# Select All The Correct Answers.Which Statements Are True About Function F?f(x)={1/3x, -x^2+2x-1,As X

**Select All The Correct Answers. Which Statements Are True About Function F? f(x) = { 1/3x, -x^2 + 2x - 1, As X }**

Understanding the properties and behaviors of functions is fundamental in mathematics. When dealing with a piecewise function like F, which is defined differently over various intervals or conditions, it becomes essential to analyze each part carefully. In this article, we will explore the function F, composed of two parts: a linear component $\frac{1}{3}x$ and a quadratic component $-x^2 + 2x - 1$. Our goal is to identify which statements about this function are true, based on their mathematical properties, graphs, and behaviors.

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## Breaking Down Function F: Definition and Components

Before delving into true or false statements, it's crucial to understand the structure of function F.

### Piecewise Definition of F

The notation suggests that F is a piecewise function:

- For some domain where $x$ satisfies certain conditions, $F(x) = \frac{1}{3}x$.
- For other parts of the domain, $F(x) = -x^2 + 2x - 1$.

Note: The exact domain segments are not explicitly provided in the initial statement. For comprehensive analysis, we will assume that the function is defined as:

$$
F(x) = \begin{cases}
\frac{1}{3}x, & \text{for } x \leq a \\
-x^2 + 2x - 1, & \text{for } x > a
\end{cases}
$$

where $a$ is some real number that determines the point of transition.

Alternatively, if the original notation indicates the function is a set of two possible expressions, perhaps indicating a union rather than a piecewise definition, then the analysis shifts to the properties of each component independently.

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# Analyzing the Components of Function F

Let's analyze each part of the function separately.

## Linear Part: $f(x) = \frac{1}{3}x$

- Type: Linear
- Slope: $\frac{1}{3}$
- Y-intercept: 0
- Behavior:
- The graph is a straight line passing through the origin.
- For every increase of 3 units in $x$, $f(x)$ increases by 1 unit.
- The function is increasing over its entire domain.
- Properties:
- Continuous everywhere
- Differentiable everywhere
- No maximum or minimum (unbounded in both directions)

## Quadratic Part: $f(x) = -x^2 + 2x - 1$

- Type: Quadratic (parabola)
- Leading coefficient: $-1$ (negative), which indicates the parabola opens downward.
- Vertex:
- To find the vertex, use the vertex formula $x = -\frac{b}{2a}$:
- $a = -1$, $b=2$
- $x_{vertex} = -\frac{2}{2 \times -1} = -\frac{2}{-2} = 1$
- Plug $x=1$ into the function:

$$
f(1) = - (1)^2 + 2(1) - 1 = -1 + 2 - 1 = 0
$$

- Vertex coordinates: $(1, 0)$

- Axis of symmetry: $x=1$

- Y-intercept: When $x=0$,

$$
f(0) = -0 + 0 - 1 = -1
$$

- Behavior:

- Opens downward
- Has a maximum point at the vertex $(1,0)$
- The parabola is symmetric about $x=1$

- Properties:
- Continuous everywhere
- Differentiable everywhere
- Has a global maximum at the vertex

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# Key Questions About Function F

To determine which statements about F are true, consider the following aspects:

- Domain and range
- Points of intersection between the components
- Continuity and differentiability
- Increasing/decreasing intervals
- Extrema (maxima and minima)
- Symmetry and other properties

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# Statements About Function F: Evaluation and Analysis

Let's evaluate common statements that might be posed about such a function.

## 1. Is Function F continuous everywhere?

Analysis:

- If F is defined as a union of two functions over different domains, then continuity depends on whether these functions connect smoothly at their boundary points.
- If F is piecewise with a transition point $a$, and both parts are continuous at $a$, then F is continuous everywhere.
- Since both components are continuous over their domains:

- $\frac{1}{3}x$ is continuous everywhere.
- $-x^2 + 2x - 1$ is continuous everywhere.

- The only potential discontinuity occurs at the boundary point $a$. If the definitions meet at $a$ with equal values, F is continuous everywhere.

Conclusion:
If the function is defined as such with matching values at the boundary, then F is continuous everywhere.
True Statement: Function F is continuous over its entire domain.

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## 2. Is Function F differentiable everywhere?

Analysis:

- Both components are differentiable everywhere; thus, differentiability depends on whether the functions connect smoothly at transition points.
- At boundary points where the definition switches, check if the derivatives match:

- Derivative of $\frac{1}{3}x$ is $\frac{1}{3}$.
- Derivative of $-x^2 + 2x - 1$ is $-2x + 2$.

- At the boundary $x = a$, for differentiability, the derivatives should be equal:

$$
\frac{1}{3} = -2a + 2
$$

Solving for $a$:

$$
-2a + 2 = \frac{1}{3} \Rightarrow -2a = \frac{1}{3} - 2 = \frac{1}{3} - \frac{6}{3} = -\frac{5}{3}
$$

$$
a = \frac{5}{6}
$$

- If the boundary point is at $a = \frac{5}{6}$ and the function values match at this point, then F is differentiable at $a$.

Conclusion:
If the boundary point is at $a = \frac{5}{6}$, and the function values match there, then F is differentiable everywhere.

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## 3. Does Function F have a maximum or minimum?

Analysis:

- The linear part $\frac{1}{3}x$ has no maximum or minimum (it's unbounded).
- The quadratic part $-x^2 + 2x - 1$ has a global maximum at $(1,0)$.

Implications:

- If F includes the quadratic part over some interval, then F has a maximum at $x=1$.
- If the linear part is defined over the entire domain, F may be unbounded in the positive or negative directions.

Conclusion:
The quadratic component has a maximum at $(1, 0)$. The overall maximum of F depends on the domain considered.

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## 4. Is the function increasing or decreasing over certain intervals?

Linear part $f(x) = \frac{1}{3}x$:

- Always increasing, since slope $> 0$.

Quadratic part $f(x) = -x^2 + 2x - 1$:

- Increasing on $(-\infty, 1)$
- Decreasing on $(1, \infty)$

Implication:

- The behavior depends on the interval and the specific piece of the function.

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# Summary of True Statements About Function F

Based on the detailed analysis, here are key truths:

- F is continuous everywhere if the piecewise parts meet smoothly at transition points.
- F is differentiable everywhere provided the derivatives match at the boundary point.
- The quadratic component has a maximum at $(1, 0)$.
- The linear component is always increasing across its domain.
- The overall behavior of F depends on the domain segment definitions and boundary conditions.

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# Common Misconceptions and Clarifications

- Misconception: The quadratic function always dominates the linear one.
- Clarification: The linear function could surpass the quadratic outside the parabola’s vertex depending on the domain.

- Misconception: The function has both a maximum and a minimum.
- Clarification: The quadratic part has a maximum; the linear part has no extrema.

- Misconception: The entire

# Frequently Asked Questions

## Which of the following statements correctly describe the function f(x) = {1/3x, -x^2 + 2x - 1} as a function of x?

f(x) is defined as a piecewise function with two different expressions depending on the value of x.

## Is f(x) = 1/3x linear?

Yes, because 1/3x is a linear function of x.

## Does the quadratic part of f(x), -x^2 + 2x - 1, represent a parabola opening upward or downward?

It opens downward since the coefficient of x^2 is negative.

## Can f(x) be considered a continuous function over its entire domain?

Not necessarily, because the piecewise parts may not connect continuously unless specified.

## Is the domain of f(x) all real numbers?

Assuming the piecewise functions are defined for all real x, then yes, the domain is all real numbers.

## Are both parts of the function, 1/3x and -x^2 + 2x - 1, polynomial functions?

No, only -x^2 + 2x - 1 is a polynomial; 1/3x is a linear function, which is a specific type of polynomial.

## Can the function f(x) have multiple outputs for a single input x?

No, because it is defined as a function with a unique output for each input, assuming the piecewise conditions are mutually exclusive or properly defined.

## Which statements are true about the behavior of f(x) as x approaches positive or negative infinity?

As x approaches infinity, the linear part 1/3x grows without bound, while the quadratic part $-x^2 + 2x - 1$ tends to negative infinity.

# Additional Resources

Comprehensive Analysis of the Function F: Identifying Correct Statements About $f(x)$

Understanding the properties and behaviors of functions is a foundational aspect of mathematical analysis, especially in calculus and algebra. In this detailed review, we delve into the function $f(x)$, given as:

$$
f(x) =
\begin{cases}
\frac{1}{3}x, & \text{for some domain segment} \\
-x^2 + 2x - 1, & \text{for another domain segment}
\end{cases}
$$

This function appears to be piecewise-defined, combining a linear component $\frac{1}{3}x$ with a quadratic component $-x^2 + 2x - 1$. Our goal is to analyze this function comprehensively, explore its properties, and determine which statements about $f(x)$ are true.

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Clarifying the Definition of $f(x)$

Before proceeding, it is essential to explicitly understand how $f(x)$ is defined. Typically, a piecewise function like this is specified over certain intervals. For example:

$$
f(x) =
\begin{cases}
\frac{1}{3}x, & x \in [a, b] \\
-x^2 + 2x - 1, & x \in (b, c]
\end{cases}

\]

However, as the problem statement provides only the two expressions without explicit domain restrictions, we will assume the standard interpretation:

- Domain of $f(x)$: The entire real line $\mathbb{R}$, with the function defined piecewise over certain intervals.
- Potential intervals:
- $f(x) = \frac{1}{3}x$ for $x \leq 1$ (or another interval)
- $f(x) = -x^2 + 2x - 1$ for $x > 1$ (or another interval)

In many typical problems, the quadratic form $-x^2 + 2x - 1$ is defined over $\mathbb{R}$, and the linear part is defined over some domain segment.

Assumption for clarity:

Let's assume the function is defined as:

$$
f(x) =
\begin{cases}
\frac{1}{3}x, & x \leq 1 \\
-x^2 + 2x - 1, & x > 1
\end{cases}
$$

This assumption allows us to analyze the function's behaviors over specific intervals, examine continuity, differentiability, extrema, and other properties.

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Analyzing the Function $f(x)$

1. Graphical Structure and Domain

Linear Part: $f(x) = \frac{1}{3}x$

- Domain: $x \leq 1$ (assuming as above)
- Range: All real numbers scaled by $\frac{1}{3}$, so $\left(-\infty, \frac{1}{3}\right]$

Quadratic Part: $f(x) = -x^2 + 2x - 1$

- Domain: $x > 1$
- Range: Since it is a downward-opening parabola (coefficient of $x^2$ is negative), the maximum point occurs at its vertex.

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2. Continuity and Junction Point at $x=1$

A crucial aspect is whether the function is continuous at the junction point $x=1$.

- At $x=1$:

$$
f(1) = \frac{1}{3} \times 1 = \frac{1}{3}
$$

- Check the quadratic side limit:

$$
\lim_{x \to 1^+} f(x) = -1^2 + 2 \times 1 - 1 = -1 + 2 - 1 = 0
$$

Since $\lim_{x \to 1^+} f(x) = 0 \neq f(1) = \frac{1}{3}$, the function is discontinuous at $x=1$ unless explicitly defined to be continuous.

Implication:

- If the function is intended to be continuous, then the quadratic branch must be defined such that:

$$
f(1) = \lim_{x \to 1^+} f(x) = 0
$$

- Otherwise, the function exhibits a jump discontinuity at $x=1$.

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3. Differentiability Analysis

Differentiability on Each Segment:

- Linear segment $f(x) = \frac{1}{3}x$:

$$
f'(x) = \frac{1}{3}
$$

- Quadratic segment $f(x) = -x^2 + 2x - 1$:

$$
f'(x) = -2x + 2
$$

At the junction point $x=1$:

$$
\text{Left derivative} (x \to 1^-) = \frac{1}{3}
$$
$$
\text{Right derivative} (x \to 1^+) = -2(1) + 2 = 0

\]

Since these are not equal ($\frac{1}{3} \neq 0$), the function is not differentiable at $x=1$ unless the derivatives match.

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4. Key Properties to Determine

Based on the above, the following are common statements to evaluate for truthfulness:

- Is the function continuous at certain points?
- Is the function differentiable at those points?
- What are the extrema (maxima and minima)?
- What is the overall behavior (increasing/decreasing)?
- Does the function have any asymptotes?
- What is the range of $f(x)$?

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Deep Dive into Each Aspect

# Continuity and Discontinuity

At $x=1$:
- As shown, the function values from each side do not match unless explicitly defined to do so, leading to a discontinuity.

Other points:

- For $x < 1$, the function is linear, which is continuous.
- For $x > 1$, the quadratic function is continuous everywhere.

Conclusion:

- Statement: "The function $f(x)$ is continuous at $x=1$."
- False, if the piecewise definitions are as assumed, because the limits do not match.

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# Differentiability and Critical Points

At $x=1$:

- Differentiability depends on the derivatives from both sides matching.
- Since $f'(1^-) = \frac{1}{3}$ and $f'(1^+) = 0$, the derivatives do not match, so:

Statement: "The function $f(x)$ is differentiable at $x=1$."
- False in this context.

Within each segment:

- For $x < 1$, $f(x) = \frac{1}{3}x$ is linear, with a constant derivative.
- For $x > 1$, $f(x) = -x^2 + 2x - 1$, which has a vertex at $x = 1$:

$$
f'(x) = 0 \Rightarrow -2x + 2 = 0 \Rightarrow x=1
$$

- At $x=1$, the quadratic reaches its maximum (since parabola opens downward):

$$
f(1) = -1 + 2 - 1 = 0
$$

Implication:

- The quadratic segment has a local maximum at $x=1$ with value 0.
- The linear segment at $x=1$ has value $\frac{1}{3}$.

Conclusion:

- At $x=1$, the quadratic segment reaches a maximum, which is higher than the linear segment's value.
- The global maximum depends on the entire domain but can be inferred.

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# Extrema (Maxima and Minima)

Quadratic Part:

- Since the parabola opens downward, the vertex at $x=1$ is a local maximum with:

$$
f(1) = 0
$$

- The quadratic tends to $-\infty$ as $x \to \pm \infty$.

Linear Part:

- The slope is positive ($\frac{1}{3}$), so $f(x) = \frac{1}{3}x$ increases without bound as $x \to \infty$.

Overall:

- The maximum value on the quadratic branch is at the vertex \(x=1\), where \(f(1) = 0\).

- The linear branch increases indefinitely, so no maximum there.

Critical points:

- For the quadratic: at \(x=1\).
- For the linear: no critical points (constant slope).

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## Range of \(f(x)\)

- Quadratic segment:

\[
f(x) = -

## Select All The Correct Answers Which Statements Are True About Function F F X 1 3x X 2 2x 1 As X

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