Sketch The Region Enclosed By X + Y2 30 And + Y = = 0. Decide Whether To Integrate With Respect To A

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Understanding the process of sketching regions enclosed by curves and determining the appropriate variable of integration is fundamental in calculus. In this article, we will explore how to analyze and sketch the region bounded by the curves represented by the equations $x + y^2 = 30$ and $y = 0$. We will also discuss how to decide whether to integrate with respect to x or y, depending on the problem's context and the region's shape. This comprehensive guide aims to help students and enthusiasts visualize complex regions and choose the most efficient integration method.

Understanding the Given Equations

Before sketching the region, it is essential to understand the nature of the given equations:

The Equation $x + y^2 = 30$

- This is a parabola opening to the left, shifted along the x-axis.
- For each value of y, the corresponding x is given by $x = 30 - y^2$.
- The parabola's vertex is at $(30, 0)$.

The Equation $y = 0$

- This is the x-axis.
- It acts as the lower boundary for the region.

Understanding these equations helps visualize the area enclosed by the curves and guides the integration approach.

Sketching the Region Enclosed by the Curves

Step 1: Find the Intersection Points

To sketch the region, determine where the curves intersect:

- Set $y = 0$ in the parabola:

$$
x + 0^2 = 30 \Rightarrow x = 30
$$

- So, the parabola intersects the x-axis at $(30, 0)$.

- The parabola opens to the left, and the region of interest is between $y = 0$ and the points where the parabola intersects the upper boundary.

- Since the problem involves only the parabola $x + y^2 = 30$ and $y=0$, the region's boundary on top is the parabola, and on the bottom, it is the x-axis.

Step 2: Determine the Range of y

- For the parabola, $x = 30 - y^2$.
- Because x must be real, $y^2 \leq 30$, so:

$$
- \sqrt{30} \leq y \leq \sqrt{30}
$$

- However, since the boundary is $y=0$ (the x-axis), and the parabola extends into the negative y-region, the region is bounded between $y=0$ and $y= \sqrt{30}$.
- The lower boundary is $y=0$, and the upper boundary is the parabola.

Step 3: Sketch the Curves

- Draw the x-axis.
- Plot the parabola $x = 30 - y^2$:

- For $y=0$, $x=30$.
- For $y=\pm \sqrt{30}$, $x=0$.

- Shade the region enclosed between $y=0$ and $y=\sqrt{30}$, bounded on the sides by the parabola.

Deciding Whether To Integrate With Respect To x or y

Choosing the variable of integration depends on the shape of the region and the simplicity of setting up the integral.

When to Integrate With Respect To x

- If the region is "vertically simple," meaning it can be expressed as the area between two functions of y.
- When the bounds for x are functions of y, as in the case of the parabola $x=30 - y^2$.

When to Integrate With Respect To y

- If the region is "horizontally simple," and the bounds for y are functions of x.
- When the curves are better expressed as functions of x.

Analyzing the Region for Integration Choice

- The parabola $x + y^2 = 30$ can be written as $y = \pm \sqrt{30 - x}$.
- The region is bounded between $y=0$ and $y=\sqrt{30 - x}$, for x between 0 and 30.

Thus, it is often more straightforward to integrate with respect to y because:

- The upper boundary is $y = \sqrt{30 - x}$.
- The lower boundary is $y=0$.
- The x-bounds are from 0 to 30.

In this case, integrating with respect to y involves setting limits for y from 0 to $\sqrt{30}$ and expressing x as a function of y:

$$
x \text{ varies from } 0 \text{ to } 30 - y^2
$$

Hence, the integration process is simplified by choosing y as the variable of integration.

Setting Up the Integral

Based on the analysis, the integral to find the area A enclosed by the curves is:

$$
A = \int_{y=0}^{\sqrt{30}} \left(\text{length of horizontal slice at } y \right) dy
$$

- The length of each horizontal slice at height y is from $x=0$ to $x=30 - y^2$.

Thus,

$$

$$
A = \int_{0}^{\sqrt{30}} \left(30 - y^2 - 0 \right) dy
$$

$$
A = \int_{0}^{\sqrt{30}} (30 - y^2) dy
$$

Calculating the Area:

$$
A = \left[30y - \frac{1}{3} y^3 \right]_0^{\sqrt{30}}
$$

$$
A = 30 \times \sqrt{30} - \frac{1}{3} (\sqrt{30})^3
$$

Since $(\sqrt{30})^3 = (\sqrt{30}) \times 30$,

$$
A = 30 \sqrt{30} - \frac{1}{3} \times (\sqrt{30} \times 30) = 30 \sqrt{30} - \frac{30 \sqrt{30}}{3}
$$

Simplify:

$$
A = 30 \sqrt{30} - 10 \sqrt{30} = (30 - 10) \sqrt{30} = 20 \sqrt{30}
$$

Final Area:

$$
\boxed{
A = 20 \sqrt{30}
}
$$

Summary of the Process

- Identify the curves and their intersections.
- Sketch the region to visualize the enclosed area.
- Determine the bounds for the variables.
- Decide the most straightforward variable for integration based on the shape and bounds.
- Set up and evaluate the integral accordingly.

Additional Tips for Similar Problems

- Always find intersection points first to determine bounds.
- Sketching is crucial for visual understanding.
- Express the region in the form that simplifies the integral setup.
- Consider the symmetry of the region for potential simplifications.
- Double-check the bounds after choosing the variable of integration.

Conclusion

Sketching the region enclosed by $x + y^2 = 30$ and $y=0$ involves understanding the shape of the parabola and the x-axis, identifying intersection points, and visualizing the bounded area. Deciding whether to integrate with respect to x or y depends on the simplicity of the bounds and the ease of expressing the region. In this case, integrating with respect to y provides a straightforward approach, leading to an elegant calculation of the enclosed area, which is $20 \sqrt{30}$.

Mastering these techniques enhances your ability to solve complex calculus problems involving regions and multiple curves, paving the way for accurate area computations and more advanced applications in mathematics and engineering.

Frequently Asked Questions

What is the main goal when sketching the region enclosed by the curves $x + y^2 = 30$ and $y = 0$?

The main goal is to graph both curves to identify the area where they overlap, which is the region enclosed between the parabola $x + y^2 = 30$ and the x-axis $y=0$, then determine the limits of integration for setting up the integral.

How do you find the points of intersection between the curves $x + y^2 = 30$ and $y = 0$?

Substitute $y=0$ into $x + y^2=30$, resulting in $x=30$. The intersection point is at $(30, 0)$. This helps define the bounds of the enclosed region.

What does the inequality $x + y^2 \leq 30$ represent in this context?

It represents the region on or inside the parabola, which is the area enclosed between the parabola and the line $y=0$.

Why is it important to decide whether to integrate with

respect to x or y when setting up the integral?

Choosing the variable of integration simplifies the limits of integration and the integrand; the choice depends on the shape and orientation of the region. For regions bounded by functions of x, integrating with respect to x is often easier, and vice versa.

How do you determine whether to integrate with respect to x or y for the given region?

Examine the region's boundaries: since y is bounded between 0 and the parabola, and x varies from the parabola's left to right, integrating with respect to y is typically more straightforward, integrating vertically.

What are the limits of integration if integrating with respect to y?

For y between 0 and the upper boundary (the parabola), y varies from 0 to $\sqrt{30 - x}$. For x, it varies from 0 to 30, considering the intersection point.

How do you set up the integral to find the area of the region?

If integrating with respect to y, set up the integral as: Area = $\int$ (from x=0 to 30) [integral of dy from y=0 to $\sqrt{30 - x}$] dx. Alternatively, if integrating with respect to y, express x in terms of y.

What is the significance of choosing the correct variable of integration in calculating the area?

Choosing the correct variable simplifies the limits and the integrand, making the calculation more straightforward and reducing potential errors.

Can the region be integrated with respect to x? If so, how would the limits be determined?

Yes, by expressing y in terms of x from the parabola: $y = \pm\sqrt{30 - x}$. The limits for x are from 0 to 30, and for y, from 0 up to $\sqrt{30 - x}$. The region can be integrated horizontally accordingly.

What is the final step after setting up the integral for the region?

Evaluate the integral to find the enclosed area, ensuring the limits and integrand are correctly set based on the chosen variable of integration.

Additional Resources

Sketch The Region Enclosed By X + Y^2 = 30 And + Y = 0: A Comprehensive Guide to Visualizing and Integrating Over the Region

Understanding how to sketch the region enclosed by $x + y^2 = 30$ and $y = 0$ is fundamental in calculus, especially when setting up integrals for area, volume, or other applications. This process involves analyzing the graphs of the given equations, determining their intersection points, and deciding whether to integrate with respect to x or y. This guide provides an in-depth walkthrough of these steps, offering clarity for students and professionals tackling similar problems.

Introduction

When dealing with regions in the xy-plane defined by curves, visualizing the area is a crucial step before performing any integrations. The equations $x + y^2 = 30$ and $y = 0$ (the x-axis) form a closed region that can be complex to interpret at first glance. Accurately sketching this region requires understanding these curves' shapes, their intersections, and the bounds of the region.

This tutorial will cover:

- The nature of the given curves
- Finding the points of intersection
- Deciding whether to integrate with respect to x or y
- Sketching the region
- Setting up the integral

Understanding the Curves

The Equation $x + y^2 = 30$

This is a parabola opening towards the x-axis because, for different y values, x varies as:

$$ x = 30 - y^2 $$

- Shape: A sideways parabola opening leftward, since x decreases as y increases in magnitude.
- Vertex: At the maximum x-value when y = 0, which is at $x = 30$.

The Equation y = 0

This is the x-axis, serving as the lower boundary of the region.

Finding the Intersection Points

To determine the enclosed region, identify where the parabola and the x-axis intersect.

Solving for Intersections

Set $y = 0$ into the parabola:

$$x + (0)^2 = 30 \Rightarrow x = 30$$

- Point of intersection: $(30, 0)$

Since y = 0 is the x-axis, the parabola intersects it at $(30, 0)$. Because the parabola opens to the left and the vertex is at $(30, 0)$, the region is bounded between y-values from 0 up to the maximum y-value where the parabola intersects itself or is bounded.

However, the parabola does not intersect the x-axis at any other point because:

$$y^2 = 30 - x$$

and for $x \leq 30$, $y^2 \geq 0$. The maximum y occurs when x is at the vertex $x=30$, which corresponds to $y=0$.

But for the enclosed region, we should consider points where the parabola dips below the x-axis or extends to other bounds.

Key insight: Since the parabola is opening to the left and the minimal x-value is unbounded in the negative direction, but for the region to be enclosed with y=0, the region must be between the parabola and the x-axis, from the vertex at $(30, 0)$ down to the points where the parabola intersects y=0 again.

Given that, the parabola only touches y=0 at $(30, 0)$, so the region is essentially a "slice" from y=0 up to the maximum y-value of the parabola.

Deciding Whether To Integrate With Respect To x or y

Consideration of the region's shape:

- The parabola is expressed as $x = 30 - y^2$, which suggests that integrating with respect to y might be more straightforward if the limits are functions of y.
- The region is bounded below by y=0 and above by the parabola.

Choosing the variable of integration:

- Integrate with respect to y: This is often easier when y is bounded between constant limits, and x varies between functions of y.
- Integrate with respect to x: This is suitable if the bounds are more naturally expressed as functions of x.

Given the shape, integrating with respect to y is preferable because:

- The parabola provides x as a function of y
- The region is "stacked" vertically between y=0 and the maximum y-value

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Sketching the Region

Step 1: Draw the coordinate axes.

Step 2: Plot the parabola $x = 30 - y^2$

- At $y=0$, $x=30$, so plot $(30, 0)$.
- At $y= \pm \sqrt{30}$, $x=0$. These points are $(0, \pm \sqrt{30})$.

Step 3: Plot the x-axis $y=0$

- The parabola intersects the x-axis at $(30, 0)$.

Step 4: Shade the region

- The region enclosed is between the parabola $x = 30 - y^2$ and the line $y=0$, from $y=0$ up to $y= \sqrt{30}$.

Visual summary:

- The region is a "curved wedge" starting at $(30, 0)$, extending upward to $y = \sqrt{30}$, bounded on the left by $x=0$ (since at $y=\pm \sqrt{30}$, $x=0$), and on the right by the parabola.

Setting Up the Integral

Vertical slices (integrating with respect to y):

- For each y between 0 and $\sqrt{30}$:

- The x-boundaries are:

- Left boundary: $x=0$ (the y-axis)
- Right boundary: $x = 30 - y^2$ (parabola)

- The area element dA:

$$dA = (\text{right boundary} - \text{left boundary}) \, dy = (30 - y^2 - 0) \, dy$$

Total area:

$$\text{Area} = \int_0^{\sqrt{30}} [30 - y^2] \, dy$$

Calculating the integral step-by-step:

1. Integrate $30 - y^2$ with respect to y:

$$
\int (30 - y^2) \, dy = 30y - \frac{1}{3} y^3 + C
$$

2. Apply limits from 0 to $\sqrt{30}$:

$$
\text{Area} = \left[30 y - \frac{1}{3} y^3 \right]_0^{\sqrt{30}} = \left(30 \sqrt{30} - \frac{1}{3} (\sqrt{30})^3 \right) - 0
$$

3. Simplify:

- $(\sqrt{30})^3 = (\sqrt{30}) \times (\sqrt{30})^2 = \sqrt{30} \times 30 = 30 \sqrt{30}$

- Therefore:

$$
\text{Area} = 30 \sqrt{30} - \frac{1}{3} \times 30 \sqrt{30} = 30 \sqrt{30} - 10 \sqrt{30} = (30 - 10) \sqrt{30} = 20 \sqrt{30}
$$

Summary of the Approach

- Identify the curves: parabola $x + y^2 = 30$ and line $y=0$
- Find intersection points: at $(30, 0)$ and $(0, \pm \sqrt{30})$
- Determine the region: bounded between y=0 and $y = \sqrt{30}$, x from 0 to the parabola
- Decide variable of integration: integrate with respect to y
- Set limits: y from 0 to $\sqrt{30}$
- Express integrand: width of each vertical slice $(30 - y^2)$
- Compute the integral: yields the total enclosed area

Additional Tips and Variations

When to choose different variables:

- If the region is better described horizontally, consider integrating with respect to x.
- For more complex regions, sketching and analyzing the graphs is crucial to avoid errors.

Generalization:

This method applies broadly to regions bounded by curves where one variable can be expressed explicitly as a function of the other, facilitating straightforward setup of integrals.

Final Remarks

Sketching the region enclosed by \(x + y^2 = 30 \) and \(y=0 \) provides a visual foundation for setting up integrals in calculus. Recognizing the shape of the parabola, the intersection points, and the bounds allows for an accurate and efficient calculation of areas, volumes, or other quantities of interest. Always verify the bounds and choose the variable of integration that simplifies the process, leading to clear and manageable integrals.

By mastering these

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