Solve Y'' 8y' + 16y = 0, Y(0) = 3, Y'(0) = 16 Y(t) = Upload Your Work For This Problem To Gradescope

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Solving second-order differential equations is a fundamental skill in mathematics, engineering, and physics. One such problem that often appears in coursework and exams involves solving the differential equation Y'' + 8y' + 16y = 0 with initial conditions Y(0) = 3 and Y'(0) = 16. This article provides a comprehensive step-by-step guide to solving this problem, including methods to find the solution, interpret the results, and submit your work on Gradescope. Whether you're a student preparing for exams or a professional brushing up on differential equations, this detailed explanation will help clarify the process.

Understanding the Differential Equation

The Standard Form

The given differential equation is:

```plaintext
Y'' + 8y' + 16y = 0
```

This is a homogeneous linear second-order differential equation with constant coefficients. The general approach involves solving the characteristic equation associated with it.

Initial Conditions

The initial conditions specify the value of the function and its derivative at t=0:

```plaintext
Y(0) = 3
Y'(0) = 16
```

These conditions are essential for determining the particular solution once the general solution to the differential equation is found.

Step-by-Step Solution Method

1. Find the Characteristic Equation

To solve the differential equation, assume a solution of the form:

```plaintext
Y(t) = e^{rt}
```

Substituting into the differential equation yields the characteristic equation:

```plaintext
r^2 + 8r + 16 = 0
```

This quadratic equation determines the nature of the solutions.

2. Solve the Characteristic Equation

Solve for r:

```plaintext
r^2 + 8r + 16 = 0
```

Using quadratic formula:

```plaintext
r = [-b ± sqrt(b^2 - 4ac)] / (2a)
```

where a=1, b=8, c=16:

```plaintext
r = [-8 ± sqrt(64 - 64)] / 2
```

Simplifies to:

```plaintext
r = [-8 ± 0] / 2
```

```

Thus, the repeated root:

```plaintext
r = -4
```

Since the discriminant is zero, the roots are repeated, indicating a solution involving exponential functions multiplied by polynomials.

3. Write the General Solution

For a repeated root r = -4, the general solution is:

```plaintext
Y(t) = (A + Bt) e^{-4t}
```

where A and B are constants to be determined by initial conditions.

Applying Initial Conditions

4. Find Derivative Y'(t)

Differentiate Y(t):

```plaintext
Y(t) = (A + Bt) e^{-4t}
```

Applying the product rule:

```plaintext
Y'(t) = d/dt [(A + Bt) e^{-4t}] = (A + Bt)(-4 e^{-4t}) + B e^{-4t}
```

Simplify:

```plaintext
Y'(t) = e^{-4t} [-4(A + Bt) + B] = e^{-4t} [-4A - 4Bt + B]
```
```

5. Apply initial conditions

At t=0:

```plaintext
Y(0) = (A + B0) e^{0} = A = 3
```

So,

```plaintext
A = 3
```

Next, for Y'(0):

```plaintext
Y'(0) = e^{0} [-4A - 4B0 + B] = -4A + B
```

Given that Y'(0) = 16:

```plaintext
-4A + B = 16
```

Substitute A=3:

```plaintext
-43 + B = 16
-12 + B = 16
B = 28
```

Final Solution

Putting it all together, the particular solution to the differential equation with the initial conditions is:

```plaintext
Y(t) = (3 + 28 t) e^{-4t}
```

This function satisfies both the differential equation and the initial conditions.

How to Upload Your Work to Gradescope

Preparing Your Submission

Once you've solved the differential equation and verified your work, the next step is to submit it to Gradescope. Here are some tips:

- Write your solution clearly, showing all steps, including the characteristic equation, roots, general solution, and how initial conditions are applied.
- Use a computer algebra system or handwriting and scan your work if needed.
- Ensure your final answer is highlighted or boxed for clarity.

Uploading Your Work

Follow these steps:

1. Log into your Gradescope account.
2. Select the relevant assignment or problem set.
3. Click on "Upload Work" or "Submit" as instructed.
4. Upload your document (PDF, image, or other accepted formats).
5. Confirm that your work is visible and legible before final submission.

Tips for a Successful Submission

- Double-check your work for accuracy before uploading.
- Include your name and relevant details on the document.
- Use clear handwriting or high-quality scans to ensure graders can read your work.
- Follow any specific instructions provided by your instructor regarding submission format or method.

Summary and Key Takeaways

- The differential equation Y'' + 8y' + 16y = 0 has a characteristic root r = -4 with multiplicity 2.
- The general solution for this differential equation is Y(t) = (A + Bt) e^{-4t}.
- Using initial conditions Y(0) = 3 and Y'(0) = 16, the specific solution is Y(t) = (3 + 28 t) e^{-4t}.
- Properly documenting your solution process and uploading your work to Gradescope ensures clarity and successful grading.

By following these steps, you can confidently solve second-order differential equations with repeated roots and accurately submit your work through Gradescope. Whether for homework, exams, or professional projects, mastering these techniques is essential for success in differential equations analysis.

Remember: Practice makes perfect. Keep practicing similar problems, and you'll become proficient in solving second-order linear differential equations with constant coefficients.

Frequently Asked Questions

What is the differential equation given in the problem?

The differential equation is 8y'' + 16y = 0.

What are the initial conditions specified for the differential equation?

The initial conditions are Y(0) = 3 and Y'(0) = 16.

How do you simplify the differential equation 8y'' + 16y = 0?

Divide both sides by 8 to get y'' + 2y = 0.

What is the characteristic equation associated with y'' + 2y = 0?

The characteristic equation is r^2 + 2 = 0.

What are the roots of the characteristic equation $r^2 + 2 = 0$?

The roots are $r = \pm i\sqrt{2}$.

What is the general solution to the differential equation based on the roots?

The general solution is $Y(t) = C1 \cos(\sqrt{2}\, t) + C2 \sin(\sqrt{2}\, t)$.

How do you use the initial conditions to find constants C1 and C2?

Substitute $t=0$ into the general solution and its derivative to solve for C1 and C2 using $Y(0)=3$ and $Y'(0)=16$.

What are the values of C1 and C2 after applying the initial conditions?

$C1 = 3$ and $C2 = 8\sqrt{2}$.

What is the final solution Y(t) for the differential equation?

$Y(t) = 3 \cos(\sqrt{2}\, t) + 8\sqrt{2} \sin(\sqrt{2}\, t)$.

Additional Resources

Differential Equations

Solving the Second-Order Homogeneous Differential Equation: An In-Depth Analysis of \(Y'' - 8Y' + 16Y = 0 \) with Initial Conditions \(Y(0) = 3 \), \(Y'(0) = 16 \)

Introduction

In the realm of differential equations, second-order linear homogeneous equations hold a central position, especially due to their wide array of applications across physics, engineering, and applied mathematics. They often describe phenomena such as oscillations, wave propagation, and stability analysis. This article aims to provide a comprehensive, step-by-step solution to the differential equation:

\[

$$
Y'' - 8Y' + 16Y = 0
$$

with initial conditions:

$$
Y(0) = 3,\quad Y'(0) = 16
$$

We will walk through the process, from identifying the characteristic equation to applying initial conditions, ensuring an understanding that is both thorough and accessible.

Understanding the Differential Equation

The equation

$$
Y'' - 8Y' + 16Y = 0
$$

is a second-order linear homogeneous differential equation with constant coefficients. Such equations are characterized by their characteristic equations, which translate the differential equation into an algebraic polynomial.

The general form:

$$
aY'' + bY' + cY = 0
$$

where a, b, c are constants. In our case:

- $a = 1$
- $b = -8$
- $c = 16$

Step 1: Find the Characteristic Equation

To solve the differential equation, we start with the characteristic equation:

$$
r^2 + br + c = 0
$$

Plugging in the coefficients:

$$
r^2 - 8r + 16 = 0
$$

This quadratic determines the nature of the solutions.

Step 2: Solve the Characteristic Equation

The quadratic:

$$
r^2 - 8r + 16 = 0
$$

can be factored or solved using the quadratic formula:

$$
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Calculating the discriminant:

$$
\Delta = (-8)^2 - 4 \times 1 \times 16 = 64 - 64 = 0
$$

Since the discriminant is zero, the roots are real and repeated.

Applying the quadratic formula:

$$
r = \frac{8 \pm \sqrt{0}}{2} = \frac{8}{2} = 4
$$

The repeated root:

$$
r = 4
$$

Step 3: Write the General Solution

For repeated roots r, the general solution to the differential equation is:

$$
Y(t) = (A + Bt) e^{rt}
$$

where A and B are constants determined by initial conditions.

Substituting $r = 4$:

$$
Y(t) = (A + Bt) e^{4t}
$$

Step 4: Apply Initial Conditions

Initial conditions enable us to find specific values for A and B.

Given:

$$
Y(0) = 3
$$

$$
Y'(0) = 16
$$

Step 5: Compute $Y(t)$ and $Y'(t)$

Calculate $Y(t)$:

$$
Y(t) = (A + Bt) e^{4t}
$$

At $t = 0$:

$$
Y(0) = (A + B \times 0) e^{0} = A \times 1 = A
$$

So:

$$
A = 3
$$

Calculate $Y'(t)$:

Using the product rule:

$$
Y'(t) = \frac{d}{dt}[(A + Bt) e^{4t}] = (A + Bt) \times 4e^{4t} + B e^{4t}
$$

Simplify:

$$
Y'(t) = 4(A + Bt) e^{4t} + B e^{4t} = e^{4t} [4A + 4Bt + B]
$$

At $t = 0$:

$$
Y'(0) = e^{0} [4A + 4B \times 0 + B] = 1 \times (4A + B) = 4A + B
$$

Recall $A = 3$, thus:

$$
16 = 4 \times 3 + B = 12 + B
$$

Solve for B:

$$
B = 16 - 12 = 4
$$

Final Solution

Combining all the components, the specific solution to the differential equation, given the initial conditions, is:

$$
\boxed{
Y(t) = (3 + 4t) e^{4t}
}
$$

Analysis and Interpretation

This solution describes an exponential growth modulated by a linear term in t. The repeated root $r=4$ indicates that the solution exhibits exponential behavior combined with polynomial factors, typical of repeated roots in characteristic equations.

The constants $A=3$ and $B=4$ are directly linked to the initial displacement and initial velocity (or rate of change), respectively, anchoring the solution to the initial physical or mathematical conditions.

Additional Considerations

- Stability: Since e^{4t} grows exponentially, the solution indicates an unstable system if interpreted physically, such as an unbounded oscillation or growth scenario.

- Homogeneity: The absence of a non-homogeneous term simplifies the solution process; if a forcing function were present, the approach would involve particular solutions.

- Extension: The methodology extends to other second-order linear differential equations, especially those with constant coefficients and repeated roots.

Conclusion

Solving the differential equation $Y'' - 8Y' + 16Y = 0$ with initial conditions $Y(0) = 3$ and $Y'(0) = 16$ involves a systematic approach: identify the characteristic roots, construct the general solution, and then apply initial conditions to find specific constants. The resulting function:

$$
\boxed{
Y(t) = (3 + 4t) e^{4t}
}
$$

accurately models the behavior dictated by the differential equation, illustrating the power and elegance of classical analytical methods in differential equations.

Note for Uploading to Gradescope

Ensure all steps, calculations, and reasoning are clearly documented and neatly organized before uploading your work to Gradescope. This not only verifies your understanding but also facilitates easier grading and feedback.

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End of Article
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