Solve The System Of Linear Equations Using The Gauss-Jordan Elimination Method. 2x + 4y - 6 = X + 2y

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Understanding how to solve systems of linear equations is fundamental in algebra and various applied sciences such as engineering, physics, and computer science. The Gauss-Jordan elimination method is a powerful and systematic technique used to find solutions efficiently. This article explores how to solve a specific system of equations using the Gauss-Jordan method, with detailed steps and explanations to enhance your comprehension.

Introduction to Systems of Linear Equations

Before delving into the solution process, it is essential to understand what a system of linear equations entails.

What Is a System of Linear Equations?

A system of linear equations consists of multiple equations with several variables, where each equation is linear. The goal is to find the values of the variables that satisfy all equations simultaneously.

Example:

$$
\begin{cases}
a_1x + b_1y + c_1z = d_1 \\
a_2x + b_2y + c_2z = d_2 \\
a_3x + b_3y + c_3z = d_3
\end{cases}
$$

Relevance of Solving Systems

- Determining unknown quantities in real-world problems.
- Analyzing electrical circuits.

- Optimizing functions in operations research.
- Modeling physical phenomena.

Understanding the Given Equation

The initial equation provided is:

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At first glance, it appears as a single linear equation, but the phrase suggests there is a system to be solved. Notice that the equation contains both lowercase and uppercase variables, which could imply different variables or perhaps a typo. Assuming the intended system involves variables x and y, and the goal is to convert the equation into a standard form suitable for solving.

Let's analyze and manipulate the given expression:

$$
2x + 4y - 6 = X + 2y
$$

Note: The usage of 'X' and 'x' could be a typo or denote different variables; for clarity, we'll assume 'X' is the same as 'x.' If they are different, the problem involves more variables. For this article, we'll treat 'X' as 'x.'

Rearranging all terms to one side:

$$
2x + 4y - 6 - x - 2y = 0
$$

Simplify:

$$
(2x - x) + (4y - 2y) - 6 = 0
$$

$$
x + 2y - 6 = 0
$$

Expressed as:

$$
x + 2y = 6
$$

This is a single linear equation in two variables. To have a solvable system, we need at least two equations. Therefore, perhaps the problem involves multiple equations, or the initial description is only part of the system.

Setting Up a System of Equations

Suppose the original problem involves the following system:

1. $2x + 4y - 6 = x + 2y$
2. A second equation for the system, for example, $x - y = 2$

This setup allows us to demonstrate the Gauss-Jordan elimination method effectively.

System:

$$
\begin{cases}
2x + 4y - 6 = x + 2y \\
x - y = 2
\end{cases}
$$

Rearranged into standard form:

Equation 1:
$$
2x + 4y - 6 = x + 2y \Rightarrow 2x + 4y - 6 - x - 2y = 0 \Rightarrow x + 2y - 6 = 0
$$
which simplifies to:
$$
x + 2y = 6
$$

Equation 2:

$$
x - y = 2
$$

Now, the system becomes:

$$
\begin{cases}
x + 2y = 6 \\
x - y = 2
\end{cases}
$$

Representing the System as an Augmented Matrix

To apply the Gauss-Jordan elimination method, convert the system into an augmented matrix:

$$
\begin{bmatrix}
1 & 2 & | & 6 \\
1 & -1 & | & 2
\end{bmatrix}
$$

The goal is to manipulate this matrix to reduced row-echelon form, where the solutions become evident.

Applying Gauss-Jordan Elimination

Gauss-Jordan elimination is a method of systematically reducing a matrix to find solutions for the variables. The steps involve:

1. Making the leading coefficient (pivot) of each row equal to 1.
2. Eliminating other entries in the pivot's column.
3. Continuing until the matrix is in reduced row-echelon form.

Step 1: Write the Augmented Matrix

\[
\begin{bmatrix}
1 & 2 & | & 6 \\
1 & -1 & | & 2
\end{bmatrix}
\]

Step 2: Make the first pivot 1 (already done).

Step 3: Eliminate the first variable in the second row

Subtract the first row from the second:

\[
R_2 \rightarrow R_2 - R_1
\]

Calculations:

\[
(1 - 1) = 0
\]
\[
(-1 - 2) = -3
\]
\[
(2 - 6) = -4
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 2 & | & 6 \\
0 & -3 & | & -4
\end{bmatrix}
\]

Step 4: Make the second pivot 1 by dividing the second row by -3

\[
R_2 \rightarrow \frac{R_2}{-3}
\]

Results:

\[
0 / -3 = 0
\]
\[
-3 / -3 = 1
\]
\[
-4 / -3 = \frac{4}{3}
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 2 & | & 6 \\
0 & 1 & | & \frac{4}{3}
\end{bmatrix}
\]

Step 5: Eliminate the second variable in the first row

Subtract \(2 \times R_2\) from \(R_1\):

\[
R_1 \rightarrow R_1 - 2 R_2
\]

Calculations:

\[
1 - 2 \times 0 = 1
\]
\[

$$
2 - 2 \times 1 = 0
$$

$$
6 - 2 \times \frac{4}{3} = 6 - \frac{8}{3} = \frac{18}{3} - \frac{8}{3} = \frac{10}{3}
$$

Final matrix:

$$
\begin{bmatrix}
1 & 0 & | & \frac{10}{3} \\
0 & 1 & | & \frac{4}{3}
\end{bmatrix}
$$

Interpreting the Solution

From the reduced matrix, the solutions for the variables are:

$$
x = \frac{10}{3}
$$

$$
y = \frac{4}{3}
$$

Thus, the solution to the system is:

$$
x = \frac{10}{3}, \quad y = \frac{4}{3}
$$

Summary of Gauss-Jordan Elimination Steps

Here's a quick recap of the process:

- Convert the system to an augmented matrix.
- Use row operations to make the leading coefficients 1.
- Use row operations to eliminate other entries in the pivot columns.
- Continue until the matrix is in reduced row-echelon form.
- Read off the solutions directly from the matrix.

Additional Tips for Using Gauss-Jordan Elimination

- Always check for zero pivots; if encountered, swap rows.
- Be systematic with row operations to avoid mistakes.
- Use fractions to maintain accuracy during calculations.
- For larger systems, consider using a calculator or computer algebra system.

Applying Gauss-Jordan to Real-World Problems

Gauss-Jordan elimination isn't just an academic exercise; it plays a crucial role in various real-world applications, including:

- Circuit analysis: Solving for currents and voltages.
- Economics: Equilibrium modeling using linear systems.
- Computer graphics: Transformations and rendering computations.
- Data science: Solving multiple equations in regression analysis.

Conclusion

Solving systems of linear equations using the Gauss-Jordan elimination method provides a clear, step-by-step approach to find solutions efficiently. Whether dealing with small systems like the example above or large, complex

Frequently Asked Questions

What is the main purpose of the Gauss-Jordan elimination method in solving systems of linear equations?

The Gauss-Jordan elimination method is used to systematically reduce a system of linear equations to its reduced row-echelon form, making it straightforward to find the solutions for the variables.

How do you set up the augmented matrix for the system 2x + 4y - 6 = x + 2y?

First, rewrite the equation in standard form: 2x + 4y - 6 = x + 2y becomes 2x + 4y - 6 - x - 2y = 0, which simplifies to (2x - x) + (4y - 2y) = 6, or x + 2y = 6. The augmented matrix is [[1, 2, 6]].

How do you convert the given equation into a standard form suitable for Gauss-Jordan elimination?

Rearrange the equation to bring all variables to one side and constants to the other side: x + 2y = 6. This standard form allows forming the coefficient matrix and applying Gauss-Jordan elimination.

Can you demonstrate the steps to solve x + 2y = 6 using Gauss-Jordan elimination?

Since it's a single equation with two variables, you can choose a variable to express in terms of the other. For example, y = (6 - x)/2. If more equations are provided, you would write the augmented matrix and perform row operations to find the solution.

What are common mistakes to avoid when applying Gauss-Jordan elimination to such systems?

Common mistakes include incorrect row operations, failing to maintain the equality during swaps, miscalculating the multipliers, and overlooking the need for consistent equations or infinite solutions.

How does the solution to the equation x + 2y = 6 relate to other potential solutions in a system?

This linear equation represents a line of solutions in the xy-plane. If combined with other equations, it could intersect at a point (unique solution), be parallel with no solution, or coincide (infinitely many solutions).

What are the advantages of using Gauss-Jordan elimination over other methods like substitution or elimination?

Gauss-Jordan elimination provides a systematic approach that can be easily implemented for larger systems, reduces the chance of algebraic mistakes, and directly yields the solutions in reduced row-echelon form.

Additional Resources

Solve The System Of Linear Equations Using The Gauss-Jordan Elimination Method

Linear algebra is a fundamental branch of mathematics that deals with vectors, matrices, and systems of linear equations. Among the various methods used to solve systems of linear equations, the Gauss-Jordan elimination method stands out for its systematic approach and efficiency in obtaining solutions directly. In this comprehensive review, we will explore the Gauss-Jordan elimination method in depth, apply it to the specific system of equations provided, and discuss its significance, procedure, advantages, and practical applications.

Understanding the System of Linear Equations

Before delving into the solution techniques, it’s crucial to understand the system of equations we are dealing with. The given problem is:

> $2x + 4y - 6 = X + 2y$

This appears to be a single equation, but in context, it suggests a need to interpret it as part of a system of linear equations or to reframe it into a standard form suitable for the Gauss-Jordan method.

Clarifying the Equation:

- The equation involves variables x and y, and constants.
- The right side, $X + 2y$, seems to relate to the left side, but the notation could be clarified. Typically, uppercase letters are used for constants or parameters; however, assuming 'X' is a typo or a variable, or perhaps meant to be a constant.

Assumption:

- To proceed meaningfully, we interpret the equation as:

\(2x + 4y - 6 = x + 2y \)

- This form is a common approach, where the goal is to solve for x and y.

Rearranged to Standard Form:

\[
2x + 4y - 6 = x + 2y
\]
\[
2x + 4y - 6 - x - 2y = 0
\]
\[
(2x - x) + (4y - 2y) - 6 = 0
\]
\[
x + 2y - 6 = 0
\]

This simplifies to:

\[
x + 2y = 6
\]

This is a single linear equation, which can be viewed as a line in the xy-plane. To apply the Gauss-Jordan elimination method, normally, we need a system with at least two equations.

Adding a Second Equation:

Suppose we introduce a second equation to form a system, as in most practical scenarios. For illustrative purposes, assume the second equation is:

\[
x - y = 2
\]

Now, the system becomes:

\[
\begin{cases}
x + 2y = 6 \\
x - y = 2

\end{cases}
\]

This system can be represented in matrix form as:

\[
\begin{bmatrix}
1 & 2 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
6 \\
2
\end{bmatrix}
\]

Gauss-Jordan Elimination Method: An Overview

What is Gauss-Jordan Elimination?

Gauss-Jordan elimination is a systematic algorithm used to solve systems of linear equations. It is an extension of Gaussian elimination, with the key difference being that Gauss-Jordan reduces the augmented matrix to reduced row echelon form (RREF), thereby directly obtaining the solutions without back substitution.

Key Features:

- Converts the matrix into a form where solutions are directly readable.
- Works efficiently for systems with a unique solution, infinitely many solutions, or no solution.
- Uses elementary row operations: row swapping, scaling rows, and adding multiples of one row to another.

Comparison with Gaussian Elimination:

- Gaussian elimination reduces the matrix to upper triangular form, then back-substitutes to find solutions.

- Gauss-Jordan goes further to produce a diagonal or identity matrix, eliminating the need for back-substitution.

Step-by-Step Procedure for Gauss-Jordan Elimination

Applying the Gauss-Jordan method involves several systematic steps:

Step 1: Write the Augmented Matrix

Construct the augmented matrix representing the system:

\[
\left[
\begin{array}{cc|c}
a_{11} & a_{12} & b_1 \\
a_{21} & a_{22} & b_2
\end{array}
\right]
\]

For our example:

\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
1 & -1 & 2
\end{array}
\right]
\]

Step 2: Make the Leading Coefficient of the First Row a 1

Identify the first pivot position (top-left element). It is already 1, so no action needed here.

Step 3: Use Row Operations to Zero Out Elements Below the Pivot

Subtract Row 1 from Row 2:

\[
R_2 \rightarrow R_2 - R_1
\]
\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
0 & -3 & -4
\end{array}
\right]
\]

Step 4: Make the Pivot of the Second Row a 1

Divide Row 2 by -3:

\[
R_2 \rightarrow \frac{R_2}{-3}
\]
\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
0 & 1 & \frac{4}{3}
\end{array}
\right]
\]

Step 5: Zero Out Elements Above the Second Pivot

Eliminate the y-term in Row 1:

\[
R_1 \rightarrow R_1 - 2 \times R_2
\]
\[
R_1: 1, 2, 6
\]
\[
2 \times R_2: 0, 2, \frac{8}{3}
\]
\[
R_1: 1, 2 - 2, 6 - \frac{8}{3} = 1, 0, \frac{10}{3}

\]

Updated matrix:

\[
\left[
\begin{array}{cc|c}
1 & 0 & \frac{10}{3} \\
0 & 1 & \frac{4}{3}
\end{array}
\right]
\]

Step 6: Read Off the Solutions

From the matrix:

\[
x = \frac{10}{3}
\]
\[
y = \frac{4}{3}
\]

Applying Gauss-Jordan to the Original Equation System

Let’s revisit the original problem, assuming the second equation is:

> x - y = 2

Our goal is to demonstrate the method thoroughly.

Step 1: Form the Augmented Matrix

\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
1 & -1 & 2

\end{array}
\right]
\]

Step 2: Make the First Pivot 1

Already 1; proceed.

Step 3: Zero Out the Element below the Pivot

Subtract Row 1 from Row 2:

\[
R_2 \rightarrow R_2 - R_1
\]

\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
0 & -3 & -4
\end{array}
\right]
\]

Step 4: Normalize the Second Row

Divide Row 2 by -3:

\[
R_2 \rightarrow R_2 / -3
\]
\[
\left[
\begin{array}{cc|c}
1 & 2 & 6 \\
0 & 1 & \frac{4}{3}
\end{array}
\right]
\]

Step 5: Eliminate the y-term in Row 1

Subtract 2 times Row 2 from Row 1:

\[
R_1 \rightarrow R_1 - 2 \times R_2
\]

\[
\text{Row 1}: 1, 2, 6
\]
\[
2 \times R_2: 0, 2, \frac{8}{3}
\]
\[
R_1: 1, 0, 6 - \frac{8}{3} = \frac{10}{3}
\]

Step 6: Final Solution

\[
x = \frac{10}{3}
\]
\[
y = \frac{4}{3}
\]

This indicates that the system has a unique solution, which can be interpreted geometrically as the intersection point of the two lines.

Advantages of Gauss-Jordan Elimination

The Gauss-Jordan elimination method offers several significant benefits:

- Direct Solution Retrieval: By reducing the matrix to reduced row echelon form, solutions are directly read off, eliminating the need for back substitution.
- Versatility: Capable of solving systems with unique solutions, infinitely many solutions, or no solutions.
- Applicability to Larger Systems: Efficiently handles larger systems involving multiple variables and equations.
- Foundation for Matrix Inversion: The method is

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