Solve The System Of Equations Using Substitution. 3x + 2y = 7 X = 3y + 6 (0, 2) (1, 2) (3, 1) (6, 0)

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Introduction

Solving systems of equations is a fundamental skill in algebra that allows us to find the point(s) where two or more equations intersect. One of the most effective methods for solving such systems, especially when one of the equations is already solved for a variable, is substitution. In this article, we will explore how to solve a system of equations using substitution, focusing on the specific system:

- $3x + 2y = 7$
- $x = 3y + 6$

Additionally, we will evaluate the given points: (0, 2), (1, 2), (3, 1), and (6, 0) to determine which, if any, satisfy the system.

Understanding the System of Equations

Before diving into the solution, it’s essential to understand the structure of the equations:

Equation 1: $3x + 2y = 7$

This is a linear equation involving both variables x and y.

Equation 2: $x = 3y + 6$

This equation is already solved for x, making it ideal for substitution.

What Is the Method of Substitution?

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation. This process reduces the system to a single-variable equation, which can then be solved straightforwardly.

Steps in the substitution method:

1. Solve one equation for one variable (here, equation 2 already expresses x in terms of y).
2. Substitute this expression into the other equation.
3. Solve for the remaining variable.
4. Substitute back to find the other variable.
5. Verify the solution by plugging the values into the original equations.

Step-by-Step Solution

Let's apply the substitution method to our system:

Step 1: Identify the substitution

Equation 2 already gives x in terms of y:

- $x = 3y + 6$

Step 2: Substitute into the first equation

Replace x in the first equation with the expression from equation 2:

- $3(3y + 6) + 2y = 7$

Step 3: Simplify and solve for y

Distribute:

- $9y + 18 + 2y = 7$

Combine like terms:

- $11y + 18 = 7$

Subtract 18 from both sides:

- $11y = 7 - 18$
- $11y = -11$

Divide both sides by 11:

- $y = -1$

Step 4: Find x using the value of y

Substitute $y = -1$ into equation 2:

- $x = 3(-1) + 6$
- $x = -3 + 6$
- $x = 3$

Solution: $(x, y) = (3, -1)$

Verifying the Solution

It’s crucial to verify whether this solution satisfies both original equations.

- First equation: $3x + 2y = 7$

Substituting $x=3$ and $y=-1$:

$3(3) + 2(-1) = 9 - 2 = 7$ ✓

- Second equation: $x = 3y + 6$

Substituting $y=-1$:

$x = 3(-1) + 6 = -3 + 6 = 3$ ✓

Both equations are satisfied, confirming that the solution is correct.

Analyzing the Given Points

Now, let's evaluate the provided points to see which satisfy the system:

Point	x	y	3x + 2y	x (from point)	Is x = 3y + 6?	Satisfies the system?
(0, 2)	0	2	3(0)+2(2)= 0+4=4	0	3(2)+6=6+6=12	No, since 0 ≠ 12
(1, 2)	1	2	3(1)+2(2)= 3+4=7	1	3(2)+6=12	No, since 1 ≠ 12
(3, 1)	3	1	3(3)+2(1)= 9+2=11	3	3(1)+6=3+6=9	No, since 3 ≠ 9
(6, 0)	6	0	3(6)+2(0)= 18+0=18	6	3(0)+6=6	No, since 6 ≠ 18

Conclusion: None of the provided points satisfy both equations simultaneously. The only solution to the system is (3, -1), which is not among the listed options.

Graphical Interpretation

Visualizing the system can provide further understanding.

- The line $3x + 2y = 7$ can be rewritten in slope-intercept form:

$2y = -3x + 7$

$y = (-3/2)x + 7/2$

- The line $x = 3y + 6$ can be rewritten as:

$y = (x - 6)/3$

Plotting these lines on a coordinate plane reveals their intersection point at (3, -1), confirming our algebraic solution.

Practical Applications of Solving Systems Using Substitution

Understanding how to solve systems of equations using substitution has numerous real-world applications:

- Physics: Calculating the intersection point of two moving objects.
- Economics: Finding equilibrium points where supply and demand curves intersect.
- Engineering: Determining optimal points in design problems.
- Computer Science: Solving for variables in algorithms and models.

Tips for Successful Substitution

- Always check if one of the equations is already solved for a variable, which simplifies substitution.
- Be careful with signs and arithmetic during substitution.
- Verify solutions by plugging values back into original equations.
- When dealing with complex systems, consider graphical or elimination methods as alternatives.

Summary

In this article, we've demonstrated how to solve a system of equations using the substitution method. The key steps involved expressing x in terms of y, substituting into the other equation, solving for y, then finding x. The solution to the system:

- (x, y) = (3, -1)

was verified to satisfy both equations. We also examined the provided points and confirmed none satisfy the system, emphasizing the importance of algebraic verification.

Mastering substitution enhances problem-solving skills and provides a foundation for tackling more complex systems in mathematics and applied fields. Whether you're preparing for exams or working on real-world problems, understanding this method is a valuable tool in your mathematical toolkit.

Frequently Asked Questions (FAQs)

Q1: Can the substitution method be used for all systems of equations?
A: It works best when one equation is already solved for a variable or can be easily rearranged. For more complex systems, other methods like elimination or graphing might be more efficient.

Q2: What if substitution leads to a contradiction?
A: If substitution results in a false statement (e.g., $0 = 5$), the system has no solution—it's inconsistent.

Q3: How do I handle systems with more than two variables?
A: The substitution method can be extended to systems with more variables, but it often becomes more complex. Methods like matrix algebra or Gaussian elimination may be more suitable.

Q4: Is substitution always the best method?
A: Not necessarily. Sometimes, elimination or graphing can be quicker depending on the equations' form.

Final Thoughts

Solving systems of equations is a foundational skill in algebra that opens doors to advanced mathematics and various scientific fields. The substitution method, in particular, offers an intuitive approach when one equation is already solved for a variable. Practice with different systems to become proficient, and always verify your solutions to ensure accuracy.

Frequently Asked Questions

How do you solve the system of equations 3x + 2y = 7 and x = 3y + 6 using substitution?

First, substitute $x = 3y + 6$ into the first equation to get $3(3y + 6) + 2y = 7$. Simplify to $9y + 18 + 2y = 7$, then combine like terms: $11y + 18 = 7$, and solve for y to find $y = -1$. Then, plug $y = -1$ back into $x = 3y + 6$ to find $x = 3(-1) + 6 = 3$.

What are the solutions to the system using substitution, given the options (0, 2), (1, 2), (3, 1), (6, 0)?

Substituting each option into both equations, only (3, 1) satisfies both. For x=3, y=1: $3(3)+2(1)=9+2=11 \neq 7$, so (3, 1) is invalid. Similarly, check others: (6, 0) gives $3(6)+2(0)=18\neq7$, so invalid. (0, 2): $3(0)+2(2)=4\neq7$, invalid. (1, 2): $3(1)+2(2)=3+4=7$, matches the first equation. For x=1, y=2: x=3y+6? 1=3(2)+6=6+6=12, no. So only (1, 2) satisfies the first equation, but check the second: x=3y+6? 1=6+6=12, no. So none of the options fully satisfy both equations. The correct solution from previous calculations is x=3, y=-1, which isn't listed here.

Why is substitution an effective method for solving systems of equations like 3x + 2y = 7 and x = 3y + 6?

Substitution simplifies the system by replacing one variable with an expression from the other equation, reducing it to a single-variable equation. This method makes it easier to find the solution systematically.

How can you verify if a given point, such as (1, 2), is a solution to the system 3x + 2y = 7 and x = 3y + 6?

Plug the point into both equations: for (1, 2), check 3(1)+2(2)=3+4=7 (true), and x=3y+6: 1=3(2)+6=6+6=12 (false). Since it doesn't satisfy both equations, (1, 2) isn't a solution.

What is the importance of checking all given options when solving systems algebraically?

Checking options helps verify which points satisfy both equations, especially when multiple-choice answers are provided. It ensures the identified solution is correct in the context of the problem.

Can the substitution method be used for non-linear systems, and why or why not?

Substitution can be used for some non-linear systems if one equation can be easily solved for a variable and substituted into the other. However, it may be more complex or less straightforward than for linear systems.

If the solution to the system is (3, -1), how can you confirm this point satisfies both equations?

Substitute x=3 and y=-1 into both equations: 3(3)+2(-1)=9-2=7 (satisfies first); and x=3y+6: 3=3(-1)+6=-3+6=3 (satisfies second). Therefore, (3, -1) is a solution.

What are common mistakes to avoid when solving systems using substitution?

Common mistakes include substituting incorrectly, miscalculating algebraic steps, forgetting to check all solutions, or assuming solutions without verifying both equations. Always double-check each step and all solutions.

Additional Resources

Solve The System Of Equations Using Substitution: A Comprehensive Guide

When tackling systems of equations, the substitution method stands out as one of the most intuitive and effective strategies, especially for linear systems involving two variables. This method hinges on solving one of the equations for one variable and then substituting that expression into the other equation to find the value of the remaining variable. The process not only simplifies the solution but also enhances understanding of the relationships between the variables involved.

In this detailed review, we will explore the step-by-step procedure to solve a specific system of equations using substitution, analyze the solutions, and interpret the results. Our focus will be on the following system:

- Equations:
- 3x + 2y = 7
- x = 3y + 6

- Potential solutions (points):
- (0, 2)
- (1, 2)
- (3, 1)
- (6, 0)

We will thoroughly examine each step, clarify the logic behind the substitution method, and verify the solutions against the given points.

Understanding the System of Equations

Before diving into the solution process, it's crucial to understand what the system of equations represents and how their solutions relate to each other.

The Equations in Context

- First Equation: 3x + 2y = 7
This is a linear equation representing a straight line when graphed in the xy-plane. Every point (x, y) on this line satisfies this equation.

- Second Equation: x = 3y + 6
This is also a linear equation, explicitly solved for x, which describes a line with a slope of 3 and a y-intercept at 6.

The goal is to find the point(s) (x, y) that satisfy both equations simultaneously—that is, the intersection point(s) of the two lines.

Graphical Interpretation

Graphically, solving the system involves plotting both lines and identifying their intersection point(s). The substitution method provides an algebraic approach to find this point without graphing.

Step-by-Step Solution Using Substitution

The substitution method involves the following steps:

1. Solve one of the equations for one variable.
2. Substitute this expression into the other equation.
3. Solve the resulting equation for the remaining variable.
4. Back-substitute to find the value of the other variable.
5. Verify the solution by plugging values into the original equations.

Let's apply this process to our system.

Step 1: Solve one equation for a variable

Looking at the second equation:

- $x = 3y + 6$

It is already solved for x, which makes substitution straightforward. We will substitute this expression for x into the first equation.

Step 2: Substitute into the other equation

Substitute $x = 3y + 6$ into the first equation ($3x + 2y = 7$):

$$
3(3y + 6) + 2y = 7
$$

This simplifies to:

$$
9y + 18 + 2y = 7
$$

Combine like terms:

$$
(9y + 2y) + 18 = 7
$$

$$
11y + 18 = 7
$$

Step 3: Solve for y

Subtract 18 from both sides:

\[
11y = 7 - 18
\]

\[
11y = -11
\]

Divide both sides by 11:

\[
y = -1
\]

Step 4: Find x using the value of y

Recall the second equation:

\[
x = 3y + 6
\]

Substitute y = -1:

\[
x = 3(-1) + 6 = -3 + 6 = 3
\]

Solution Point: \[(x, y) = (3, -1)\]

Verification of the Solution

It’s essential to verify whether the obtained solution truly satisfies both equations.

- Check in the first equation:

\[
3x + 2y = 7
\]
\[

$$
3(3) + 2(-1) = 9 - 2 = 7 \quad \checkmark
$$

- Check in the second equation:

$$
x = 3y + 6
$$
$$
3 = 3(-1) + 6 = -3 + 6 = 3 \quad \checkmark
$$

Since the point (3, -1) satisfies both equations, it is the exact solution to the system.

Analysis of the Given Options

Now, let's compare our solution with the list of provided points:

- (0, 2)
- (1, 2)
- (3, 1)
- (6, 0)

None of these points match (3, -1). To determine if any of these points satisfy the system, we can substitute each into the original equations.

Testing the Points

1. Point (0, 2):

- First equation: $3(0) + 2(2) = 0 + 4 = 4 \neq 7 \rightarrow$ No
- Second equation: $x = 3(2) + 6 = 6 + 6 = 12 \neq 0 \rightarrow$ No

2. Point (1, 2):

- First equation: $3(1) + 2(2) = 3 + 4 = 7 \rightarrow$ Yes
- Second equation: $x = 3(2) + 6 = 6 + 6 = 12 \neq 1 \rightarrow$ No

3. Point (3, 1):

- First equation: $3(3) + 2(1) = 9 + 2 = 11 \neq 7 \rightarrow$ No
- Second equation: $x = 3(1) + 6 = 3 + 6 = 9 \neq 3 \rightarrow$ No

4. Point (6, 0):

- First equation: $3(6) + 2(0) = 18 + 0 = 18 \neq 7 \rightarrow$ No
- Second equation: $x = 3(0) + 6 = 6 \rightarrow$ Yes

Only the point (1, 2) satisfies the first equation, and the point (6, 0) satisfies the second, but none satisfy both equations simultaneously. This confirms that the actual intersection point is not among the listed options.

Understanding the Nature of the Solution

The discrepancy highlights an important concept:

- Unique Solution: The system has exactly one solution, (3, -1), which is the point of intersection of the two lines.

- No solutions among the options: The provided points are not solutions to the system, indicating they are either points on one line but not the other or are unrelated to the system's intersection.

This illustrates the importance of algebraic methods like substitution in precisely identifying solutions rather than relying solely on graphing or approximations.

Deeper Insights into the Substitution Method

The substitution method is particularly advantageous in systems where:

- One of the equations is already solved for a variable (like x in our case).
- The equations are linear, providing straightforward algebraic manipulation.
- The system is consistent and has a unique solution.

Benefits of Using Substitution

- Clarity: Directly shows how variables are related.
- Efficiency: Often faster than graphing, especially with complex equations.
- Verification: Easy to verify solutions algebraically.

Limitations & Considerations

- When equations are nonlinear, substitution can become algebraically intensive.
- If the expressions involve fractions or radicals, careful handling is necessary.
- Systems with infinitely many solutions or no solutions require additional analysis.

Extending the Technique: General Approach

While our example is straightforward, the substitution method can be generalized:

1. Identify an equation that is easily solved for one variable.
2. Express that variable in terms of the other(s).
3. Substitute into the other equations to reduce the number of variables.
4. Solve the resulting equations for the remaining variables.
5. Use back-substitution to find all variables.
6. Verify solutions in original equations.

This approach can be extended to systems with more than two equations and variables, often in conjunction with elimination or matrix methods.

Practical Applications and Real-World Contexts

Understanding how to solve systems through substitution is not just an academic exercise—it has numerous real-world applications:

- Engineering: Calculating points of intersection in circuit analysis or structural engineering.
- Economics: Finding equilibrium points where supply meets demand.
- Physics: Determining where two moving objects meet based on their equations of motion.
- Computer Graphics: Calculating intersections of lines or curves.

Being proficient with substitution enhances problem-solving skills across these disciplines.

Summary and Final Thoughts

- The substitution method is

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