Solve The System By The Method Of Elimination And Check Any Solutions Algebraically. 4x - 0.3y = 5.5

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Understanding how to solve systems of equations is a fundamental skill in algebra that enables students and professionals to find the values of variables that satisfy multiple conditions simultaneously. One effective method for solving such systems is the Method of Elimination, which involves adding or subtracting equations to eliminate one variable, simplifying the process of finding solutions. Additionally, it is crucial to check solutions algebraically to ensure their accuracy and validity within the original equations. In this article, we will walk through the process of solving a system of equations using the method of elimination and demonstrate how to verify solutions algebraically, focusing on the example equation: $4x - 0.3y = 5.5$.

Understanding the System of Equations

Before applying the method of elimination, it is important to understand what a system of equations entails.

What Is a System of Equations?

A system of equations consists of two or more equations with the same set of variables. The solution to the system is the set of variable values that satisfy all equations simultaneously.

Types of Systems

- Consistent and Independent: Has exactly one solution.
- Consistent and Dependent: Has infinitely many solutions.
- Inconsistent: Has no solution.

For the purposes of this article, we focus on consistent systems with two equations and two variables, which can often be solved using the method of elimination.

Setting Up the System for Elimination

Suppose we have two equations:

1. $4x - 0.3y = 5.5$
2. $ax + by = c$ (another equation with variables x and y)

Our goal is to eliminate one variable to solve for the other.

Step 1: Express Both Equations in a Standard Form

In our example, we only have one equation explicitly: $4x - 0.3y = 5.5$. To demonstrate the elimination method, we need a second equation involving the same variables.

Let's consider a second equation for clarity:

2. $2x + y = 4$

Now, our system is:

- Equation 1: $4x - 0.3y = 5.5$
- Equation 2: $2x + y = 4$

Applying the Method of Elimination

The elimination method involves manipulating the equations so that adding or subtracting them cancels out one variable.

Step 1: Equalize the Coefficients

To eliminate a variable, align the coefficients of that variable in both equations.

In our example, notice that coefficients of x are 4 and 2. To eliminate x, we can multiply the second equation by 2:

- Equation 2 multiplied by 2: $(2x + y) \times 2 \rightarrow 4x + 2y = 8$

Now, compare this with Equation 1:

- Equation 1: $4x - 0.3y = 5.5$
- Modified Equation 2: $4x + 2y = 8$

Step 2: Subtract the Equations

Subtract Equation 1 from the modified Equation 2 to eliminate x:

$(4x + 2y) - (4x - 0.3y) = 8 - 5.5$

Simplify:

$4x + 2y - 4x + 0.3y = 2.5$

This simplifies further to:

$(2y + 0.3y) = 2.5$

Combining like terms:

$2.3y = 2.5$

Step 3: Solve for y

Divide both sides by 2.3:

$y = 2.5 / 2.3 \approx 1.087$

Now we have the value of y.

Finding the Value of x

With y known, substitute it back into one of the original equations to solve for x.

Using Equation 2: $2x + y = 4$

Substitute $y \approx 1.087$:

$2x + 1.087 = 4$

Subtract 1.087 from both sides:

$2x = 4 - 1.087 \approx 2.913$

Divide both sides by 2:

$x \approx 2.913 / 2 \approx 1.4565$

Checking the Solution Algebraically

Verifying solutions is an essential step to ensure accuracy. We substitute the approximate values of x and y into the original equations.

Step 1: Check Equation 1: $4x - 0.3y = 5.5$

Substitute $x \approx 1.4565$ and $y \approx 1.087$:

$4(1.4565) - 0.3(1.087) \approx ?$

Calculate:

- $4 \times 1.4565 \approx 5.826$
- $0.3 \times 1.087 \approx 0.326$

Now, $5.826 - 0.326 \approx 5.5$

The left side approximates the right side (5.5), confirming the solution is valid within rounding errors.

Step 2: Check Equation 2: $2x + y = 4$

Substitute $x \approx 1.4565$ and $y \approx 1.087$:

$2 \times 1.4565 + 1.087 \approx ?$

Calculate:

- $2 \times 1.4565 \approx 2.913$
- $2.913 + 1.087 \approx 4.000$

This confirms the solution satisfies both equations.

Important Tips for Solving Systems by Elimination

- **Ensure coefficients are aligned:** Multiply equations as needed to match coefficients for the variable you want to eliminate.
- **Be cautious with decimals:** When equations involve decimals, it's often helpful to clear decimals by multiplying through by factors to avoid rounding errors.
- **Always check your solutions:** Substitute your solution back into the original equations to confirm correctness.

- **Handle special cases:** Be aware of scenarios where coefficients are already aligned or where equations are dependent or inconsistent.

Additional Examples and Practice Problems

Example 1:
Solve the system:
- $3x + 2y = 7$
- $5x - y = 9$

Solution Steps:

1. Multiply the second equation by 2 to match the y coefficient:
- $10x - 2y = 18$

2. Add the first and new second equations:
- $(3x + 2y) + (10x - 2y) = 7 + 18$
- $13x = 25$
- $x = 25 / 13 \approx 1.923$

3. Substitute x into the second original equation:
- $5(1.923) - y = 9$
- $9.615 - y = 9$
- $y = 9.615 - 9 \approx 0.615$

4. Check the solution:
- $3(1.923) + 2(0.615) \approx 5.769 + 1.23 \approx 7.00$

Practice Problems:

- Solve the system:
- $2x + 3y = 8$
- $4x - y = 2$

- Solve the system:
- $7x + y = 14$
- $-3x + 2y = 1$

Conclusion

The method of elimination is a powerful and systematic approach to solving systems of linear equations. By carefully aligning coefficients and performing addition or subtraction, you can efficiently find the solution variables. Remember, once you obtain potential solutions, always verify them by substituting back into the original equations to confirm their validity. This not only helps catch calculation errors but also reinforces understanding of the relationships between variables. With practice, solving systems algebraically becomes a straightforward process that enhances your problem-solving skills in algebra and beyond.

Summary of Key Steps:

1. Write the system in standard form.
2. Multiply equations to align coefficients of the variable to eliminate.
3. Add or subtract equations to eliminate one variable.
4. Solve for the remaining variable.
5. Substitute back into original equations to check solutions.
6. Verify accuracy and validity of solutions algebraically.

Adopting these steps ensures a clear and reliable approach to solving systems using the elimination method, helping students and professionals confidently handle complex algebraic problems.

Frequently Asked Questions

What is the first step to solve the system using the elimination method?

The first step is to write both equations in standard form and align like terms to prepare for elimination.

How do you eliminate one variable in the system 4x - 0.3y = 5.5?

To eliminate a variable, multiply one or both equations by suitable numbers so that the coefficients of either x or y are opposites, then add or subtract the equations.

Can you demonstrate the elimination method by solving the system: 4x - 0.3y = 5.5 and another equation?

Yes. Suppose the second equation is $2x + y = 7$. Multiply the first equation by 2 to get $8x - 0.6y = 11$, then multiply the second equation by 0.3 to get $0.6x + 0.3y = 2.1$. Adding these results to eliminate y allows solving for x.

How do you check the solutions algebraically after solving the system?

Substitute the found values of x and y back into both original equations to verify if both equations are satisfied.

How do you handle decimal coefficients like -0.3y in the elimination method?

Multiply the entire equation by 10 (or a suitable factor) to clear decimals, making calculations easier and more straightforward.

Is it necessary to always check solutions algebraically after solving?

Yes, verifying solutions ensures they satisfy all original equations and helps identify extraneous solutions or mistakes.

Can the elimination method be used for nonlinear systems?

No, the elimination method is primarily for linear systems. Nonlinear systems often require substitution, graphing, or other methods for solutions.

Additional Resources

Solve The System By The Method Of Elimination And Check Any Solutions Algebraically: A Comprehensive Exploration

Introduction

In the realm of algebra, systems of equations serve as foundational tools for modeling real-world phenomena, analyzing relationships among variables, and developing problem-solving strategies. Among the array of methods available, the method of elimination stands out for its systematic approach to solving simultaneous equations. When applied correctly, it not only yields solutions efficiently but also provides a means to verify the accuracy of results through algebraic checking. This article thoroughly explores the process of solving a specific linear system using the method of elimination and emphasizes the importance of algebraic verification.

The System in Focus

Consider the following system of equations:

1) $4x - 0.3y = 5.5$

2) (Another equation to be determined – for example, let's analyze a second equation to form a system)

To effectively demonstrate the elimination method, it’s essential to have a complete system. For this review, we will examine the system:

$4x - 0.3y = 5.5$

$2x + 0.5y = 4.0$

This pair of equations provides a suitable context to illustrate the elimination process and subsequent algebraic verification.

Understanding the Equations

Before delving into the solution, it's important to understand the characteristics of these equations:

- Both are linear equations in two variables, x and y.
- The coefficients are real numbers, with some fractional components.
- The system appears consistent and solvable; however, this will be confirmed through calculation.

Method of Elimination: An In-Depth Approach

Step 1: Preparing the Equations for Elimination

The goal of elimination is to cancel out one variable by aligning the coefficients such that adding or subtracting the equations eliminates that variable. To do this effectively, the coefficients of one variable should be equal in magnitude and opposite in sign.

Given the equations:

(1) $4x - 0.3y = 5.5$

(2) $2x + 0.5y = 4.0$

Observe the coefficients of x: 4 and 2. To align these, multiply the second equation by 2:

Equation (2) × 2:

(2) × 2: $4x + 1.0y = 8.0$

Now, the equations are:

(1) $4x - 0.3y = 5.5$

(2') $4x + 1.0y = 8.0$

Step 2: Eliminating a Variable

Subtract equation (1) from (2'):

$(4x + 1.0y) - (4x - 0.3y) = 8.0 - 5.5$

Simplify:

$(4x - 4x) + (1.0y + 0.3y) = 2.5$

$0 + 1.3y = 2.5$

Solve for y:

$y = 2.5 / 1.3 \approx 1.9231$

To maintain precision, express as a fraction:

$1.3 = 13/10$

So,

$y = 2.5 \div (13/10) = 2.5 \times (10/13) = (25/10) \times (10/13) = (25/13)$

Simplify:

$y = 25/13 \approx 1.9231$

Step 3: Substituting Back to Find x

Now that y is known, substitute into one of the original equations to solve for x.

Using equation (1):

$4x - 0.3y = 5.5$

Plugging in $y = 25/13$:

$4x - 0.3 \times (25/13) = 5.5$

Calculate $0.3 \times (25/13)$:

$0.3 = 3/10$

So,

$(3/10) \times (25/13) = (3 \times 25) / (10 \times 13) = 75 / 130 = 15 / 26$

Now, the equation becomes:

$4x - 15/26 = 5.5$

Express 5.5 as a fraction:

$5.5 = 11/2$

Rewrite the equation:

$4x = 11/2 + 15/26$

Find common denominator (26):

$11/2 = (11 \times 13) / (2 \times 13) = 143 / 26$

So,

$4x = 143/26 + 15/26 = (143 + 15)/26 = 158/26$

Simplify numerator and denominator:

$158/26 = (79/13)$

Finally, divide both sides by 4:

$x = (79/13) \div 4 = (79/13) \times (1/4) = 79 / (13 \times 4) = 79 / 52$

Simplify if possible:

79 and 52 share no common factors, so the solution for x is:

$x = 79/52 \approx 1.5192$

Summary of the Solution

The solutions to the system are:

$x = 79/52 \approx 1.5192$

$y = 25/13 \approx 1.9231$

These fractional forms provide precise solutions, vital for algebraic verification.

Algebraic Checking: Confirming the Solution

Step 1: Substitute into Equation (1)

Original equation:

$4x - 0.3y = 5.5$

Substitute $x = 79/52$ and $y = 25/13$:

$4 \times (79/52) - 0.3 \times (25/13) =?$

Calculate:

$4 \times (79/52) = (4/1) \times (79/52) = (4 \times 79) / 52 = 316 / 52$

Simplify numerator and denominator:

$316/52 = (79/13)$

Next:

$0.3 \times (25/13) = (3/10) \times (25/13) = 75 / 130 = 15 / 26$

Express both as decimals or fractions:

Now, verify:

$(79/13) - (15/26) = ?$

Express both over a common denominator, 26:

(79/13) = (158/26)

So:

(158/26) - (15/26) = (143/26)

Calculate:

143/26 ≈ 5.5

Since the right side of the original equation is 5.5, the substitution confirms the solution satisfies Equation (1).

Step 2: Verify with Equation (2)

Equation:

2x + 0.5y = 4.0

Substitute:

2 × (79/52) + 0.5 × (25/13) = ?

Calculate:

2 × (79/52) = (2/1) × (79/52) = (2 × 79) / 52 = 158 / 52 = 79 / 26

Next:

0.5 × (25/13) = (1/2) × (25/13) = 25 / 26

Sum:

(79/26) + (25/26) = (104/26) = 4

Matching the right side of the equation, the solution satisfies Equation (2).

Conclusion: The solutions (x = 79/52, y = 25/13) are verified algebraically, confirming their accuracy.

Implications and Significance

This detailed process illustrates the robustness of the elimination method in solving linear systems. It emphasizes the importance of multiplying equations to align coefficients, simplifying expressions to fractional forms for precision, and verifying solutions algebraically to eliminate errors. This approach not only ensures the correctness of solutions but also enhances understanding of linear relationships.

Moreover, the algebraic verification process underscores the importance of meticulous calculations, especially when fractional coefficients are involved. In applied contexts, such as engineering, economics, or physics, such rigor ensures models remain reliable and predictions accurate.

Extensions and Practical Applications

While the example considered here involves a straightforward two-variable system, the elimination method scales to more complex systems with additional variables, often in conjunction with substitution or matrices. Practical applications include:

- Determining equilibrium points in economic models
- Solving circuit equations in electrical engineering
- Optimizing resource allocations in operations research
- Analyzing chemical reaction balances

In each case, the method of elimination, coupled with algebraic verification, ensures solutions are both precise and reliable.

Conclusion

The process of solving systems of equations by the method of elimination, followed by algebraic checking, remains a cornerstone of algebraic problem-solving. Its systematic nature, combined with the capacity for verification, makes it an invaluable technique for students, educators, and professionals alike. As demonstrated through the detailed example, careful manipulation and verification lead to accurate solutions, fostering a deeper understanding of linear relationships and mathematical rigor.

By mastering these techniques, learners and practitioners can confidently approach complex systems, ensuring their solutions stand up to scrutiny and serve as solid foundations for further analysis.

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