Solve The Following Exponential Equation. Express The Solution Set In Terms Of Natural Logarithms Or

Solve The Following Exponential Equation. Express The Solution Set In Terms Of Natural Logarithms Or

Understanding how to solve exponential equations is a fundamental skill in algebra and higher mathematics. These equations involve variables in the exponent and often require specific techniques such as logarithmic transformations to find solutions. In this comprehensive guide, we will explore various methods to solve exponential equations and express the solutions using natural logarithms (ln). Whether you're a student preparing for exams or a math enthusiast seeking clarity, this article aims to provide detailed explanations, step-by-step processes, and practical examples to master solving exponential equations.

What Are Exponential Equations?

Exponential equations are equations where the variable appears in the exponent position. The general form of an exponential equation is:

$$a^{x} = b$$

where:
- a is a positive real number, and $a \neq 1$,
- b is a positive real number,
- x is the variable to solve for.

These equations can be simple or complex, involving multiple exponential expressions, coefficients, or even nested exponents.

Examples of exponential equations:
- $2^{x} = 8$
- $5^{2x} = 125$
- $3^{x+2} = 27$

Why Use Natural Logarithms in Solving Exponential Equations?

Natural logarithms are particularly useful because they are the inverse functions of exponential functions with base e (Euler's number, approximately 2.71828). When the base of the exponential is not e, logarithms help in converting the exponential equation into a linear form, making it easier to isolate the variable.

Key reasons to use natural logarithms:
- To "undo" the exponential operation.
- To convert exponential equations to linear equations.
- To express solutions explicitly in terms of $\ln$.

General Method for Solving Exponential Equations

The process typically involves these steps:
1. Isolate the exponential expression if possible.
2. Take the natural logarithm (ln) of both sides of the equation.
3. Apply logarithmic properties to simplify.
4. Solve for the variable.
5. Express the solution set, including all possible solutions.

Step-by-Step Examples

Example 1: Solving a Simple Exponential Equation

Solve $2^{x} = 8$ and express the solution in terms of natural logarithms.

Step 1: Recognize that $8 = 2^{3}$, or proceed directly:

$$2^{x} = 8$$

Step 2: Take the natural logarithm of both sides:

$$\ln(2^{x}) = \ln(8)$$

Step 3: Use logarithmic properties: $\ln(a^{b}) = b \ln(a)$:

$$x \ln(2) = \ln(8)$$

Step 4: Express $\ln(8)$ as:

$$\ln(8) = \ln(2^{3}) = 3 \ln(2)$$

Step 5: Solve for x:

$$x \ln(2) = 3 \ln(2)$$

$$x = \frac{3 \ln(2)}{\ln(2)} = 3$$

Solution: $x = 3$

Expressed in terms of natural logarithms:

$$x = \frac{\ln(8)}{\ln(2)}$$

Example 2: Solving an Exponential Equation with a Coefficient

Solve $3^{2x+1} = 81$, expressing the solutions in terms of natural logarithms.

Step 1: Recognize that $81 = 3^{4}$:

$$3^{2x+1} = 3^{4}$$

Step 2: Since bases are the same and both sides are positive,

$$2x + 1 = 4$$

Step 3: Solve for x:

$$2x = 3$$

$$x = \frac{3}{2}$$

Alternatively, if the bases are not the same, we proceed with logs:

$$3^{2x+1} = 81$$

Taking natural logs:

$$\ln(3^{2x+1}) = \ln(81)$$

Applying the log property:

$$(2x + 1) \ln(3) = \ln(81)$$

Express $\ln(81)$ as:

$$\ln(3^{4}) = 4 \ln(3)$$

Thus:

$$ (2x + 1) \ln(3) = 4 \ln(3) $$

Divide both sides by $\ln(3)$:

$$ 2x + 1 = 4 $$

Solve for x:

$$ 2x = 3 $$

$$ x = \frac{3}{2} $$

Solution: $x = \frac{3}{2}$

Expressed in terms of natural logarithms:

$$ x = \frac{\ln(81)}{2 \ln(3)} - \frac{1}{2} $$

but since $\ln(81) = 4 \ln(3)$:

$$ x = \frac{4 \ln(3)}{2 \ln(3)} - \frac{1}{2} = 2 - \frac{1}{2} = \frac{3}{2} $$

Solving More Complex Exponential Equations

Some exponential equations involve multiple exponential terms, coefficients, or variables in exponents. In such cases, the approach involves algebraic manipulations and logarithmic transformations.

Example 3: Exponential Equation with Multiple Terms

Solve $5^{x} + 5^{2x} = 130$.

Step 1: Recognize that $5^{2x} = (5^{x})^{2}$.

Let $y = 5^{x}$. The equation becomes:

$$ y + y^{2} = 130 $$

Step 2: Rewrite as a quadratic:

$$ y^{2} + y - 130 = 0 $$

Step 3: Solve for y:

$$ y = \frac{-1 \pm \sqrt{1^{2} - 4 \times 1 \times (-130)}}{2} $$

$$ y = \frac{-1 \pm \sqrt{1 + 520}}{2} $$

$$ y = \frac{-1 \pm \sqrt{521}}{2} $$

Since $y = 5^{x} > 0$, discard negative solutions:

$$ y = \frac{-1 + \sqrt{521}}{2} $$

Step 4: Solve for x:

$$ 5^{x} = y $$

$$ x = \frac{\ln y}{\ln 5} $$

Expressed in terms of natural logarithms:

$$ x = \frac{\ln \left(\frac{-1 + \sqrt{521}}{2} \right)}{\ln 5} $$

Note: Only the positive root is valid because the exponential function is always positive.

Special Cases and Tips for Solving Exponential Equations

- When the bases are the same: Equate the exponents directly if the bases are identical and both positive.

- When bases are different but positive: Use logarithms to bring both sides to a common form.

- When the exponential is multiplied by a coefficient: Isolate the exponential term before applying logs.

- Handling equations involving e: If the base is e, the natural logarithm simplifies calculations directly.

- Check for extraneous solutions: When solving for x, verify solutions in the original equation.

Expressing the Solution Set in Terms of Natural

Logarithms

Expressing solutions in terms of natural logarithms enhances clarity and generality, especially when dealing with more abstract equations. The key is recognizing that:

$$ a^{x} = b \Rightarrow x = \frac{\ln b}{\ln a} $$

This formula applies universally for positive $a \neq 1$ and positive b.

Example:

Solve $4^{x} = 10$ in terms of natural logarithms:

$$ x = \frac{\ln 10}{\ln 4} $$

Similarly, for equations involving more complex terms, the solution may involve multiple logarithmic expressions, but the core idea remains the same.

Summary of Key Points

- To solve exponential equations, isolate the exponential term whenever possible.
- Use natural logarithms to linearize the exponential expressions.
- Apply logarithmic properties such as $\ln(a^{b}) = b \ln a$.
- Express solutions explicitly in terms of $\ln$ for clarity.
- Always verify solutions to avoid extraneous roots introduced during transformations.

Conclusion

Mastering the technique of solving exponential equations with natural logarithms is essential for advancing in

Frequently Asked Questions

How do you solve an exponential equation like 2^x = 8 using natural logarithms?

Take the natural logarithm of both sides: ln(2^x) = ln(8). Then, bring down the exponent: x ln(2) = ln(8). Solve for x: x = ln(8) / ln(2).

What is the general approach to solving equations of the form a^{x} = b using natural logarithms?

Apply natural logarithm to both sides: $\ln(a^{x}) = \ln(b)$. Simplify to $x \ln(a) = \ln(b)$, then solve for x: $x = \ln(b) / \ln(a)$.

How can I express the solution set of an exponential equation involving multiple solutions in terms of natural logarithms?

Identify the exponential form, take natural logarithms on both sides, and solve for the variable. The solution set is expressed as $x = (\ln(b) + 2\pi i k) / \ln(a)$ for complex solutions or $x = \ln(b) / \ln(a)$ for real solutions, depending on the context.

Why is it necessary to express solutions in terms of natural logarithms when solving exponential equations?

Expressing solutions in terms of natural logarithms allows us to linearize the exponential equations, making it easier to isolate and solve for the variable, especially when dealing with exponents involving variables.

Can you provide an example of solving log-based exponential equations using natural logarithms?

Yes. For example, solve $5^{2x} = 125$. Take natural logarithm: $\ln(5^{2x}) = \ln(125)$. Simplify: $2x \ln(5) = \ln(125)$. Solve for x: $x = \ln(125) / (2 \ln(5))$.

What is the solution set for the exponential equation e^{x} = 7 in terms of natural logarithms?

Take natural logarithm: $\ln(e^{x}) = \ln(7)$. Simplify: $x = \ln(7)$. So, the solution set is $\{ x = \ln(7) \}$.

Additional Resources

Solve The Following Exponential Equation. Express The Solution Set In Terms Of Natural Logarithms Or

When faced with exponential equations, the ability to manipulate and solve them effectively hinges on understanding the properties of exponents and logarithms. The phrase "Solve The Following Exponential Equation. Express The Solution Set In Terms Of Natural Logarithms Or" signifies a common instructional directive in algebra and calculus, emphasizing the importance of natural logarithms (ln) when solving equations involving variables in exponents. This comprehensive guide explores the methods for solving exponential equations, highlights the role of natural logarithms, and discusses how to express solutions clearly and accurately.

Understanding Exponential Equations

Exponential equations are equations where the variable appears in the exponent. They take forms such as:

- $a^x = b$
- $e^{kx} = c$
- $2^{3x+1} = 16$

where a, b, c are constants, and x is the variable to be solved for.

Key features of exponential equations:

- The base a is usually a positive real number not equal to 1.
- The solutions often involve logarithms because they are inverse functions of exponential functions.
- When the equation involves the natural exponential function e^x, natural logarithms become particularly useful.

The Role of Natural Logarithms in Solving Exponential Equations

Natural logarithms ($\ln$) are logarithms with base e, where $e \approx 2.71828$. They serve as the inverse of the exponential function e^x. When solving exponential equations, especially those with base e, natural logarithms allow us to rewrite and isolate the variable.

Why use natural logarithms?

- They convert exponents into multiplicative factors, turning the problem into a linear equation.
- They simplify the process of solving equations where the variable is in the exponent.
- They provide exact solutions expressed in terms of known constants.

General Method for Solving Exponential Equations

The process of solving exponential equations typically involves these steps:

1. Isolate the exponential expression: Ensure the exponential term is alone on one side of the equation.
2. Apply logarithms: Take the natural logarithm (or log base a, if appropriate) of both sides to leverage the inverse property.
3. Use logarithmic properties: Simplify the equation using properties such as:
- $\ln(ab) = \ln a + \ln b$

- $\ln \left(\frac{a}{b}\right) = \ln a - \ln b$
- $\ln a^k = k \ln a$

4. Solve for the variable: Rearrange the resulting linear or algebraic expression to find the solution(s).
5. Express the solution set: Present the solutions clearly, often in terms of natural logarithms, especially when multiple solutions exist.

Case Study 1: Solving Simple Exponential Equations

Consider the equation:

$$3^x = 7$$

Solution:

- Recognize that the base 3 is positive and not equal to 1.
- Take natural logarithms on both sides:

$$\ln(3^x) = \ln(7)$$

- Apply the power rule:

$$x \ln(3) = \ln(7)$$

- Solve for x:

$$x = \frac{\ln(7)}{\ln(3)}$$

Solution set:

$$\boxed{x = \frac{\ln(7)}{\ln(3)}}$$

This solution expresses x explicitly in terms of natural logarithms, making it exact and calculable with a calculator.

Case Study 2: Equations with the Base e

Suppose we have:

$$e^{2x + 1} = 5$$

Solution:

- Isolate the exponential:

$$ e^{2x + 1} = 5 $$

- Take the natural logarithm:

$$ \ln(e^{2x + 1}) = \ln(5) $$

- Simplify:

$$ 2x + 1 = \ln(5) $$

- Solve for x:

$$ 2x = \ln(5) - 1 $$

$$ x = \frac{\ln(5) - 1}{2} $$

Solution set:

$$ \boxed{x = \frac{\ln(5) - 1}{2}} $$

This method highlights the straightforward application of natural logarithms when the exponential involves e.

Complex Equations: Multiple Solutions and Domains

Some exponential equations can have multiple solutions, especially when dealing with equations involving both e^x and other terms, or when considering the domain restrictions.

Example:

$$ 2^x = -4 $$

This equation has no solutions because 2^x is always positive for real x, and cannot equal a negative number.

Example:

$$ e^{x} = e^{2x} $$

- Take natural logarithms:

$$ \ln(e^{x}) = \ln(e^{2x}) $$

$$ x = 2x $$

- Solve:

$$ x - 2x = 0 \Rightarrow -x = 0 \Rightarrow x=0 $$

- Verify:

$$ e^{0} = 1 $$

$$ e^{2 \times 0} = e^{0} = 1 $$

Both sides equal 1; hence, the solution is $x=0$.

Expressing the Solution Set in Terms of Natural Logarithms

When solving exponential equations, the goal is often to express solutions explicitly using natural logarithms. This provides clarity, especially when the solutions involve ratios or multiple steps.

Example:

Solve:

$$ 5^{x+2} = 20 $$

Solution:

- Rewrite:

$$ 5^{x+2} = 20 $$

- Take natural logarithm:

$$ \ln(5^{x+2}) = \ln(20) $$

- Apply power rule:

$$ (x+2) \ln(5) = \ln(20) $$

- Solve for x:

$$ x + 2 = \frac{\ln(20)}{\ln(5)} $$

$$ x = \frac{\ln(20)}{\ln(5)} - 2 $$

Final answer:

$$\boxed{x = \frac{\ln(20)}{\ln(5)} - 2}$$

This form neatly expresses the solution in terms of natural logarithms, making it easier to evaluate numerically or analyze.

Features and Benefits of Using Logarithms in Solutions

Pros:

- Exact solutions: Logarithms convert exponentials into linear forms, enabling precise algebraic solutions.
- Universal applicability: Logarithms can handle a wide range of bases, especially when the base isn't e.
- Simplification: Logarithmic properties simplify complex exponential equations, reducing them to manageable forms.
- Insight into growth/decay: Logarithms help analyze exponential growth or decay processes in sciences and engineering.

Cons:

- Requires familiarity with properties: To manipulate logarithms effectively, one must be comfortable with their properties.
- Potential for domain issues: Logarithms are only defined for positive arguments, so solutions must be checked for validity.
- Approximate numerical solutions: When exact forms involve complex logs, numerical approximation may be necessary for practical applications.

Conclusion

Solving exponential equations and expressing their solutions in terms of natural logarithms is a fundamental skill in algebra and calculus. The key steps involve isolating the exponential expression, applying logarithms, leveraging properties of logarithms, and solving for the variable. This approach provides exact solutions that are easily interpretable and evaluable. Whether dealing with simple equations like $3^x = 7$ or more complex forms involving e, natural logarithms serve as an essential tool for unlocking the solutions.

By mastering these techniques, students and mathematicians can confidently approach a broad class of exponential equations, understand their solutions in a clear and precise manner, and apply these methods across various fields such as physics, engineering, economics, and beyond. Remember, the power of logarithms lies in their ability to transform exponential growth and decay into linear problems, simplifying the path to solutions and deepening your understanding of exponential functions.

In summary:

- Always isolate the exponential term.
- Take the natural logarithm of both sides.
- Use logarithmic properties to simplify.
- Solve for the variable.
- Express the solution set explicitly in terms of natural logarithms.
- Verify solutions within the domain constraints.

With practice, solving exponential equations and expressing solutions in natural logarithmic form will become an intuitive and valuable skill in your mathematical toolkit.

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