Select The Correct Answer From Each Drop-down Menu. Cos (7 Pi/12) + Cos (Pi / 12) = Cos (7 Pi/12) -

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Understanding trigonometric functions and their properties is essential for mastering advanced mathematics, especially topics involving angles, identities, and their applications. The given expression, involving cosine functions with specific angles, provides an excellent opportunity to explore angle addition and subtraction formulas, symmetry properties, and simplification techniques in trigonometry. This article aims to guide you step-by-step through the process of evaluating the expression:

Cos (7π/12) + Cos (π/12) = Cos (7π/12) – ___

by selecting the correct answer from each drop-down menu and understanding the underlying mathematical principles.

Introduction to Cosine Functions and Angle Measures

Before diving into the specific problem, it's important to review some fundamental concepts related to cosine functions and angle measures in radians.

Understanding Radians and Degrees

Angles can be measured in degrees or radians. Radians are often preferred in calculus and higher mathematics because of their natural relationship with arc lengths and the unit circle.

- 1 full circle = 360 degrees = 2π radians
- Common angles in radians:
- 0 = 0 radians
- π/6 = 30 degrees
- π/4 = 45 degrees
- π/3 = 60 degrees
- π/2 = 90 degrees
- π = 180 degrees
- 2π = 360 degrees

The angles in the problem involve π/12, which is 15 degrees, and 7π/12, which is 105 degrees.

Cosine Function Recap

The cosine function, $\cos(\theta)$, measures the x-coordinate of a point on the unit circle at an angle θ from the positive x-axis.

Key properties:
- Cosine is an even function: $\cos(-\theta) = \cos(\theta)$
- Range: $-1 \leq \cos(\theta) \leq 1$
- Periodic with period 2π: $\cos(\theta + 2\pi) = \cos(\theta)$

Understanding the Problem: The Expression Involving Cosines

The core of the problem involves evaluating the sum and difference of two cosine functions:

$\mathrm{Cos}\ (7\pi/12) + \mathrm{Cos}\ (\pi/12)$

and relating it to:

$\mathrm{Cos}\ (7\pi/12) -$ ___

The goal is to find the value that completes the expression, based on trigonometric identities.

Key Trigonometric Identities for Simplification

To evaluate and manipulate the expression, certain identities are instrumental:

Sum-to-Product and Difference-to-Product Formulas

- Sum of cosines:

$\cos A + \cos B = 2 \cos [(A + B)/2] \cdot \cos [(A - B)/2]$

- Difference of cosines:

$\cos A - \cos B = -2 \sin [(A + B)/2] \cdot \sin [(A - B)/2]$

Applying these identities simplifies complex expressions into products of sines and cosines, often easier to evaluate.

Step-by-Step Solution Process

Let’s now analyze the given expression and find the correct answer from each drop-down menu.

Step 1: Express the sum using the sum-to-product formula

Given:

$\cos(7\pi/12) + \cos(\pi/12)$

Applying the sum-to-product identity:

$\cos A + \cos B = 2 \cos [(A + B)/2] \cdot \cos [(A - B)/2]$

Calculate:

$A = 7\pi/12$

$B = \pi/12$

Sum:

$(A + B)/2 = (7\pi/12 + \pi/12)/2 = (8\pi/12)/2 = (2\pi/3)/2 = \pi/3$

Difference:

$(A - B)/2 = (7\pi/12 - \pi/12)/2 = (6\pi/12)/2 = (\pi/2)/2 = \pi/4$

Therefore,

$\cos(7\pi/12) + \cos(\pi/12) = 2 \cdot \cos(\pi/3) \cdot \cos(\pi/4)$

Recall:

$\cos(\pi/3) = 1/2$

$\cos(\pi/4) = \sqrt{2}/2$

Thus,

$\cos(7\pi/12) + \cos(\pi/12) = 2 \cdot (1/2) \cdot (\sqrt{2}/2) = (1) \cdot (\sqrt{2}/2) = \sqrt{2}/2$

Step 2: Express the difference using the difference-to-product formula

Now, examine:

$\cos(7\pi/12) - ?$

We need to find a term such that:

$\cos(7\pi/12) - \_\_\_ = ?$

Since the sum is $\sqrt{2}/2$, the difference likely involves similar angles or identities.

Alternatively, the difference of cosines formula:

$\cos A - \cos B = -2 \sin [(A + B)/2] \cdot \sin [(A - B)/2]$

Applying to $\cos(7\pi/12) - C$, where C is the unknown, the goal is to find C such that the difference simplifies in a meaningful way.

Step 3: Recognize the pattern or symmetry

Given that the sum of the two cosines equals $\sqrt{2}/2$ and the original expression involves subtracting a cosine, a common approach is to consider the cosine of complementary or supplementary angles, or to look for an angle C such that:

$\cos(7\pi/12) - \cos C = ?$

and the result relates to the previous sum.

Identifying the Correct Drop-down Choices

Based on the identities and calculations:

- The sum: $\cos(7\pi/12) + \cos(\pi/12) = \sqrt{2}/2$
- The difference likely involves a similar structure, possibly involving $\cos(\pi/12)$, $\cos(5\pi/12)$, or other related angles.

Given the pattern, the most probable choice for the missing cosine is $\cos(\pi/12)$, because it appears naturally in the context.

Final Expression and Answer Selection

The original statement is:

Cos (7π/12) + Cos (π/12) = Cos (7π/12) – ___

From the calculations:

- Sum = √2/2
- The difference probably equals the negative of the same value, or involves cos(π/12).

Thus, the complete statement is:

Cos (7π/12) + Cos (π/12) = Cos (7π/12) – Cos (π/12)

which simplifies to:

√2/2 = Cos (7π/12) – Cos (π/12)

Answer:

The missing term is Cos (π/12).

Summary of Key Points

- The sum-to-product formula simplifies the sum of cosines into a product involving cosines of average and half-difference angles.
- Evaluating cos(7π/12) and cos(π/12) directly is straightforward once the angles are recognized.
- The difference of cosines formula helps relate the difference to products of sine functions.
- The symmetry and complementary nature of angles in the unit circle often guide the selection of the correct cosine term in such problems.

Practical Applications of These Identities

Understanding how to manipulate sums and differences of cosine functions is crucial in various fields, including:

- Signal processing: Analyzing wave interference patterns.
- Physics: Studying oscillations and vibrations.
- Engineering: Designing filters and analyzing periodic signals.
- Mathematics: Simplifying complex trigonometric expressions in calculus.

Conclusion

By carefully applying the sum-to-product and difference-to-product identities, evaluating specific cosine values, and understanding the symmetry of angles, we find that:

Cos (7π/12) + Cos (π/12) = Cos (7π/12) – Cos (π/12)

The correct choice from the drop-down menu is Cos (π/12). This problem illustrates the importance of mastering fundamental identities and recognizing patterns in trigonometric expressions, which can simplify complex problems and deepen understanding of the unit circle.

Additional Tips for Mastering Trigonometry Problems

- Always convert angles into radians or degrees consistently.
- Memorize key cosine and sine values at common angles.
- Practice applying sum-to-product and difference-to-product formulas regularly.
- Use the unit circle to verify the signs and approximate values.
- Recognize symmetrical properties of the unit circle to simplify expressions.

By honing these skills, you'll be better equipped to handle a wide range of trigonometric problems confidently and accurately.

Frequently Asked Questions

What is the value of Cos(7π/12) + Cos(π/12)?

2 Cos(4π/12) Cos(3π/12), which simplifies to 2 Cos(π/3) Cos(π/4), resulting in √2/2.

How can you simplify Cos(7π/12) + Cos(π/12)?

Using the sum-to-product formula: 2 Cos((7π/12 + π/12)/2) Cos((7π/12 - π/12)/2), which simplifies to 2 Cos(π/2) Cos(π/3).

What is the value of Cos(7π/12) - Cos(π/12)?

-2 Sin((7π/12 + π/12)/2) Sin((7π/12 - π/12)/2), which simplifies to -2 Sin(π/2) Sin(π/3), resulting in -√3.

Which formula is used to combine Cos(7π/12) + Cos(π/12) and Cos(7π/12) - Cos(π/12)?

The sum-to-product formulas for cosine: Cos A + Cos B = 2 Cos((A+B)/2) Cos((A−B)/2) and Cos A - Cos B = -2 Sin((A+B)/2) Sin((A−B)/2).

If Cos(7π/12) + Cos(π/12) = 0, what is the value of Cos(7π/12) - Cos(π/12)?

It can be calculated using the difference formula; the exact value depends on the specific angles, but generally, the two are related through these identities.

What is the simplified form of Cos(7π/12) + Cos(π/12) using known cosine values?

It simplifies to √2/2, since Cos(π/3) = 1/2 and Cos(π/4) = √2/2, leading to the final value after applying the formulas.

Additional Resources

Select The Correct Answer From Each Drop-down Menu. Cos (7 Pi/12) + Cos (Pi / 12) = Cos (7 Pi/12) -

Understanding trigonometric identities and their applications is fundamental for students and professionals dealing with mathematics, physics, engineering, and related fields. One intriguing problem involves simplifying the sum of two cosine functions and expressing it as a difference, where the key lies in selecting the correct options from dropdown menus. This article explores this problem in depth, providing a comprehensive guide to solving such trigonometric expressions, emphasizing the importance of identities, and illustrating the process step-by-step in a clear, reader-friendly manner suitable for learners and enthusiasts alike.

Introduction to Cosine Sum and Difference Identities

Before delving into the specific problem, it is essential to grasp the foundational concepts involving cosine sum and difference formulas. These identities serve as powerful tools to simplify complex trigonometric expressions.

Cosine Sum Identity:

$$
\cos A + \cos B = 2 \cos \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right)
$$

Cosine Difference Identity:

$$
\cos A - \cos B = -2 \sin \left(\frac{A + B}{2}\right) \sin \left(\frac{A - B}{2}\right)
$$

These identities allow us to convert sums or differences of cosines into products, which often simplifies calculations and reveals underlying relationships.

Analyzing the Given Expression

The problem presents the expression:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \cos \left(\frac{7\pi}{12} \right) - \text{(something)}
$$

The task is to select the correct term from a dropdown menu to complete the equation, transforming the sum into a difference of cosines. The question challenges us to identify the value or expression that, when substituted, maintains the equality.

Step-by-Step Solution Approach

To resolve this, we need to:

1. Express the sum using the cosine sum identity.
2. Identify the corresponding cosine or sine expression that matches the pattern.
3. Use the identities to rewrite the sum as a difference, matching the structure of the target expression.
4. Determine the correct dropdown choice based on the derived expression.

Let's walk through these steps carefully.

Step 1: Applying the Cosine Sum Identity

Given the sum:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right)
$$

Using the identity:

$$
\cos A + \cos B = 2 \cos \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right)
$$

Set:

$$
A = \frac{7\pi}{12}, \quad B = \frac{\pi}{12}
$$

Calculate:

$$
\frac{A + B}{2} = \frac{\frac{7\pi}{12} + \frac{\pi}{12}}{2} = \frac{\frac{8\pi}{12}}{2} = \frac{\frac{2\pi}{3}}{2} = \frac{\pi}{3}
$$

$$
\frac{A - B}{2} = \frac{\frac{7\pi}{12} - \frac{\pi}{12}}{2} = \frac{\frac{6\pi}{12}}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}
$$

Thus:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = 2 \cos \left(\frac{\pi}{3} \right) \cos \left(\frac{\pi}{4} \right)
$$

Evaluate these cosines:

$$
\cos \left(\frac{\pi}{3} \right) = \frac{1}{2}
$$

$$
\cos \left(\frac{\pi}{4} \right) = \frac{\sqrt{2}}{2}
$$

So:

$$
\text{Sum} = 2 \times \frac{1}{2} \times \frac{\sqrt{2}}{2} = 1 \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}
$$

Result of the sum:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \frac{\sqrt{2}}{2}
$$

Step 2: Expressing the sum as a difference

Now, the original expression asks for an equivalent form:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \cos \left(\frac{7\pi}{12} \right) - \text{(something)}
$$

From the previous calculation, the sum equals $\frac{\sqrt{2}}{2}$. To write this as a difference involving $\cos \left(\frac{7\pi}{12} \right)$, consider the identity involving cosine difference.

Recall:

$$
\cos A - \cos B = -2 \sin \left(\frac{A + B}{2} \right) \sin \left(\frac{A - B}{2} \right)
$$

Alternatively, since the sum equals a known value, perhaps we should look for an expression that directly links the sum to a difference involving cosine.

Another approach is to find a value X such that:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \cos \left(\frac{7\pi}{12} \right) - X
$$

which implies:

$$
X = \cos \left(\frac{7\pi}{12} \right) - \left(\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) \right) = - \cos \left(\frac{\pi}{12} \right)
$$

Thus, the missing term is:

$$
X = - \cos \left(\frac{\pi}{12} \right)
$$

From the dropdown options, the correct answer should be $- \cos \left(\frac{\pi}{12} \right)$.

Final Expression and Interpretation

Putting it all together, the equation becomes:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \cos \left(\frac{7\pi}{12} \right) - \left(- \cos \left(\frac{\pi}{12} \right) \right)
$$

which simplifies to:

$$
\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right) = \cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right)
$$

This confirms the consistency of the choice.

Summary of the Key Takeaways

- The sum $\cos \left(\frac{7\pi}{12} \right) + \cos \left(\frac{\pi}{12} \right)$ simplifies to $\frac{\sqrt{2}}{2}$ using the sum identity.
- Recognizing the pattern allows rewriting the sum as a difference involving cosine.
- The correct dropdown answer is $- \cos \left(\frac{\pi}{12} \right)$, aligning with the algebraic manipulation.
- Understanding these identities enhances problem-solving skills in trigonometry, enabling the transformation of complex expressions into manageable forms.

Conclusion

Mastering the art of selecting the correct options from dropdown menus in trigonometric problems hinges on a solid grasp of identities and their applications. This exercise illustrates how a seemingly straightforward sum of cosines can be elegantly expressed as a difference, provided the identities are applied correctly. Whether you're a student preparing for exams or a professional dealing with mathematical modeling, this approach enhances analytical thinking and problem-solving efficiency. Remember to always verify your steps and understand the underlying identities — they are the keys to unlocking many mathematical challenges.

References & Further Reading

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- Larson, R., & Edwards, B. H. (2013). Precalculus with Limits. Ceng

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