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Suppose That F(x)= (3x3) And G(x)=6x^28.For Each Function H Given Below, Find A Formula For H(x) And

Understanding how to manipulate and combine functions is a fundamental aspect of algebra and calculus. In this article, we will explore the process of determining explicit formulas for composite functions based on given functions F(x) and G(x). Specifically, when F(x) = 3x^3 and G(x) = 6x^28, we will examine various ways to form new functions H(x), such as sums, differences, products, quotients, and compositions. Our goal is to provide clear, step-by-step instructions to find formulas for each H(x) based on the operations involved, and to deepen your understanding of function operations for improved problem-solving skills and SEO-friendly content.

Understanding the Given Functions and Basic Operations

Given Functions

- **F(x) = 3x^3**
- **G(x) = 6x^28**

These two functions are polynomial functions involving powers of x. F(x) is a cubic function scaled by 3, while G(x) involves a high-degree polynomial scaled by 6. When working with these functions, common operations include addition, subtraction, multiplication, division, and composition, each leading to a different form of H(x).

Function Operations and Their Significance

- **Sum of functions:** H(x) = F(x) + G(x)
- **Difference of functions:** H(x) = F(x) - G(x)

- **Product of functions:** H(x) = F(x) G(x)
- **Quotient of functions:** H(x) = F(x) / G(x), provided G(x) ≠ 0
- **Composition of functions:** H(x) = F(G(x)) or G(F(x))

Each of these operations results in a new function H(x) with a unique formula derived from the original functions. Let’s explore each case in detail.

Formulas for Sum and Difference of F(x) and G(x)

Sum of F(x) and G(x): H(x) = F(x) + G(x)

To find this, simply add the expressions for F(x) and G(x):

Calculation:

1. F(x) = 3x^3
2. G(x) = 6x^28
3. H(x) = 3x^3 + 6x^28

Final Formula:

H(x) = 3x^3 + 6x^28

Difference of F(x) and G(x): H(x) = F(x) - G(x)

Similarly, subtract G(x) from F(x):

Calculation:

1. F(x) = 3x^3
2. G(x) = 6x^28

3. $H(x) = 3x^3 - 6x^{28}$

Final Formula:
$H(x) = 3x^3 - 6x^{28}$

Formulas for Product and Quotient of F(x) and G(x)

Product: $H(x) = F(x)\ G(x)$

Multiplying the two functions:

Calculation:

1. $F(x) = 3x^3$
2. $G(x) = 6x^{28}$
3. $H(x) = (3x^3)\ (6x^{28})$

Simplification Steps:

- Multiply the coefficients: 3 6 = 18
- Add the exponents of x: 3 + 28 = 31

Final Formula:
$H(x) = 18x^{31}$

Quotient: $H(x) = F(x) / G(x)$, $G(x) \neq 0$

Dividing F(x) by G(x):

Calculation:

1. F(x) = 3x^3

2. G(x) = 6x^28

3. H(x) = (3x^3) / (6x^28)

Simplification Steps:

- Divide the coefficients: 3 / 6 = 1/2

- Subtract the exponents of x: 3 - 28 = -25

Final Formula:

H(x) = (1/2) x^{-25} = \frac{1}{2} \cdot \frac{1}{x^{25}}

Formulas for Composition of Functions

Function composition involves plugging one function into another. There are two primary compositions to consider:

1. H(x) = F(G(x))

This means replacing every x in F(x) with G(x):

Calculation:

1. F(x) = 3x^3

2. G(x) = 6x^28

3. H(x) = 3 [G(x)]^3

Step-by-step:

- First, find $G(x)$: $6x^{28}$
- Then, cube $G(x)$: $[6x^{28}]^3 = 6^3 x^{28 3} = 216 x^{84}$
- Finally, multiply by 3: $H(x) = 3 216 x^{84} = 648 x^{84}$

Final Formula:
$H(x) = 648x^{84}$

2. $H(x) = G(F(x))$

Replace x in $G(x)$ with $F(x)$:

Calculation:

1. $G(x) = 6x^{28}$
2. Replace x with $F(x)$: $6 [F(x)]^{28}$

Step-by-step:

- $F(x) = 3x^3$
- Calculate $[F(x)]^{28} = (3x^3)^{28} = 3^{28} x^{3 28} = 3^{28} x^{84}$
- Multiply by 6: $H(x) = 6 3^{28} x^{84}$

Final Formula:
$H(x) = 6 3^{28} x^{84}$

Practical Applications and Tips for Working with Function Formulas

Why Understanding These Formulas Matters

Knowing how to derive formulas for combined functions allows mathematicians, engineers, and scientists to analyze complex systems more effectively. Whether modeling physical phenomena, analyzing data, or solving equations, understanding these operations is crucial.

Tips for Simplifying Function Expressions

- Always simplify coefficients first.
- Use exponent rules: x^a x^b = x^{a + b}, (x^a)^b = x^{a b}, x^a / x^b = x^{a - b}.
- When composing functions, carefully substitute and simplify step-by-step.
- Check for domain restrictions, especially in division and composition, to avoid undefined expressions.

Common Mistakes to Avoid

- Forgetting to raise coefficients to the power when exponentiating products.
- Mixing up addition and multiplication rules for exponents.
- Neglecting to consider the domain restrictions in division and composition.

Conclusion

Mastering the process of finding formulas for functions like H(x) based on given functions F(x) and G(x) is

Frequently Asked Questions

Given $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, how do you find the formula for $H(x) = F(x) + G(x)$?

$H(x) = F(x) + G(x) = 3x^3 + (6x^2 + 8) = 3x^3 + 6x^2 + 8$.

If $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, what is the formula for $H(x) = F(x) - G(x)$?

$H(x) = 3x^3 - (6x^2 + 8) = 3x^3 - 6x^2 - 8$.

Given the functions $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, how do you find $H(x) = F(x) G(x)$?

$H(x) = (3x^3) (6x^2 + 8) = 3x^3 6x^2 + 3x^3 8 = 18x^5 + 24x^3$.

With $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, what is the formula for $H(x) = F(G(x))$ (composition of functions)?

$H(x) = F(G(x)) = 3(6x^2 + 8)^3$.

Given $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, how to find $H(x) = G(F(x))$?

$H(x) = G(F(x)) = 6(3x^3)^2 + 8 = 69x^6 + 8 = 54x^6 + 8$.

If $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, how do you determine the inverse of $H(x) = F(x) + G(x)$?

Since $H(x) = 3x^3 + 6x^2 + 8$, solving for x in terms of y ($H(x) = y$) involves solving the cubic equation $y = 3x^3 + 6x^2 + 8$ for x, which typically requires numerical methods or factoring techniques depending on specific y values.

Additional Resources

Suppose That $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$. For Each Function H Given Below, Find A Formula For H(x) And

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Investigating Composite Functions: Deriving Formulas for H(x) from Given F(x) and G(x)

Mathematics, especially algebra and calculus, fundamentally revolves around understanding how functions interact, combine, and transform. When given specific functions, such as $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, one of the most intriguing tasks is to determine how these functions can be composed or combined to form a new function H(x). This process not only solidifies comprehension of function operations—such as addition, subtraction, multiplication, division, and composition—but also offers insights into their behavior and properties.

This comprehensive review explores the methodology for deriving formulas for various functions H(x) based on the given functions F(x) and G(x). We will analyze common functional combinations, establish formulas, and exemplify the process with detailed calculations, enabling a thorough understanding of function composition and combination in algebraic contexts.

Understanding the Given Functions

Before delving into specific H(x) formulas, it is crucial to thoroughly understand the given functions:

Function $F(x) = 3x^3$

- Type: Cubic polynomial scaled by 3.
- Domain: All real numbers (since polynomial functions are defined everywhere).
- Range: All real numbers (since cubic functions are unbounded).
- Behavior: Monotonically increasing (for $x > 0$), decreasing (for $x < 0$), with inflection point at $x=0$.

Function $G(x) = 6x^2 + 8$

- Type: Quadratic polynomial shifted vertically.
- Domain: All real numbers.
- Range: $[8, \infty)$ because the quadratic term is always ≥ 0, scaled by 6, then shifted up by 8.
- Behavior: Parabola opening upwards; minimum at $x=0$, where $G(0)=8$.

Understanding these functions' nature allows us to anticipate how their combinations may behave.

Common Types of Function Combinations and Their Formulas

When analyzing H(x) derived from F and G, typical operations include:

- Addition: $H(x) = F(x) + G(x)$
- Subtraction: $H(x) = F(x) - G(x)$
- Multiplication: $H(x) = F(x)\ G(x)$
- Division: $H(x) = F(x) / G(x)$, with $G(x) \neq 0$

- Composition: $H(x) = F(G(x))$ or $H(x) = G(F(x))$
- Other combinations: Linear combinations like $aF(x) + bG(x)$

In this review, we'll focus chiefly on these common forms, especially composition, which often presents the most interesting challenges.

Deriving H(x) for Specific Combinations

Suppose the problem specifies a particular H(x) based on F and G. For clarity, let's examine typical cases.

Case 1: $H(x) = F(x) + G(x)$

Formula Derivation:

Given $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$,

$$
H(x) = F(x) + G(x) = 3x^3 + (6x^2 + 8) = 3x^3 + 6x^2 + 8
$$

Analysis:

- The resulting function is a cubic polynomial with added quadratic and constant terms.
- Degree: 3 (since highest degree term is $3x^3$).
- Behavior: Polynomial with inflection, smooth curve.

Case 2: $H(x) = F(x) - G(x)$

Formula Derivation:

$$
H(x) = 3x^3 - (6x^2 + 8) = 3x^3 - 6x^2 - 8
$$

Analysis:

- Cubic polynomial shifted downward by the quadratic term and constant.

Case 3: $H(x) = F(x) G(x)$

Formula Derivation:

Multiplying the functions:

$$
H(x) = (3x^3) \times (6x^2 + 8)
$$

Apply distributive property:

$$
H(x) = 3x^3 \times 6x^2 + 3x^3 \times 8 = 18x^5 + 24x^3
$$

Simplified Formula:

$$
H(x) = 18x^5 + 24x^3
$$

Analysis:

- The resulting function is a polynomial of degree 5.
- Behavior: Dominated by the x^5 term; odd degree, so it extends from negative to positive infinity.

Case 4: $H(x) = F(x) / G(x)$, with $G(x) \neq 0$

Formula Derivation:

$$
H(x) = \frac{3x^3}{6x^2 + 8}
$$

Domain restrictions:

- $G(x) \neq 0$
- Solve for $G(x) \neq 0$:

$$
6x^2 + 8 \neq 0 \Rightarrow 6x^2 \neq -8
$$

Since $6x^2 \geq 0$ for all real x, and $-8 < 0$, $G(x) \neq 0$ for all real x. So, the domain is all real numbers.

Simplification:

Factor numerator and denominator where possible:

$$
H(x) = \frac{3x^3}{6x^2 + 8}
$$

No common factors, so the expression remains as is.

Behavior:

- Rational function with vertical asymptotes only if the denominator is zero, which it isn't in the real domain here.

Case 5: Composition: $H(x) = F(G(x))$

Formula Derivation:

Calculate $G(x)$:

$$
G(x) = 6x^2 + 8
$$

Now, plug into F:

$$
H(x) = F(G(x)) = 3 \times (G(x))^3 = 3 \times (6x^2 + 8)^3
$$

Final formula:

$$
H(x) = 3(6x^2 + 8)^3
$$

Expansion:

While expanding is possible but algebraically intensive, expressing $H(x)$ as above suffices for most purposes.

Case 6: Composition: $H(x) = G(F(x))$

Formula Derivation:

Calculate $F(x)$:

$$
F(x) = 3x^3
$$

Substitute into G:

\[

$$
H(x) = 6 \times (F(x))^2 + 8 = 6 \times (3x^3)^2 + 8 = 6 \times 9x^6 + 8 = 54x^6 + 8
$$

Final formula:

$$
H(x) = 54x^6 + 8
$$

Advanced Analysis: Behavior and Graphical Implications

Understanding the formulas derived above allows us to analyze the behavior of each $H(x)$:

Polynomial Combinations

- Degree and Leading Coefficient: The degree indicates the end behavior of the polynomial. For example, $H(x) = 18x^5 + 24x^3$ has degree 5, so as $x \to \pm\infty$, $H(x) \to \pm\infty$ accordingly.
- Roots: Solving $H(x) = 0$ can reveal points where the function crosses the x-axis.

Rational Functions

- Domain Restrictions: Ensuring the denominator isn't zero is key. In our example, $G(x) \neq 0$ for all real x, simplifying domain considerations.

Composition Functions

- Complexity: Composition often results in higher-degree polynomials, making their analysis more involved.
- Graphical Behavior: The shape can be significantly different from the base functions, especially when compositions involve raising functions to powers.

Practical Applications and Significance

Understanding how to derive formulas for $H(x)$ based on given F and G is vital in numerous mathematical and real-world contexts:

- Engineering: Modeling complex systems where multiple variables interact.
- Physics: Analyzing composite motions or transformations.
- Economics: Combining functions to model relationships between variables.
- Computer Science: Function composition is foundational in programming and algorithms.

Moreover, the ability to manipulate and derive such formulas hones problem-solving skills vital for higher-level mathematics.

Summary and Conclusions

In summary, given the functions $F(x) = 3x^3$ and $G(x) = 6x^2 + 8$, the process of finding formulas for various H(x) functions involves:

- Applying basic algebraic operations: addition, subtraction, multiplication, division.
- Understanding function composition: substituting one function into another.
- Simplifying expressions where possible.
- Analyzing the resulting functions' behavior, domain, and range.

The core takeaway is that these methods are systematic and applicable across a broad spectrum of functions, providing a foundational toolset for mathematical analysis, problem-solving, and practical application.

By mastering these techniques, students and professionals alike can effectively navigate complex function interactions, paving the way for deeper insights into mathematical models and real-world phenomena.

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