Original Area (A_1): $L_1 \times W_1 = 24 \times 14 = 336$ square inches
- Original Perimeter (P_1): $2(L_1 + W_1) = 2(24 + 14) = 2(38) = 76$ inches

The Dilation Process and Its Effect on the Rectangle

The problem states: It is dilated, and the image has a width of 21 inches. This indicates that:

- The original width (14 inches) is scaled to 21 inches after dilation.

Given this, we can find the scale factor for the dilation:

Scale factor (k):

$$k = \frac{\text{Dilated width}}{\text{Original width}} = \frac{21}{14} = 1.5$$

This means the rectangle was scaled uniformly by a factor of 1.5 about a specific point (assuming the dilation is centered appropriately).

Calculating the New Dimensions

Since dilation affects all linear dimensions proportionally:

- New length (L_2):

$$L_2 = L_1 \times k = 24 \times 1.5 = 36 \text{ inches}$$

- New width (W_2):

$$W_2 = W_1 \times k = 14 \times 1.5 = 21 \text{ inches}$$

The given information confirms the width after dilation, and the length naturally scales to 36 inches.

New Area and Perimeter Calculations

Understanding how dilation impacts area and perimeter:

- Area after dilation (A_2):

$$A_2 = A_1 \times k^2 = 336 \times (1.5)^2 = 336 \times 2.25 = 756 \text{ square inches}$$

- Perimeter after dilation (P_2):

$$P_2 = P_1 \times k = 76 \times 1.5 = 114 \text{ inches}$$

These calculations demonstrate the direct proportionality of linear measures and the quadratic relationship of area to the scale factor.

Summary of Changes Due to Dilation

Property	Original	Dilation ($k=1.5$)	Dilated Image
Length	24 in	$\times 1.5$	36 in
Width	14 in	$\times 1.5$	21 in
Area	336 sq in	$\times 2.25$	756 sq in
Perimeter	76 in	$\times 1.5$	114 in

Understanding the Significance of the Scale Factor

The scale factor, in this case, 1.5, plays a crucial role in determining the size of the dilated rectangle. It indicates a 50% increase in all linear dimensions.

Implications:

- The shape remains similar; only its size changes.
- The area increases by the square of the scale factor, which is 2.25 here.
- The perimeter increases linearly by the scale factor.

This consistent proportional change ensures that the dilated rectangle retains the same shape but is larger in size.

Practical Applications of Dilation

Dilation isn't just a theoretical concept; it has numerous real-world applications:

- Architecture and Design: Scaling blueprints or models to different sizes.
- Manufacturing: Adjusting component sizes proportionally.
- Art and Graphics: Enlarging or reducing images while maintaining proportions.
- Mathematical Modeling: Understanding how systems change in scale.

Understanding how to calculate and interpret the effects of dilation allows professionals and students to accurately predict how changes in one dimension affect the entire figure.

Additional Considerations: Center of Dilation and Its Effect

In this problem, the exact center of dilation isn't specified. Typically, when a figure is dilated, the scale factor applies relative to a specific point:

- If the dilation is centered at the rectangle's center, all points move along rays through the center.
- If centered at a vertex or corner, the shape shifts accordingly.

In most problems like this, unless otherwise specified, the dilation is assumed to be centered at a point that maintains the figure's proportionality relative to the original shape.

Summary and Final Thoughts

In conclusion, analyzing the dilation of a rectangle with an initial length of 24 inches and a width of 14 inches, resulting in an image with a width of 21 inches, reveals:

- The scale factor is 1.5.
- The length scales to 36 inches.
- The area increases from 336 to 756 square inches.
- The perimeter expands from 76 to 114 inches.

This process exemplifies fundamental principles of similar figures and proportional reasoning in geometry. Mastery of dilation concepts allows for accurate scaling in various mathematical, engineering, and artistic applications, highlighting the importance of understanding how transformations influence geometric figures.

Remember: Always identify the scale factor when analyzing dilation, and apply it consistently to all linear dimensions to understand the full impact on the figure's size and area.

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