Suppose That F(x, Y) = X Xy + Y 5x + 5y With X + Y 25. 1. Absolute Minimum Of F(x, Y) Is 2. Absolute

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Introduction

In the realm of multivariable calculus, understanding how to find the minimum or maximum of a function subject to certain constraints is fundamental. The problem at hand involves analyzing the function $F(x, y) = x^2 y + y 5x + 5y$, with the given constraint $x + y = 25$. The key objectives are to determine the absolute minimum of $F(x, y)$ under this constraint and to understand the nature of this minimum.

This article will provide a detailed exploration of the problem, including methods to solve constrained optimization problems, application of Lagrange multipliers, and step-by-step calculations to identify the minimum value of the function. Additionally, we will discuss the significance of these results in mathematical and applied contexts.

Understanding the Function and Constraints

The Function $F(x, y)$

The function is given by:

$$
F(x, y) = x^2 y + 5x y + 5 y
$$

Note that the original expression appears to have some inconsistent notation, but for clarity, we interpret it as:

$$
F(x, y) = x^2 y + 5x y + 5 y
$$

This is a multivariable function involving quadratic and linear terms.

The Constraint $x + y = 25$

The problem specifies that $x + y = 25$. This is a linear constraint that restricts the domain of the function to a line in the xy-plane.

Goal

- Find the absolute minimum value of $F(x, y)$ subject to the constraint.
- Determine the point(s) where this minimum occurs.

Approaches to Constrained Optimization

To find the extrema of a function under a constraint, two primary methods are commonly used:

1. Substitution Method

- Solve the constraint for one variable.
- Substitute into the function to reduce it to a single variable.
- Find the minimum or maximum of the resulting single-variable function.

2. Lagrange Multipliers

- Set up the Lagrangian function incorporating the constraint.
- Find the critical points by solving the resulting system of equations.

Given the nature of the problem, the substitution method provides a straightforward approach, but Lagrange multipliers offer a more general framework.

Step-by-Step Solution

1. Express y in terms of x

From the constraint:

$$
x + y = 25 \implies y = 25 - x
$$

2. Substitute into $F(x, y)$

Express F as a function of x:

$$
F(x) = x^2 (25 - x) + 5x (25 - x) + 5 (25 - x)
$$

Simplify each term:

- $x^2 (25 - x) = 25x^2 - x^3$
- $5x (25 - x) = 125x - 5x^2$
- $5 (25 - x) = 125 - 5x$

Now, combine:

$$
F(x) = (25x^2 - x^3) + (125x - 5x^2) + (125 - 5x)
$$

Simplify further:

$$
F(x) = 25x^2 - x^3 + 125x - 5x^2 + 125 - 5x
$$

Combine like terms:

$$
F(x) = (25x^2 - 5x^2) + (-x^3) + (125x - 5x) + 125
$$
$$
F(x) = 20x^2 - x^3 + 120x + 125
$$

3. Find critical points by differentiation

Compute the derivative with respect to x:

$$
F'(x) = \frac{d}{dx} (20x^2 - x^3 + 120x + 125) = 40x - 3x^2 + 120
$$

Set $F'(x) = 0$:

$$
40x - 3x^2 + 120 = 0
$$

Rewrite:

$$
-3x^2 + 40x + 120 = 0
$$

Divide through by -1:

$$
3x^2 - 40x - 120 = 0
$$

4. Solve the quadratic equation

Using the quadratic formula:

$$
x = \frac{40 \pm \sqrt{(-40)^2 - 4 \times 3 \times (-120)}}{2 \times 3}
$$

Compute discriminant:

$$
D = 1600 - 4 \times 3 \times (-120) = 1600 + 1440 = 3040
$$

Calculate:

$$
x = \frac{40 \pm \sqrt{3040}}{6}
$$

Simplify $\sqrt{3040}$:

- $3040 = 16 \times 190$, so:

$$
\sqrt{3040} = 4 \sqrt{190}
$$

Thus:

$$
x = \frac{40 \pm 4 \sqrt{190}}{6} = \frac{4(10 \pm \sqrt{190})}{6} = \frac{2(10 \pm \sqrt{190})}{3}
$$

5. Find corresponding y values

Recall:

$$
y = 25 - x
$$

So:

$$
y = 25 - \frac{2(10 \pm \sqrt{190})}{3}
$$

Determining the Absolute Minimum

1. Evaluate $F(x)$ at critical points

Calculate $F(x)$ at:

$$
x_1 = \frac{2(10 + \sqrt{190})}{3}
$$
$$
x_2 = \frac{2(10 - \sqrt{190})}{3}
$$

Corresponding y:

$$
y_1 = 25 - x_1
$$
$$
y_2 = 25 - x_2
$$

2. Calculate $F(x, y)$ at these points

Alternatively, since we've expressed F in terms of x, we can directly compute $F(x)$ at these critical x-values to compare.

Numerical Approximation

Given that $\sqrt{190} \approx 13.784$, compute:

$$
x_1 \approx \frac{2(10 + 13.784)}{3} = \frac{2 \times 23.784}{3} \approx \frac{47.568}{3} \approx 15.856
$$
$$
x_2 \approx \frac{2(10 - 13.784)}{3} = \frac{2 \times (-3.784)}{3} \approx \frac{-7.568}{3} \approx -2.523
$$

Corresponding y:

$$
y_1 = 25 - 15.856 \approx 9.144
$$
$$
y_2 = 25 - (-2.523) \approx 27.523
$$

Now, evaluate F at these points:

- For $x \approx 15.856$:

$$
F(x) = 20x^2 - x^3 + 120x + 125
$$

Calculate:

$$
x^2 \approx (15.856)^2 \approx 251.9
$$
$$
x^3 \approx 15.856 \times 251.9 \approx 3994.9
$$
$$
F \approx 20 \times 251.9 - 3994.9 + 120 \times 15.856 + 125
$$
$$
= 5038 - 3994.9 + 1902.72 + 125
$$
$$
\approx 5038 - 3994.9 + 2027.72
$$
$$
= (5038 - 3994.9) + 2027.72 \approx 1043.1 + 2027.72 \approx 3070.82
$$

- For $x \approx -2.523$:

$$
x^2 \approx 6.365
$$
$$
x^3 \approx -2.523 \times 6.365 \approx -16.065
$$
$$
F \approx 20 \times 6.365 - (-16.065) + 120 \times (-2.523) + 125
$$
$$
= 127.3 + 16.065 - 302.76 + 125
$$
$$
= (127.3 + 16.065) + (125 - 302.76) \approx 143.365 - 177.76 \approx -34.395
$$

3. Conclusion on

Frequently Asked Questions

What is the function F(x, y) given in the problem?

The function is $F(x, y) = x \cdot x\, y + y \cdot 5x + 5y$, with the condition that $x + y = 25$.

How do you interpret the constraint $x + y = 25$ in this problem?

The constraint $x + y = 25$ restricts the domain of the function to points where the sum of x and y equals 25, enabling us to find the minimum of F(x, y) along this line.

What method can be used to find the absolute minimum of F(x, y) under the given constraint?

Lagrange multipliers or substitution methods can be used to reduce the problem to a single variable and find the minimum of F(x, y) subject to $x + y = 25$.

How do you simplify F(x, y) using the constraint $x + y = 25$?

Express y as $y = 25 - x$ and substitute into F(x, y) to get a function of a single variable x, then analyze to find the minimum.

What is the first step to find the absolute minimum of F(x, y)?

Substitute $y = 25 - x$ into F(x, y) to obtain a single-variable function F(x) and then differentiate with respect to x.

What is the derivative of the simplified function F(x) with respect to x?

The derivative is obtained by differentiating the expression for F(x) after substitution, setting it to zero to find critical points.

How do you determine the minimum value of F(x, y) after finding critical points?

Evaluate F(x) at the critical points and compare these values, ensuring they satisfy the original constraint to identify the absolute minimum.

What is the significance of the absolute minimum value of F(x, y)?

The absolute minimum represents the lowest possible value of the function F(x, y) along the line $x + y = 25$, which is important in optimization problems.

Based on the problem statement, what are the known values related to the minimum of F(x, y)?

The problem indicates that the absolute minimum of F(x, y) is 2, and it is the smallest value attainable under the given constraint.

How can the absolute minimum value of 2 be verified for the function F(x, y)?

By solving for x and y that satisfy the critical point conditions and the constraint, then substituting back into F(x, y) to confirm the minimum value is 2.

Additional Resources

Suppose That F(x, Y) = X Xy + Y 5x + 5y With X + Y 25. 1. Absolute Minimum Of F(x, Y) Is 2. Absolute

In the world of calculus and optimization, understanding the behavior of functions—particularly their minima and maxima—is fundamental. Today, we delve into a specific problem involving a bi-variable function, exploring its properties, constraints, and the process to find its absolute minimum value. This problem not only highlights core concepts such as partial derivatives and constrained optimization but also demonstrates how mathematical reasoning can be applied systematically to solve complex-looking problems.

Introduction to the Function and the Problem Statement

The function in question is defined as:

\[F(x, y) = X \times Xy + Y \times 5x + 5y \]

with the constraint:

\[X + Y = 25 \]

The goal is to determine the absolute minimum value of \(F(x, y)\), which, according to the problem statement, is 2. This task involves understanding the function's structure, applying calculus techniques, and considering the constraint to identify the critical points where the minimum occurs.

Clarifying the Function and Variables

Interpreting the Function

At first glance, the notation appears somewhat ambiguous, likely due to formatting issues. To analyze it effectively, we need to interpret the function correctly.

- The function is:

\[F(x, y) = X \times X y + Y \times 5x + 5y \]

- The variables are \(X\) and \(Y\), but the notation suggests that \(X\) and \(Y\) might be functions of \(x\) and \(y\), or perhaps simply variables themselves.

Given the presence of the constraint \(X + Y = 25\), it's reasonable to interpret \(X\) and \(Y\) as variables rather than functions.

Likely Correction and Assumption

Suppose the original function was meant to be:

\[F(X, Y) = X \times X \times Y + Y \times 5X + 5Y \]

which simplifies to:

$$F(X, Y) = X^2 Y + 5X Y + 5Y$$

Alternatively, perhaps the original expression was:

$$F(X, Y) = X \times X Y + Y \times 5x + 5y$$

but considering the context and the standard notation, it's most plausible that the function is:

$$F(X, Y) = X^2 Y + 5XY + 5Y$$

with the constraint:

$$X + Y = 25$$

for clarity, we will proceed under this assumption, as it aligns with typical function forms and makes the problem tractable.

Formulating the Optimization Problem

The problem reduces to minimizing:

$$F(X, Y) = X^2 Y + 5XY + 5Y$$

subject to the linear constraint:

$$X + Y = 25$$

This is a constrained optimization problem, which can be approached using methods such as substitution, Lagrange multipliers, or direct calculus techniques.

Strategy Overview

1. Express one variable in terms of the other using the constraint.
2. Substitute into the function to obtain a single-variable function.
3. Find critical points by differentiating and setting derivatives to zero.
4. Verify whether these points correspond to minima, maxima, or saddle points.
5. Confirm the absolute minimum value and its location.

Step-by-Step Solution Approach

Step 1: Express Y in terms of X

From the constraint:

$$X + Y = 25 \Rightarrow Y = 25 - X$$

Step 2: Substitute into $F(X, Y)$

Substituting $Y = 25 - X$:

$$
\begin{aligned}
F(X) &= X^2 (25 - X) + 5X (25 - X) + 5 (25 - X) \\
&= X^2 \times 25 - X^3 + 125X - 5X^2 + 125 - 5X
\end{aligned}
$$

$$

Simplify term by term:

- $X^2 \times 25 = 25X^2$
- $- X^3$
- $125X$
- $- 5X^2$
- 125
- $- 5X$

Combine like terms:

$$
F(X) = -X^3 + (25X^2 - 5X^2) + (125X - 5X) + 125
$$

$$
F(X) = -X^3 + 20X^2 + 120X + 125
$$

Now, the problem reduces to finding the minimum of:

$$
F(X) = -X^3 + 20X^2 + 120X + 125
$$

where X is constrained such that $Y = 25 - X$, but since Y must be real, X must satisfy:

$$ 0 \leq X \leq 25 $$

Finding Critical Points

Step 3: Derive $F'(X)$ and set to zero

Calculate the first derivative:

$$
F'(X) = -3X^2 + 40X + 120
$$

Set derivative to zero to find critical points:

$$
-3X^2 + 40X + 120 = 0
$$

Divide through by -1:

$$
3X^2 - 40X - 120 = 0
$$

Step 4: Solve the quadratic equation

Using quadratic formula:

$$
x = \frac{40 \pm \sqrt{(-40)^2 - 4 \times 3 \times (-120)}}{2 \times 3}
$$

Calculate discriminant:

$$
D = 1600 - 4 \times 3 \times (-120) = 1600 + 1440 = 3040
$$

Compute the roots:

$$
x = \frac{40 \pm \sqrt{3040}}{6}
$$

Approximate $\sqrt{3040}$:

$$
\sqrt{3040} \approx 55.14
$$

Thus,

$$
x \approx \frac{40 \pm 55.14}{6}
$$

Calculate both:

- For the positive root:

$$
x \approx \frac{40 + 55.14}{6} \approx \frac{95.14}{6} \approx 15.86
$$

- For the negative root:

$$
x \approx \frac{40 - 55.14}{6} \approx \frac{-15.14}{6} \approx -2.52
$$

Since x must be in the interval $[0, 25]$, discard the negative root.

Step 5: Evaluate $f(x)$ at critical points and endpoints

Critical point:

- $x \approx 15.86$
- Corresponding $y = 25 - 15.86 \approx 9.14$

Endpoints:

- $x = 0$, then $y = 25$
- $x = 25$, then $y = 0$

Calculate $f(x)$ at these points:

At $x \approx 15.86$:

$$
F(15.86) = -(15.86)^3 + 20 \times (15.86)^2 + 120 \times 15.86 + 125
$$

Approximate:

- $(15.86)^3 \approx 3990.5$
- $(15.86)^2 \approx 251.5$
- $20 \times 251.5 \approx 5030$
- $120 \times 15.86 \approx 1903.2$

Now,

$$
F(15.86) \approx -3990.5 + 5030 + 1903.2 + 125 \approx 1067.7
$$

At $X = 0$:

$$
F(0) = -0 + 0 + 0 + 125 = 125
$$

At $X = 25$:

$$
F(25) = -(25)^3 + 20 \times (25)^2 + 120 \times 25 + 125
$$

Calculate:

- $25^3 = 15625$
- $20 \times 625 = 12500$
- $120 \times 25 = 3000$

Thus,

$$
F(25) = -15625 + 12500 + 3000 + 125 = -15625 + 15725 = 100
$$

Analyzing Results and Confirming the Minimum

From the calculations:

Point	X	Y	$F(X,Y)$ Approximate Value
Critical point	15.86	9.14	1067.7
Endpoint 0	0	25	125
Endpoint 25	25	0	100

The smallest value among these is approximately 100 at \(X=25

Suppose That F X Y X Xy Y 5x 5y With X Y 25 1 Absolute Minimum Of F X Y Is 2 Absolute

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