Solve Using The Quadratic Formula. Show All Work. Write Each Solution In Simplest Form. No Decimals.

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Understanding how to solve quadratic equations is a fundamental skill in algebra. The quadratic formula offers a systematic method to find solutions for any quadratic equation of the form $ax^2 + bx + c = 0$. This article provides a comprehensive guide on how to apply the quadratic formula step-by-step, ensuring clarity and accuracy. We emphasize showing all work, writing each solution in its simplest form, and avoiding decimals to maintain exactness. Whether you're a student preparing for exams or someone seeking to strengthen your algebra skills, this guide will help you master solving quadratic equations efficiently.

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation in a single variable x, generally written as:

$ax^2 + bx + c = 0$

where:
- a, b, and c are coefficients with $a \neq 0$
- x represents the variable or unknown

Quadratic equations can be solved through various methods, including factoring, completing the square, graphing, and the quadratic formula. When the coefficients are such that factoring is difficult or impossible, the quadratic formula is the most reliable method.

The Quadratic Formula: An Overview

The quadratic formula provides a direct way to find the roots (solutions) of any quadratic equation:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

Key components:
- Discriminant: $D = b^2 - 4ac$
- The ± sign indicates two possible solutions

The nature of the solutions depends on the discriminant:
- $D > 0$: Two real solutions
- $D = 0$: One real solution (a repeated root)
- $D < 0$: No real solutions (complex roots)

This formula is derived from completing the square and is a universal solution method applicable to all quadratic equations.

Step-by-Step Guide to Solving Quadratic Equations Using the Formula

Below is a detailed process to solve quadratic equations with the quadratic formula, emphasizing clarity and showing all work.

Step 1: Write the Equation in Standard Form

Ensure the quadratic equation is written as:

$ax^2 + bx + c = 0$

If necessary, rearrange the equation to match this form.

Step 2: Identify Coefficients a, b, and c

Extract the coefficients directly:
- a: coefficient of x^2
- b: coefficient of x
- c: constant term

Step 3: Calculate the Discriminant (D)

Use the formula:

$D = b^2 - 4ac$

Calculate this value carefully, keeping in mind to avoid decimals.

Step 4: Plug Values into the Quadratic Formula

Insert a, b, and D into:

$x = [\ -b \pm \sqrt{D}\ ] / 2a$

Remember, if $D < 0$, the solutions will be complex, involving imaginary numbers.

Step 5: Simplify the Square Root ($\sqrt{D}$)

Factor D if possible to simplify $\sqrt{D}$ into simplest radical form. This is crucial for writing solutions in exact form without decimals.

Step 6: Write the Two Solutions

Apply the ± sign to obtain both solutions:

$x_1 = [\ -b + \sqrt{D}\ ] / 2a$

$x_2 = [\ -b - \sqrt{D}\ ] / 2a$

Simplify each solution by reducing fractions and simplifying radicals.

Step 7: Present Solutions in Simplest Form

Express each root fully simplified, avoiding decimals, and write in exact radical form unless the radical simplifies to a rational number.

Practical Examples of Solving Quadratic Equations

Below are illustrative examples demonstrating each step.

Example 1: Solving a Quadratic with Real Roots

Solve: $2x^2 + 3x - 2 = 0$

Step 1: Standard form (already in standard form).

Step 2: Coefficients:
- $a = 2$
- $b = 3$
- $c = -2$

Step 3: Discriminant:
$D = b^2 - 4ac$
$D = 3^2 - 4\ 2\ (-2)$
$D = 9 - (-16)$
$D = 9 + 16$
$D = 25$

Step 4: Plug into quadratic formula:
$x = [\ -b \pm \sqrt{D}\ ] / (2a)$
$x = [\ -3 \pm \sqrt{25}\ ] / (2\ 2)$
$x = [\ -3 \pm 5\ ] / 4$

Step 5: Simplify solutions:

- For the plus sign:
$x = (\ -3 + 5\ ) / 4 = 2 / 4 = 1 / 2$

- For the minus sign:
$x = (\ -3 - 5\ ) / 4 = -8 / 4 = -2$

Final solutions:
$x = 1/2$ and $x = -2$

Example 2: Solving a Quadratic with Complex Roots

Solve: $x^2 + 4x + 5 = 0$

Step 1: Standard form.

Step 2: Coefficients:
- a = 1
- b = 4
- c = 5

Step 3: Discriminant:
$D = 4^2 - 4\ 1\ 5 = 16 - 20 = -4$

Since $D < 0$, solutions are complex.

Step 4: Compute $\sqrt{D}$:
$\sqrt{D} = \sqrt{-4} = \sqrt{4}\ \sqrt{-1} = 2i$

Step 5: Plug into formula:
$x = [-4 \pm 2i] / (2\ 1) = [-4 \pm 2i] / 2$

Step 6: Simplify:
$x = (-4 / 2) \pm (2i / 2) = -2 \pm i$

Final solutions:
$x = -2 + i$ and $x = -2 - i$

Tips for Simplifying Solutions in Radical Form

- Always factor the radicand (the number under the radical) to identify perfect squares.
- Simplify radicals by extracting perfect square factors.
- Write solutions as exact radicals, avoiding decimals unless explicitly required.
- For complex solutions, express imaginary parts clearly with i.

Common Mistakes to Avoid

- Forgetting to write the quadratic in standard form.
- Miscalculating the discriminant.
- Not simplifying radicals properly.
- Including decimals when exact radical form is possible.
- Mixing solutions with radicals and decimals, reducing clarity.

Additional Practice Problems

To reinforce your understanding, try solving these quadratic equations using the quadratic formula, following each step:

1. $3x^2 - 5x + 2 = 0$
2. $x^2 - 6x + 13 = 0$
3. $4x^2 + 4x + 1 = 0$
4. $5x^2 + 2x - 3 = 0$
5. $x^2 + 2x + 2 = 0$

Remember to always show all work, simplify radicals, and write solutions in simplest form without decimals.

Conclusion

Mastering the quadratic formula is essential for solving quadratic equations accurately and efficiently. By carefully identifying coefficients, calculating the discriminant, simplifying radicals, and writing solutions in their simplest form, students and learners can confidently tackle a wide range of quadratic problems. Practice diligently with various equations to become proficient, and always double-check your work for accuracy. With consistent effort, applying the quadratic formula will become a straightforward and reliable method in your algebra toolkit.

SEO Keywords for Optimization

- Solve quadratic equations using quadratic formula
- Step-by-step quadratic formula solutions
- Write solutions in simplest form
- No decimals in quadratic solutions
- Discriminant calculation in quadratic equations
- Simplify radicals in quadratic solutions
- Quadratic formula practice problems
- How to solve quadratic equations exactly
- Algebra quadratic formula tutorial
- Solving complex roots in quadratics

By incorporating these keywords naturally throughout your content, this article aims to rank well in search engine results related to solving quadratic equations with the quadratic formula.

Frequently Asked Questions

Solve the quadratic equation 2x^2 - 4x - 6 = 0 using the quadratic formula. Show all work and write the solutions in simplest form, without decimals.

Given quadratic: $2x^2 - 4x - 6 = 0$

$a = 2$, $b = -4$, $c = -6$

Discriminant: $D = b^2 - 4ac = (-4)^2 - 4(2)(-6) = 16 + 48 = 64$

$x = [-b \pm \sqrt{D}] / (2a) = [4 \pm \sqrt{64}] / (4)$

$= [4 \pm 8] / 4$

Solution 1: $(4 + 8)/4 = 12/4 = 3$

Solution 2: $(4 - 8)/4 = -4/4 = -1$

Final solutions: $x = 3, -1$

Use the quadratic formula to solve x^2 + 5x + 6 = 0. Show all steps and write solutions in simplest form, no decimals.

Given quadratic: $x^2 + 5x + 6 = 0$

$a=1$, $b=5$, $c=6$

Discriminant: $D = 5^2 - 4(1)(6) = 25 - 24 = 1$

$x = [-5 \pm \sqrt{1}] / (2) = [-5 \pm 1] / 2$

Solution 1: $(-5 + 1)/2 = -4/2 = -2$

Solution 2: (-5 - 1)/2 = -6/2 = -3
Solutions: x = -2, -3

Solve the quadratic equation 3x^2 + 2x - 1 = 0 using the quadratic formula. Show all work and write solutions in simplest form, avoiding decimals.

Given quadratic: 3x^2 + 2x - 1 = 0
a=3, b=2, c=-1
Discriminant: D = 2^2 - 4(3)(-1) = 4 + 12 = 16
x = [-2 ± √16] / (6) = [-2 ± 4] / 6
Solution 1: (-2 + 4)/6 = 2/6 = 1/3
Solution 2: (-2 - 4)/6 = -6/6 = -1
Final solutions: x = 1/3, -1

Find the solutions of 4x^2 - 7x + 3 = 0 using the quadratic formula. Show all work and write solutions in simplest form, no decimals.

Given quadratic: 4x^2 - 7x + 3 = 0
a=4, b=-7, c=3
Discriminant: D = (-7)^2 - 4(4)(3) = 49 - 48 = 1
x = [7 ± √1] / (8) = [7 ± 1] / 8
Solution 1: (7 + 1)/8 = 8/8 = 1
Solution 2: (7 - 1)/8 = 6/8 = 3/4
Solutions: x = 1, 3/4

Solve 5x^2 + 4x + 1 = 0 using the quadratic formula. Show all steps and write the solutions in simplest form, no decimals.

Given quadratic: 5x^2 + 4x + 1 = 0
a=5, b=4, c=1
Discriminant: D = 4^2 - 4(5)(1) = 16 - 20 = -4
Since D is negative, solutions involve imaginary numbers:
x = [-4 ± √(-4)] / (10) = [-4 ± 2i] / 10 = [-2/5] ± (i/5)
Final solutions: x = -2/5 + i/5, -2/5 - i/5

Solve the quadratic equation x^2 - 6x + 9 = 0 using the quadratic formula. Show all work and write solutions in simplest form, without decimals.

Given quadratic: x^2 - 6x + 9 = 0
a=1, b=-6, c=9

Discriminant: $D = (-6)^2 - 4(1)(9) = 36 - 36 = 0$
$x = [6 \pm \sqrt{0}] / 2 = 6/2 = 3$
Since discriminant is zero, there is one real repeated root:
Solution: $x = 3$

Additional Resources

Solve Using The Quadratic Formula. Show All Work. Write Each Solution In Simplest Form. No Decimals.

The quadratic formula is an essential tool in algebra, providing a systematic way to solve quadratic equations of the form $ax^2 + bx + c = 0$. Mastering this method not only enhances problem-solving skills but also deepens understanding of polynomial equations and their roots. This article offers a comprehensive review of solving quadratic equations using the quadratic formula, emphasizing clarity, thoroughness, and adherence to instructions such as showing all work, simplifying solutions, and avoiding decimals. Whether you're a student preparing for exams or a math enthusiast seeking to strengthen foundational skills, understanding the nuances of this method is invaluable.

Understanding the Quadratic Formula

What Is the Quadratic Formula?

The quadratic formula is a direct solution method for quadratic equations. It is derived from the process of completing the square and is applicable to any quadratic equation in standard form:

$ax^2 + bx + c = 0$

where a, b, and c are coefficients with $a \neq 0$.

The formula itself is:

$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$

This formula calculates the roots (solutions) of the quadratic equation, which may be real or complex depending on the discriminant ($b^2 - 4ac$).

Key Features of the Quadratic Formula

- Universal applicability: Valid for all quadratic equations, regardless of coefficients.
- Explicit solutions: Provides exact solutions without approximation, especially when radicals are involved.
- Discriminant insight: The value under the square root (discriminant) indicates the nature of roots—real and distinct, real and repeated, or complex conjugates.

Step-by-Step Approach to Solving Quadratics Using the Formula

1. Write the Equation in Standard Form

Ensure the quadratic equation is in the form $ax^2 + bx + c = 0$. If not, rearrange terms accordingly.

2. Identify Coefficients a, b, and c

Extract the numerical coefficients from the quadratic. For example, in $3x^2 - 5x + 2 = 0$, $a=3$, $b=-5$, $c=2$.

3. Calculate the Discriminant (D)

Find $D = b^2 - 4ac$. This value determines the nature of the roots and guides the next steps.

4. Analyze the Discriminant

- If $D > 0$: Two real and distinct solutions.
- If $D = 0$: One real repeated solution.
- If $D < 0$: Two complex conjugate solutions.

5. Apply the Quadratic Formula

Substitute a, b, and c into the formula:

$x = (-b \pm \sqrt{D}) / (2a)$

Perform each step carefully, showing all work.

6. Simplify the Radical (If Possible)

Express the radical in simplest radical form, factoring out perfect squares from the radical if applicable.

7. Write the Solutions in Simplest Form

Ensure the solutions are fully simplified algebraically, avoiding decimals, and express radical parts clearly.

Detailed Example: Solving a Quadratic Equation

Suppose we want to solve:

$2x^2 - 4x - 6 = 0$

Step 1: Write in standard form (already done).

Step 2: Identify coefficients:

$a = 2, b = -4, c = -6$

Step 3: Calculate the discriminant:

$D = (-4)^2 - 4(2)(-6) = 16 - (-48) = 16 + 48 = 64$

Step 4: Analyze discriminant:

$D = 64 > 0$, so two real solutions.

Step 5: Apply the quadratic formula:

$x = (-(-4) \pm \sqrt{64}) / (22)$
$x = (4 \pm \sqrt{64}) / 4$

Step 6: Simplify radicals:

$\sqrt{64} = 8$

Step 7: Write solutions:

$x = (4 + 8) / 4 = 12 / 4 = 3$

$x = (4 - 8) / 4 = -4 / 4 = -1$

Final solutions: $x = 3$, $x = -1$

All work is shown, solutions are in simplest form, and no decimals are involved.

Handling Special Cases and Common Pitfalls

When the Discriminant Is Zero

The quadratic has exactly one real root, repeated twice. The solution simplifies to:

$x = -b / (2a)$

Show this explicitly to demonstrate understanding.

When the Discriminant Is Negative

Solutions involve complex numbers:

$x = (-b \pm \sqrt{(-D)}i) / (2a)$

Expressed as:

$x = (-b / 2a) \pm (\sqrt{|D|} / 2a)i$

Ensure radicals are simplified, and complex solutions are written properly.

Common Mistakes to Avoid

- Forgetting to include the ± sign, resulting in only one solution.
- Failing to simplify radicals fully.
- Mixing decimal approximations when instructed to avoid decimals.
- Dividing by zero if $a = 0$ (in which case, the equation isn't quadratic).

Pros and Cons of Using the Quadratic Formula

Pros:

- Universality: Works for all quadratic equations.
- Clarity: Provides a clear, step-by-step solution process.
- Exact Solutions: Yields precise answers in radical form without approximation.
- Insightful: The discriminant offers information about the roots' nature.

Cons:

- Algebraic Complexity: Involves radical expressions that may be cumbersome to simplify.
- Time-consuming: May be longer than factoring for simple quadratics.
- Limited for non-quadratic equations: Not applicable outside quadratic form.
- Potential for errors: Mistakes in algebraic manipulation can occur if steps are not carefully followed.

Features of Solving Quadratic Equations via the Formula

- Show All Work: Demonstrates understanding and allows for easier error checking.
- Simplify Each Solution: Ensures solutions are in their simplest radical form.
- No Decimals: Adheres strictly to exact solutions, often necessary in higher-level math.
- Systematic Approach: Can be applied universally to any quadratic equation.

Conclusion

Mastering the quadratic formula is a fundamental skill in algebra, providing a reliable method to solve any quadratic equation. Carefully following each step—identifying coefficients, calculating the discriminant, applying the formula, simplifying radicals, and writing solutions in simplest form—ensures accuracy and clarity. While the process may seem algebraically intensive, especially when radicals are involved, the discipline of showing all work and avoiding decimals fosters a deeper understanding of quadratic equations and their roots. Whether dealing with real or complex solutions, the quadratic formula remains an indispensable tool, and proficiency in its application opens the door to more advanced mathematical concepts and problem-solving techniques.

Remember: Always check your solutions by substituting back into the original equation to verify correctness, and practice with varied problems to reinforce understanding and efficiency.

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