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Show That The Function Defined By Df(x,y)=(y^2 xy) dxx^2 Dy Is Inexact. Test The Integrating Factor 1/xy^2

Understanding the nature of differential equations is fundamental to solving many problems in mathematics, physics, and engineering. Among these, inexact differential equations are particularly interesting because they require special methods, such as integrating factors, to solve. In this article, we will analyze the given differential function, determine whether it is inexact, and explore how to use the integrating factor \(\frac{1}{xy^2} \) to make it exact. This comprehensive guide aims to clarify these concepts with detailed explanations and step-by-step procedures.

Understanding Differential Equations and Exactness

Before diving into the specific problem, it is important to understand the basics of differential equations, especially the concept of exact equations.

What Is a Differential Equation?

A differential equation involves derivatives of a function and aims to find that function. For example, an equation involving \(x \), \(y \), and derivatives like \(\frac{\partial y}{\partial x} \) describes how \(y \) changes with \(x \).

Exact Differential Equations

A differential equation of the form:
\[
M(x,y) dx + N(x,y) dy = 0
\]
is called exact if there exists a function \(\Psi(x, y) \) such that:
\[
\frac{\partial \Psi}{\partial x} = M(x,y), \quad \frac{\partial \Psi}{\partial y} = N(x,y)
\]
and the differential form is integrable directly from \(\Psi \).

Condition for Exactness:
\[
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}
\]

If this condition holds, the differential equation is exact.

Analyzing the Given Function

The function provided is:
\[
Df(x,y) = (y^2 xy) \, dx^2 \, Dy
\]
However, this notation appears to be somewhat ambiguous or incorrectly formatted. Based on standard conventions, this seems to be a differential form involving functions of \(x \) and \(y \), likely intended as:

\[
M(x,y) dx + N(x,y) dy
\]
where:
- \(M(x,y) \) is associated with \(dx \),
- \(N(x,y) \) is associated with \(dy \).

Interpreting the given expression:

Suppose the intended differential form is:
\[
\boxed{
(y^2 xy) dx + \text{(something)} dy
}
\]
Given the notation, perhaps the function is:

\[
M(x,y) = y^2 xy
\]
which simplifies to:
\[
M(x,y) = x y^3
\]

Now, the expression mentions \(dxx^2 Dy \), which seems to be a typographical or formatting error. Possibly, it means the differential form:

\[
M(x,y) dx + N(x,y) dy
\]
with:
\[
M(x,y) = x y^3
\]
and
\[

$$
N(x,y) = \text{some function}
$$

Since the initial description is ambiguous, for the purpose of this analysis, let's assume the differential form is:

$$
\boxed{
x y^3 dx + y^2 xy dy
}
$$
which simplifies to:

$$
x y^3 dx + x y^3 dy
$$

In this case:

$$
M(x,y) = x y^3
$$
$$
N(x,y) = x y^3
$$

This symmetric form suggests the differential form:

$$
x y^3 dx + x y^3 dy
$$

Note: If this interpretation is not what was intended, please clarify. For now, we proceed with this assumption.

Checking Whether the Differential Equation Is Exact

To determine if the differential equation:

$$
M(x,y) dx + N(x,y) dy = 0
$$
is exact, compute:

$$
\frac{\partial M}{\partial y} \quad \text{and} \quad \frac{\partial N}{\partial x}
$$

Given:

$$
M(x,y) = x y^3
$$
$$
N(x,y) = x y^3
$$

Calculate:

$$
\frac{\partial M}{\partial y} = x \cdot 3 y^2 = 3 x y^2
$$
$$
\frac{\partial N}{\partial x} = y^3
$$

Since:

$$
\frac{\partial M}{\partial y} = 3 x y^2
$$
$$
\frac{\partial N}{\partial x} = y^3
$$
and these are not equal unless $3 x y^2 = y^3$, which is only true under specific conditions (e.g., $y=0$ or $y=3x$). Therefore, the differential form is not exact in general.

Summary:
- The differential form is not exact because $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.
- To solve this differential equation, we can attempt to find an integrating factor to make it exact.

Using Integrating Factors to Make the Equation Exact

Since the original differential form isn't exact, an integrating factor $\mu(x,y)$ can be used to multiply through and achieve exactness.

Choosing an Integrating Factor

Common choices for integrating factors depend on the form of M and N. Typical options include:

- $\mu(x)$, a function of x alone.
- $\mu(y)$, a function of y alone.
- $\mu(x,y)$, a function of both, but more complex.

In this case, we're testing the integrating factor:

$$
\mu(x,y) = \frac{1}{x y^2}
$$

Applying the Integrating Factor $\frac{1}{x y^2}$

Multiply the original differential form $M dx + N dy$ by $\mu(x,y)$:

$$
\tilde{M} = \frac{1}{x y^2} \cdot M = \frac{1}{x y^2} \cdot x y^3 = y
$$
$$
\tilde{N} = \frac{1}{x y^2} \cdot N = y
$$

New differential form:

$$
\tilde{M} dx + \tilde{N} dy = y dx + y dy
$$

Checking if the Modified Equation Is Exact

Calculate:

$$
\frac{\partial \tilde{M}}{\partial y} = \frac{\partial y}{\partial y} = 1
$$
$$
\frac{\partial \tilde{N}}{\partial x} = \frac{\partial y}{\partial x} = 0
$$

Since:

$$
\frac{\partial \tilde{M}}{\partial y} \neq \frac{\partial \tilde{N}}{\partial x}
$$

the differential form remains not exact after multiplication by the proposed integrating factor.

Re-evaluating the Integrating Factor Choice

Given that the attempt with $\frac{1}{x y^2}$ did not produce an exact form, perhaps this integrating factor is not suitable for this specific differential equation. Alternatively, the initial form of the differential equation might differ from our assumptions.

Conclusion: Is the Differential Equation Inexact?

Based on the calculations:

- The original differential form is not exact.
- Multiplying by $\frac{1}{x y^2}$ simplifies the form but does not make it exact.
- Additional analysis or different integrating factors are necessary to solve the differential equation.

Final Remarks and Recommendations

- When dealing with differential equations, always verify exactness by checking if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
- If the form isn't exact, explore different integrating factors, such as functions of x alone, y alone, or more complex functions.
- For the specific case of $\frac{1}{x y^2}$, if it doesn't produce an exact form, consider alternative strategies like substitution or more advanced integrating factors.

Summary of Key Steps:

1. Identify the differential form $M(x,y) dx + N(x,y) dy$.

2. Verify if the form is exact by comparing $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$.

3. If not exact, consider various integrating factors to make it exact.

4. Test candidate integrating factors by multiplying through and checking for

exactness.

5. Once an exact form is achieved, solve for the potential function \(\Psi(x,y) \) to

Frequently Asked Questions

What does it mean for a differential form to be inexact?

A differential form is inexact if it is not the differential of some scalar potential function, meaning it does not satisfy the exactness condition and cannot be integrated directly to find a potential.

Given the differential form Df(x,y) = (y^2 + xy) dx + x^2 Dy, how can we determine if it is exact?

We check if the mixed partial derivatives are equal, i.e., whether $\partial/\partial y$ of M equals $\partial/\partial x$ of N, where M and N are the coefficients of dx and dy respectively. If they are equal, the form is exact; otherwise, it is inexact.

How do we test whether the differential form Df(x,y) is inexact?

Calculate $\partial/\partial y$ of M (the coefficient of dx) and $\partial/\partial x$ of N (the coefficient of dy). If these partial derivatives are not equal, the form is inexact.

What is the integrating factor when testing the form with the factor 1/(x y^2)?

The integrating factor 1/(x y^2) is used to multiply the differential form to potentially make it exact, by transforming the coefficients accordingly.

How do you verify if 1/(x y^2) is an integrating factor for the differential form?

Multiply the original form by $1/(x y^2)$ and then check if the resulting form satisfies the exactness condition, i.e., if $\partial/\partial y$ of the new M equals $\partial/\partial x$ of the new N.

What are the steps to show that the form becomes exact after applying the integrating factor?

First, multiply M and N by $1/(x y^2)$, then compute their partial derivatives with respect to y and x respectively. If these derivatives are equal, the form is now exact.

Why is it important to find an integrating factor for inexact differential forms?

An integrating factor transforms an inexact form into an exact one, allowing us to find a potential function and solve the differential equation more easily.

Can you provide a brief summary of how to demonstrate that the given form is inexact and that the integrating factor 1/(xy^2) makes it exact?

First, verify that the form is inexact by comparing the mixed partial derivatives. Then, multiply the form by $1/(xy^2)$ and confirm that the new derivatives are equal, establishing that the form becomes exact with this integrating factor.

Additional Resources

Show That The Function Defined By $Df(x,y)=(y^2xy)$ dxx^2 Dy Is Inexact. Test The Integrating Factor $1/xy^2$

Introduction

In the realm of differential equations, especially those involving multiple variables, understanding whether a given differential form is exact or inexact is fundamental. This distinction influences the methods used to find solutions and the types of techniques applicable. Today, we explore a specific differential form:

$$Df(x, y) = (y^2 xy) \, dx + x^2 \, Dy$$

and demonstrate its inexactness. Additionally, we will examine the role of an integrating factor—in this case, $\frac{1}{xy^2}$—to transform an inexact differential form into an exact one, thereby simplifying the process of solving the associated differential equation.

Understanding Exact and Inexact Differential Forms

Before delving into the specific function, it's essential to clarify what makes a differential form exact or inexact.

- Exact Differential Form: A differential form $M(x, y) \, dx + N(x, y) \, dy$ is exact if there exists a function $F(x, y)$ such that:

$$dF = M(x, y) \, dx + N(x, y) \, dy$$

which implies:

$$\frac{\partial F}{\partial x} = M(x, y) \quad \text{and} \quad \frac{\partial F}{\partial y} = N(x, y)$$

- Inexact Differential Form: If no such F exists, the form is inexact. These forms often require an integrating factor to become exact, making them easier to integrate.

The key criterion to test whether a differential form is exact involves the equality of mixed partial derivatives:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

If this condition holds, the form is exact; if not, it is inexact.

Analyzing the Given Differential Form

Let's examine the differential form provided:

$$Df(x, y) = (y^2 xy) \, dx + x^2 \, Dy$$

First, clarify the notation and interpret the form properly.

- The term $y^2 xy$ simplifies to $y^2 \times x \times y = x y^3$.

- The second term appears as $x^2 \, Dy$, which suggests a differential involving y.

However, the notation is somewhat ambiguous; typically, a differential form involving two variables x and y is written as:

$$M(x, y) \, dx + N(x, y) \, dy$$

Given the context, it seems the original function is:

$$ M(x, y) = x y^3 $$
$$ N(x, y) = x^2 $$

To confirm this, assume the differential form:

$$ \omega = M(x, y) \, dx + N(x, y) \, dy = x y^3 \, dx + x^2 \, dy $$

Now, the goal is to determine whether ω is exact.

Testing for Exactness

Recall the criterion:

$$ \frac{\partial M}{\partial y} \stackrel{?}{=} \frac{\partial N}{\partial x} $$

Compute each partial derivative:

- $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} (x y^3) = x \times 3 y^2 = 3 x y^2$

- $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} (x^2) = 2 x$

Compare:

$$ 3 x y^2 \quad \text{and} \quad 2 x $$

These are equal only if:

$$ 3 x y^2 = 2 x $$

which simplifies to:

$$ 3 y^2 = 2 $$

or

$$ y^2 = \frac{2}{3} $$

But since this equality must hold for all y, and it's not true in general, the differential form is not exact.

Conclusion: The form $\omega = x y^3 \, dx + x^2 \, dy$ is inexact.

The Role of an Integrating Factor

When faced with an inexact differential form, a common strategy is to find an integrating factor: a function $\mu(x, y)$ such that multiplying the entire form by μ makes it exact.

In our case, the suggested integrating factor is:

$$\mu(x, y) = \frac{1}{x y^2}$$

The motivation for choosing this particular integrating factor often comes from analyzing the structure of the derivatives or from systematic methods, such as examining the form's coefficients.

Applying the Integrating Factor

Multiply the original differential form ω by $\mu(x, y)$:

$$\omega' = \frac{1}{x y^2} \left(x y^3 \, dx + x^2 \, dy \right)$$

Simplify each term:

- $\frac{1}{x y^2} \times x y^3 = y \, dx$

- $\frac{1}{x y^2} \times x^2 = \frac{x^2}{x y^2} = \frac{x}{y^2}$

Thus,

$$\omega' = y \, dx + \frac{x}{y^2} \, dy$$

Define:

- $M'(x, y) = y$

- $N'(x, y) = \frac{x}{y^2}$

Now, verify if ω' is exact:

Compute:

$$\frac{\partial M'}{\partial y} = \frac{\partial}{\partial y} (y) = 1$$

$$\frac{\partial N'}{\partial x} = \frac{\partial}{\partial x} \left(\frac{x}{y^2} \right) = \frac{1}{y^2}$$

Since:

$$
\frac{\partial M'}{\partial y} = 1 \quad \text{and} \quad \frac{\partial N'}{\partial x} = \frac{1}{y^2}
$$

these are equal only if $1 = \frac{1}{y^2}$, which is true only when:

$$
y^2 = 1
$$

This indicates $y = \pm 1$, which is not valid for a general y. Therefore, the form is still not exact after multiplying by the integrating factor.

Reassessing the Integrating Factor

Since the initial integrating factor $\frac{1}{x y^2}$ does not make the differential form exact in general, it hints that perhaps the integrating factor should depend solely on y or x, or perhaps a different function altogether.

In practice, for such forms, common integrating factors include:

- Functions of x alone: $\mu(x)$
- Functions of y alone: $\mu(y)$
- Product of functions of x and y

Given the above, the most systematic approach involves:

1. Checking if an integrating factor depending only on y can work.

2. Checking if an integrating factor depending only on x can work.

3. Using the known criteria:

$$ \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \quad \text{or} \quad \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} $$

to identify potential integrating factors.

Systematic Search for an Integrating Factor

Let's check whether an integrating factor depending solely on y exists.

Define:

\[
\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{2 x - 3 x y^2}{x y^3} = \frac{2x}{x y^3} - \frac{3 x y^2}{x y^3} = \frac{2}{y^3} - \frac{3 y^2}{y^3} = \frac{2}{y^3} - \frac{3}{y}
\]

Simplify:

\[
\frac{2}{y^3} - \frac{3}{y} = \frac{2 - 3 y^2}{y^3}
\]

Since this expression depends only on \(y \), it suggests that an integrating factor \(\mu(y) \) can be found with:

\[
\frac{d \mu}{dy} = \mu \times \left(\frac{2}{y^3} - \frac{3}{y} \right)
\]

which leads to:

\[
\frac{d \mu}{dy} / \mu = \frac{2 - 3

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