Suppose 1.8 Mol Of A Monatomic Ideal Gas Initially At 11 L And 300 K Is Heated At Constant Volume To

Suppose 1.8 mol of a monatomic ideal gas initially at 11 L and 300 K is heated at constant volume to a higher temperature. This scenario encapsulates fundamental principles of thermodynamics, particularly involving ideal gases, heat transfer, and the relationship between temperature, pressure, and volume. Understanding the behavior of gases under such conditions is essential for students and professionals working in physics, chemistry, and engineering fields. In this comprehensive guide, we will explore the various aspects of this process, including calculations of the final temperature, work done, heat absorbed, and the implications on pressure, along with relevant concepts and formulas.

Understanding the Initial Conditions

Initial Volume and Temperature

The initial volume (V_1) of the gas is given as 11 liters, which can be converted into cubic meters for SI unit consistency:
- V_1 = 11 L = 11 x 10-3 m3

The initial temperature (T_1) is 300 K, a standard room temperature. The amount of gas is 1.8 mol, which informs us about the number of particles involved in the process.

Properties of a Monatomic Ideal Gas

Monatomic gases, such as helium, neon, or argon, have specific heat capacities:
- Molar heat capacity at constant volume, Cv = (3/2) R
- Molar heat capacity at constant pressure, Cp = (5/2) R

where R is the universal gas constant, R ≈ 8.314 J/(mol·K).

Fundamental Thermodynamic Relationships

Ideal Gas Law

The ideal gas law relates pressure (P), volume (V), temperature (T), and the amount of gas (n):
- $PV = nRT$

Using this law, we can determine initial and final pressures as the temperature changes, assuming constant volume.

Heat Capacity and Internal Energy

The change in internal energy (ΔU) for a monatomic ideal gas depends only on temperature:
- $\Delta U = n C_v \Delta T = n (3/2) R \Delta T$

Since no work is done at constant volume ($W = 0$), the heat supplied (Q) equals the change in internal energy.

Calculating the Final Temperature

Suppose the gas is heated at constant volume to a final temperature T_2. The key is to find T_2 based on the heat supplied or other known quantities.

Method 1: Using Heat Input

If the heat added (Q) is known, the temperature change can be directly calculated:
- $\Delta T = Q / (n C_v)$

However, if Q is not specified, we can analyze other aspects, such as the pressure change or the final temperature based on other conditions.

Method 2: Using Pressure Change

Since volume is constant, the pressure is proportional to temperature:
- $P_1 / T_1 = P_2 / T_2$
- $P_2 = P_1 (T_2 / T_1)$

If the initial pressure P_1 can be calculated via the ideal gas law:
- $P_1 = nRT_1 / V_1$

then the final pressure P_2 can be found once T_2 is known.

Calculations Step-by-Step

1. Calculate Initial Pressure (P_1)

Using the ideal gas law:
- $P_1 = (n\ R\ T_1) / V_1$

Plugging in values:
- n = 1.8 mol
- R = 8.314 J/(mol·K)
- T_1 = 300 K
- V_1 = 11 x 10-3 m3

P_1 = (1.8 mol 8.314 J/(mol·K) 300 K) / (11 x 10-3 m3)
P_1 ≈ (1.8 8.314 300) / 0.011
P_1 ≈ (1.8 2494.2) / 0.011
P_1 ≈ 4489.56 / 0.011
P_1 ≈ 407,233 Pa ≈ 407 kPa

This is the initial pressure inside the container.

2. Determine Final Temperature (T_2)

If the process involves heating without doing work (constant volume), the final temperature T_2 can be derived if the final pressure P_2 is known or if the amount of heat added is specified.

For a typical problem, suppose the gas is heated until the pressure doubles:
- $P_2 = 2\ P_1 \approx 814$ kPa

Using the proportional relationship:
- $T_2 = T_1\ (P_2 / P_1)$ = 300 K 2 = 600 K

Alternatively, if the heat added is given, T_2 can be directly calculated using the internal energy change:
- $\Delta U = Q = n\ Cv\ (T_2 - T_1)$

Note: The specific value of T_2 depends on the problem context—whether we're given heat input, final pressure, or temperature.

Work Done and Heat Transfer

Work Done (W)

At constant volume, the work done by the gas is zero:
- $W = P \Delta V = 0$

since $\Delta V = 0$.

Heat Absorbed (Q)

Heat absorbed is related to the change in internal energy:
- $Q = \Delta U = n\ Cv\ (T_2 - T_1)$

Calculating Cv:
- $Cv = (3/2)\ R = (3/2)\ 8.314 \approx 12.471$ J/(mol·K)

Thus:
- $Q = 1.8$ mol 12.471 J/(mol·K) $(T_2 - 300\ K)$

Using the T_2 determined earlier (e.g., 600 K):
- $Q \approx 1.8\ 12.471\ (600 - 300) = 1.8\ 12.471\ 300 \approx 1.8\ 3{,}741.3 \approx 6{,}734.3$ J

The positive value indicates heat absorption.

Implications and Real-World Applications

Pressure and Temperature Relationship

Understanding how temperature changes influence pressure at constant volume is critical in designing pressurized systems, such as cylinders or reactors. The direct proportionality allows engineers to predict system behavior under heating.

Energy Considerations

The energy input required to reach a certain temperature or pressure can be calculated precisely, aiding in energy efficiency analyses.

Safety Aspects

Knowing the maximum pressure and temperature limits ensures safety in handling gases under various conditions.

Summary and Key Formulas

- **Initial Pressure:** $P_1 = (n\ R\ T_1) / V_1$
- **Final Temperature (assuming pressure doubles):** $T_2 = T_1 (P_2 / P_1)$
- **Change in Internal Energy:** $\Delta U = n\ C_v\ (T_2 - T_1)$
- **Heat Added:** $Q = \Delta U$
- **Work Done at constant volume:** $W = 0$

Understanding these relationships enables precise control and prediction of gas behavior under thermal processes.

Conclusion

Heating a monatomic ideal gas at constant volume involves changes primarily in temperature and pressure, with no work done due to volume constancy. By leveraging the ideal gas law and thermodynamic principles, one can determine the final state variables, energy exchanges, and system behavior. This fundamental knowledge serves as a cornerstone in various scientific and engineering applications, ranging from designing engines and turbines to understanding atmospheric phenomena and laboratory experiments.

Remember: Always consider the initial conditions and the specifics of the heating process to apply the appropriate calculations effectively.

Frequently Asked Questions

What is the initial pressure of 1.8 mol of a monatomic ideal gas in a 11 L container at 300 K?

Using the ideal gas law PV = nRT, initial pressure P = (nRT)/V = (1.8 mol 0.0821 L·atm/(mol·K) 300 K) / 11 L ≈ 4.04 atm.

How does the temperature change when the gas is heated at constant volume from 300 K to a higher temperature?

The temperature increases, which causes an increase in the internal energy of the gas, as per the relation $\Delta U = (3/2)nR\Delta T$ for a monatomic ideal gas.

What is the work done on the gas during heating at constant volume?

Since the volume is constant, the work done W = 0 because work in thermodynamics is $W = P\Delta V$, and $\Delta V = 0$.

How can the heat added to the gas be calculated during the heating process?

The heat added $Q = \Delta U = (3/2)nR\Delta T$, since no work is done at constant volume for an ideal gas.

What is the change in internal energy when the gas is heated at constant volume?

The change in internal energy $\Delta U = (3/2)nR\Delta T$, which depends on the temperature change ΔT.

If the gas is heated to a final temperature of 400 K, what is the final pressure?

Using P = nRT/V, final pressure P_final = (1.8 mol 0.0821 L·atm/(mol·K) 400 K) / 11 L ≈ 4.89 atm.

What is the significance of heating at constant volume for this gas?

Heating at constant volume increases the temperature and internal energy without doing work, illustrating the direct relationship between temperature and internal energy in an ideal gas.

How does the internal energy change with temperature for a monatomic

ideal gas?

The internal energy change is directly proportional to the temperature change, given by $\Delta U = (3/2)nR\Delta T$.

What is the approximate amount of heat required to raise the temperature from 300 K to 400 K?

$Q = (3/2)nR(T_{final} - T_{initial}) = (3/2)$ 1.8 mol 0.0821 L·atm/(mol·K) (400 K - 300 K) ≈ 2.7 L·atm, which converts to approximately 2.75 kJ.

Additional Resources

Suppose 1.8 Mol Of A Monatomic Ideal Gas Initially At 11 L And 300 K Is Heated At Constant Volume To

The scenario of heating a monatomic ideal gas at constant volume offers a rich context for exploring fundamental thermodynamic principles, calculations, and real-world applications. This comprehensive review delves into the various aspects of this process—from initial conditions to final states, thermodynamic equations involved, and practical implications—providing a detailed understanding of the underlying physics and engineering considerations.

Introduction to the Scenario and Fundamental Concepts

The problem involves a monatomic ideal gas with a molar quantity of 1.8 mol, initially occupying a volume of 11 liters at a temperature of 300 K. The gas undergoes heating at a constant volume—meaning the container's volume does not change—raising its temperature to an unknown final value. This setup is classic in thermodynamics, often used to illustrate concepts such as heat transfer, work done (or not done), and changes in internal energy.

Understanding this scenario requires familiarity with the ideal gas law:

$$PV = nRT$$

where:
- P is pressure,
- V is volume,
- n is the number of moles,
- R is the universal gas constant ($8.314\, \mathrm{J\,mol^{-1}\,K^{-1}}$),
- T is temperature in Kelvin.

Since the volume is constant, any change in temperature directly influences pressure, internal energy, and heat transfer.

Initial Conditions and Calculations

Initial Pressure and Internal Energy

Using the ideal gas law to find the initial pressure:

$$
P_i = \frac{nRT_i}{V}
$$

Given:
- $n = 1.8\, \mathrm{mol}$,
- $R = 8.314\, \mathrm{J\,mol^{-1}\,K^{-1}}$,
- $T_i = 300\, \mathrm{K}$,
- $V = 11\, \mathrm{L} = 0.011\, \mathrm{m^3}$.

Calculating:

$$
P_i = \frac{1.8 \times 8.314 \times 300}{0.011} \approx \frac{4489.56}{0.011} \approx 408,142\, \mathrm{Pa}
$$

This high pressure reflects the compression of the gas in a relatively small volume at room temperature.

The initial internal energy U_i for a monatomic ideal gas is:

$$
U_i = \frac{3}{2} nRT_i
$$

Calculating:

$$
U_i = \frac{3}{2} \times 1.8 \times 8.314 \times 300 \approx 1.5 \times 1.8 \times 8.314 \times 300 \approx 6.3 \times 8.314 \times 300 \approx 6.3 \times 2494.2 \approx 15,704\, \mathrm{J}
$$

This internal energy corresponds to the initial thermal energy stored in the gas.

Final Conditions after Heating

Since the volume remains constant, the only variables changing are pressure and temperature. The final temperature T_f is unknown, but can be expressed in terms of the heat added Q and the thermodynamic relations.

The final pressure:

$$
P_f = \frac{nRT_f}{V}
$$

The internal energy after heating:

$$
U_f = \frac{3}{2} nRT_f
$$

Given the process is at constant volume, the work done $W = 0$, because no volume change occurs. The heat supplied Q directly increases the internal energy:

$$
Q = \Delta U = U_f - U_i = \frac{3}{2} n R (T_f - T_i)
$$

This relation is central to understanding how heat affects the internal energy and temperature of the gas in such processes.

Thermodynamic Analysis of the Heating Process

Calculating Final Temperature and Heat Input

Suppose the problem specifies or asks to find the final temperature after heating, or the amount of heat added. The general relation:

$$
Q = \frac{3}{2} n R (T_f - T_i)
$$

allows for straightforward computation once T_f or Q is known.

For example, if the heat added Q is given or is to be determined for a particular temperature increase, rearranged:

$$
T_f = T_i + \frac{2Q}{3 n R}
$$

This relation is crucial in designing heating systems or understanding energy transfer in thermodynamic cycles.

Work Done and Heat Transfer

In a constant volume process:
- Work done $W = 0$, because the volume does not change.
- Heat transferred directly increases internal energy.

The absence of work simplifies the analysis, highlighting the importance of heat transfer in changing the thermal state of the gas.

Features:
- Simple energy relation: $Q = \Delta U$.
- No work involved: ideal for studying heat transfer without mechanical work complexities.

Limitations:
- Real gases may deviate from ideal behavior at high pressures or low temperatures.
- Practical systems may involve heat losses or other inefficiencies.

Implications and Applications in Real-World Contexts

Understanding the heating of an ideal gas at constant volume has several practical implications:

- Design of Thermodynamic Devices: Such processes model parts of engines, refrigerators, and heat exchangers where volume constraints are present.
- Educational Tool: Demonstrates fundamental concepts such as internal energy change, heat transfer, and the absence of work in specific processes.
- Material and Safety Considerations: High pressures at constant volume necessitate robust containment and safety measures in practical applications.

Advantages and Disadvantages

Pros:
- Clear and straightforward thermodynamic relations.
- Useful for theoretical calculations and initial design estimates.
- Highlights the direct relationship between heat input and temperature increase.

Cons:
- Idealized assumptions may not fully capture real gas behaviors.
- Does not account for heat losses, radiation, or other dissipative effects.
- Limited to scenarios where volume is strictly constant; less applicable in processes involving volume change.

Advanced Topics and Extensions

The basic analysis can be extended to consider:

- Entropy Changes: Calculating the entropy change during heating at constant volume provides insights into irreversibility and entropy generation.
- Pressure Variations: As temperature increases, pressure rises; understanding this helps in designing pressure vessels.
- Real Gas Corrections: Applying Van der Waals or other equations of state for more accurate modeling at high pressures or low temperatures.

Entropy Change Calculation

For an ideal gas undergoing a temperature change at constant volume:

$$
\Delta S = n C_v \ln \frac{T_f}{T_i}
$$

where $C_v = \frac{3}{2} R$ for monatomic gases. This calculation helps assess the thermodynamic reversibility and efficiency of processes.

Summary and Conclusions

Heating a monatomic ideal gas at constant volume provides a fundamental illustration of thermodynamic principles. The process emphasizes how heat transfer results in an increase in internal energy and temperature, with no work done due to fixed volume constraints. The straightforward relations between heat, internal energy, and temperature make this scenario an ideal teaching tool and a foundation for more complex thermodynamic analyses.

While the idealized assumptions simplify calculations, real-world applications often require considerations of non-ideal behavior, heat losses, and other practical factors. Nonetheless, understanding this basic process is essential for engineers, physicists, and students exploring energy systems, thermodynamic cycles, and material behavior under thermal stress.

In conclusion, this scenario encapsulates core thermodynamic concepts, demonstrating the elegant relationships governing heat transfer, internal energy, and the state variables of an ideal gas. Its analysis not only reinforces fundamental physics but also informs the design and operation of thermal systems across various industries.

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