Solve The Equation Cos(x) Cot(x) + Squareroot3 Cos(x) = 0, Finding All Solutions In The Interval [0,

Solve The Equation Cos(x) Cot(x) + Squareroot3 Cos(x) = 0, Finding All Solutions In The Interval [0, is a common trigonometric problem that involves understanding the relationships between cosine, cotangent, and their properties within a specified interval. This type of equation appears frequently in advanced mathematics, physics, and engineering, especially when analyzing periodic phenomena or wave functions. In this comprehensive guide, we will explore the step-by-step process to solve this equation, analyze all possible solutions, and understand the underlying principles that lead to these solutions. The goal is to equip you with a deep understanding of solving trigonometric equations, emphasizing methods that are both effective and optimized for search engine visibility.

Understanding the Equation: Cos(x) Cot(x) + √3 Cos(x) = 0

Before diving into the solution process, it's essential to analyze and interpret the given equation:

$$
\cos(x) \cot(x) + \sqrt{3} \cos(x) = 0
$$

This equation involves two primary functions:

- Cosine function ($\cos(x)$): A fundamental trigonometric function that oscillates between -1 and 1.
- Cotangent function ($\cot(x)$): Defined as $\cot(x) = \frac{\cos(x)}{\sin(x)}$, where $\sin(x)$ must not be zero.

The presence of $\cot(x)$ indicates potential restrictions on the domain, especially at points where $\sin(x) = 0$.

Step-by-Step Solution Approach

To solve the equation effectively, we follow a systematic approach:

1. Rewrite the Equation in Terms of Basic Trigonometric Functions

Start by expressing $\cot(x)$ explicitly:

$$
\cot(x) = \frac{\cos(x)}{\sin(x)}
$$

Substitute this into the original equation:

$$
\cos(x) \times \frac{\cos(x)}{\sin(x)} + \sqrt{3} \cos(x) = 0
$$

Simplify numerator:

$$
\frac{\cos^2(x)}{\sin(x)} + \sqrt{3} \cos(x) = 0
$$

---

2. Factor Out Common Terms

Notice that both terms contain $\cos(x)$:

$$
\cos(x) \left(\frac{\cos(x)}{\sin(x)} + \sqrt{3} \right) = 0
$$

This gives us two cases to consider:

- Case 1: $\cos(x) = 0$
- Case 2: $\frac{\cos(x)}{\sin(x)} + \sqrt{3} = 0$

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3. Solve Each Case Separately

Case 1: $\cos(x) = 0$

The cosine function equals zero at:

$$
x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
$$

Within the interval $[0, 2\pi)$, the solutions are:

$$
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
$$

Note: For the interval $[0, \text{some upper limit}]$, adjust accordingly.

Case 2: $\frac{\cos(x)}{\sin(x)} + \sqrt{3} = 0$

Rearranged:

$$
\frac{\cos(x)}{\sin(x)} = -\sqrt{3}
$$

Recall that $\frac{\cos(x)}{\sin(x)} = \cot(x)$, hence:

$$
\cot(x) = -\sqrt{3}
$$

---

4. Solve $\cot(x) = -\sqrt{3}$

The cotangent function:

$$
\cot(x) = \frac{\cos(x)}{\sin(x)}
$$

has standard values at specific angles:

$\cot(x)$ value	Corresponding basic angle x
$\sqrt{3}$	$x = \frac{\pi}{6}$
$-\sqrt{3}$	$x = \frac{5\pi}{6}$ or $x= \frac{11\pi}{6}$ (since cotangent is negative in the second and fourth quadrants)

For $\cot(x) = -\sqrt{3}$, the solutions are:

$$
x = \frac{5\pi}{6} + k\pi, \quad k \in \mathbb{Z}
$$

Within the interval $[0, 2\pi)$, solutions are:

$$
x = \frac{5\pi}{6}, \quad x = \frac{11\pi}{6}
$$

Summary of All Solutions in the Interval [0, 2π]

Combining solutions from both cases, the solutions are:

- From $\cos(x) = 0$:

$$
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
$$

- From $\cot(x) = -\sqrt{3}$:

$$
x = \frac{5\pi}{6}, \quad x = \frac{11\pi}{6}
$$

Verify Solutions and Domain Restrictions

It's crucial to verify each solution within the original equation, especially considering the domain restrictions:

- The original expression involves $\frac{\cos^2(x)}{\sin(x)}$, which is undefined at points where $\sin(x) = 0$.
- Solutions where $\sin(x) = 0$ ($x = 0, \pi, 2\pi, ...$) should be checked to avoid division by zero.

Verify solutions:

- $x = \frac{\pi}{2}$:

$$
\cos\left(\frac{\pi}{2}\right) = 0 \Rightarrow \text{Equation simplifies to } 0 + \sqrt{3} \times 0 = 0
$$

Valid.

- $x = \frac{3\pi}{2}$:

$$
\cos\left(\frac{3\pi}{2}\right) = 0 \Rightarrow 0 + \sqrt{3} \times 0 = 0

\]

Valid.

- \(x = \frac{5\pi}{6}\):

\[
\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}, \quad
\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}
\]

\[
\cot\left(\frac{5\pi}{6}\right) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}
\]

Equation:

\[
\cos(x) \cot(x) + \sqrt{3} \cos(x) = \left(-\frac{\sqrt{3}}{2}\right) \times (-\sqrt{3}) + \sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right)
\]

Calculate:

\[
\left(-\frac{\sqrt{3}}{2}\right) \times (-\sqrt{3}) = \frac{\sqrt{3}}{2}
\times \sqrt{3} = \frac{\sqrt{3} \times \sqrt{3}}{2} = \frac{3}{2}
\]

And:

\[
\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) = - \frac{3}{2}
\]

Sum:

\[
\frac{3}{2} - \frac{3}{2} = 0
\]

Valid solution.

- \(x = \frac{11\pi}{6}\):

\[
\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}, \quad
\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}
\]

$$
\cot\left(\frac{11\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}
$$

Equation:

$$
\frac{\sqrt{3}}{2} \times (-\sqrt{3}) + \sqrt{3} \times \frac{\sqrt{3}}{2} = -\frac{3}{2} + \frac{3}{2} = 0
$$

Valid solution.

Final Set of Solutions in the Interval [0, 2π]

```
\[
\boxed{
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}, \quad x = \frac{5\pi}{6
```

Frequently Asked Questions

What is the given equation to solve?

The equation is cos(x) cot(x) + √3 cos(x) = 0.

How can we simplify the equation cos(x) cot(x) + √3 cos(x) = 0?

Factor out cos(x): cos(x)(cot(x) + √3) = 0.

What are the two cases to consider after factoring?

Either cos(x) = 0 or cot(x) + √3 = 0.

What is the solution when cos(x) = 0 in [0, ??]?

In the interval [0, ??], cos(x) = 0 at x = π/2 + nπ, where n is an integer within the interval bounds.

How do we solve cot(x) + √3 = 0?

Rewrite as cot(x) = -√3, then find x such that cot(x) = -√3.

What are the solutions for $\cot(x) = -\sqrt{3}$?

$\cot(x) = -\sqrt{3}$ at $x = 5\pi/6 + n\pi$, for integers n.

What is the general solution for x when $\cot(x) = -\sqrt{3}$ in [0, ??]?

$x = 5\pi/6 + n\pi$, where n is an integer such that x falls within the interval [0, ??].

How do we find all solutions in the interval $[0, 2\pi]$?

Include solutions where $x = \pi/2$ (from $\cos(x)=0$) and $x = 5\pi/6$, $11\pi/6$ (from cotangent solution), ensuring all are within $[0, 2\pi]$.

What are the final solutions in $[0, 2\pi]$ for the equation?

$x = \pi/2$, $5\pi/6$, and $11\pi/6$.

Are there any additional solutions outside $[0, 2\pi]$?

Yes, solutions repeat periodically at intervals of π, so $x = \pi/2 + n\pi$ and $x = 5\pi/6 + n\pi$ for all integers n, within the domain considered.

Additional Resources

In-Depth Analysis of the Equation: \(\cos(x) \cot(x) + \sqrt{3} \cos(x) = 0\)

Solving trigonometric equations requires a deep understanding of the fundamental identities, algebraic manipulation skills, and strategic approaches to simplify complex expressions. The equation:

\[
\cos(x) \cot(x) + \sqrt{3} \cos(x) = 0
\]

presents an interesting challenge. This detailed review will explore the equation from multiple angles, including algebraic simplification, solution strategies, domain considerations, and verification of solutions. Our goal is to not only find all solutions within the specified interval but to also provide a comprehensive understanding of the methods involved.

Understanding the Equation and Its Components

Before attempting to solve, it’s crucial to analyze the structure of the given equation:

$$
\cos(x) \cot(x) + \sqrt{3} \cos(x) = 0
$$

Key components:

- $\cos(x)$: The cosine function, which oscillates between -1 and 1.
- $\cot(x)$: The cotangent function, defined as $\cot(x) = \frac{\cos(x)}{\sin(x)}$, with domain restrictions where $\sin(x) \neq 0$.
- $\sqrt{3}$: A constant (~1.732), which often appears in special angles related to 30°, 45°, 60°.

Initial observations:

- The equation involves a product $\cos(x) \cot(x)$ plus a linear term $\sqrt{3} \cos(x)$.
- The presence of $\cot(x)$ suggests that solutions will be constrained by the domain where $\sin(x) \neq 0$.

Rewriting the Equation for Clarity

The first step in solving the equation is to express everything in terms of sine and cosine to exploit known identities:

$$
\cos(x) \cot(x) + \sqrt{3} \cos(x) = 0
$$

Recall that:

$$
\cot(x) = \frac{\cos(x)}{\sin(x)}
$$

Substituting:

$$
\cos(x) \times \frac{\cos(x)}{\sin(x)} + \sqrt{3} \cos(x) = 0
$$

which simplifies to:

$$
\frac{\cos^2(x)}{\sin(x)} + \sqrt{3} \cos(x) = 0
$$

Factorization and Simplification

To facilitate solution, factor out common terms:

$$
\cos(x) \left(\frac{\cos(x)}{\sin(x)} + \sqrt{3} \right) = 0
$$

Leading to two cases:

Case 1: $\cos(x) = 0$

Case 2: $\frac{\cos(x)}{\sin(x)} + \sqrt{3} = 0$

Case 1: $\cos(x) = 0$

Solving $\cos(x) = 0$:

- The cosine function equals zero at:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

Domain considerations:

- The problem specifies solutions within an interval $[0, \text{}]$. Since the upper bound isn't explicitly provided, for this analysis, assume the interval is $[0, 2\pi]$, a common choice in trigonometry problems.

Solutions in $[0, 2\pi]$:

$$
x = \frac{\pi}{2}, \quad \frac{3\pi}{2}
$$

Verification:

- At $x = \frac{\pi}{2}$:

$\cos(\frac{\pi}{2})=0$, and $\cot(\frac{\pi}{2})=0$.

The original equation becomes:

$$
0 \times 0 + \sqrt{3} \times 0 = 0
$$

Valid solution.

- At $x = \frac{3\pi}{2}$:

$\cos(\frac{3\pi}{2})=0$, $\cot(\frac{3\pi}{2})=0$.

The original equation:

$$
0 \times 0 + \sqrt{3} \times 0 = 0
$$

Valid solution.

Conclusion for Case 1:

$$
x = \frac{\pi}{2}, \quad \frac{3\pi}{2}
$$

Case 2: $\frac{\cos(x)}{\sin(x)} + \sqrt{3} = 0$

This simplifies to:

$$
\frac{\cos(x)}{\sin(x)} = -\sqrt{3}
$$

or

$$
\cot(x) = -\sqrt{3}
$$

Solution for $\cot(x) = -\sqrt{3}$:

- The cotangent function equals $-\sqrt{3}$ at specific angles. Recall the standard values:

$$
\cot(x) = \tan\left(\frac{\pi}{2} - x\right)
$$

- The general solution for $\cot(x) = c$ is:

$$
x = \arccot(c) + n\pi
$$

- Using known values:

$$
\cot\left(\frac{\pi}{6}\right) = \sqrt{3}
$$

$$
\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}
$$

Hence:

$$
\cot(x) = -\sqrt{3} \quad \Rightarrow \quad x = \frac{5\pi}{6} + n\pi
$$

Similarly, because cotangent is periodic with period π, the solutions in $[0, 2\pi]$ are:

$$
x = \frac{5\pi}{6}, \quad \frac{5\pi}{6} + \pi = \frac{5\pi}{6} + \pi = \frac{5\pi}{6} + \frac{6\pi}{6} = \frac{11\pi}{6}
$$

Verification:

- At $x = \frac{5\pi}{6}$:

$\cot(\frac{5\pi}{6}) = -\sqrt{3}$, matching the condition.

- At $x = \frac{11\pi}{6}$:

$\cot(\frac{11\pi}{6}) = -\sqrt{3}$, matching the condition.

Summary of Solutions in $[0, 2\pi]$

Combining both cases, the solutions are:

$$
x = \frac{\pi}{2}, \quad \frac{3\pi}{2}, \quad \frac{5\pi}{6}, \quad \frac{11\pi}{6}
$$

These four solutions satisfy the original equation within the interval $[0, 2\pi]$.

Extending Beyond the Basic Interval

If the interval extends beyond $[0, 2\pi]$, solutions repeat periodically due to the periodicity of the involved functions.

- For $\cos(x) = 0$:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

- For $\cot(x) = -\sqrt{3}$:

$$
x = \frac{5\pi}{6} + n\pi, \quad n \in \mathbb{Z}
$$

Note: When considering the entire real line, solutions are infinite, but within a specified finite interval, the above solutions are relevant.

Domain Restrictions and Validity Checks

One must verify the domain restrictions:

- The original expression involves $\cot(x) = \frac{\cos(x)}{\sin(x)}$. At points where $\sin(x) = 0$, $\cot(x)$ is undefined.

- The solutions x where $\sin(x) = 0$ are:

$$
x = 0, \pi, 2\pi, \dots
$$

- These are not solutions to the original equation because of the undefined $\cot(x)$. Our solutions are carefully chosen to avoid these points.

Summary:

- All solutions found satisfy the domain restrictions.

Graphical Interpretation and Visualization

Understanding the solutions geometrically can be highly beneficial:

- The graph of $\cos(x) \cot(x) + \sqrt{3} \cos(x)$ versus x can be plotted over the interval $[0, 2\pi]$. The zeros of this graph correspond to solutions.

- The key points identified correspond to intersections with the x-axis, which can be visually confirmed.

Special Angles and Trigonometric

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