Suppose The Derivative Of A Function Fis F'(x) = (x - 5)^6 (X + 8^)5(x - 6)^4 On What Interval(s) Is

Suppose The Derivative Of A Function Fis F'(x) = (x - 5)^6 (X + 8)^5 (x - 6)^4. On What Interval(s) Is

Understanding the behavior of a function based on its derivative is fundamental in calculus. Given the derivative $F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4$, analyzing the intervals where the original function $F(x)$ is increasing or decreasing helps in sketching its graph, identifying local maxima and minima, and understanding its overall shape. This comprehensive guide explores how to determine these intervals systematically.

1. Understanding the Derivative and Its Components

Before diving into interval analysis, it is essential to understand the structure of the derivative $F'(x)$.

1.1. The Significance of the Derivative

- The derivative $F'(x)$ indicates the slope of the function $F(x)$ at any point x.
- When $F'(x) > 0$, the function $F(x)$ is increasing.
- When $F'(x) < 0$, the function $F(x)$ is decreasing.
- When $F'(x) = 0$, the function may have a local maximum, local minimum, or a point of inflection, depending on the behavior around that point.

1.2. Components of $F'(x)$

- $F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4$
- Each factor corresponds to a critical point where the function's behavior might change:
- $(x - 5)^6$
- $(x + 8)^5$
- $(x - 6)^4$

2. Finding Critical Points and Sign Changes

Critical points occur where $F'(x) = 0$ or where $F'(x)$ is undefined.

Since $F'(x)$ is a polynomial, it is defined everywhere; thus, critical points are where $F'(x) = 0$.

2.1. Solving for Critical Points

Set each factor equal to zero:
- $x - 5 = 0 \Rightarrow x = 5$
- $x + 8 = 0 \Rightarrow x = -8$
- $x - 6 = 0 \Rightarrow x = 6$

Critical points are at:
- $x = -8$
- $x = 5$
- $x = 6$

2.2. Multiplicity and Its Effect on Sign Changes

- The multiplicity of each root affects whether $F'(x)$ changes sign at that point:
- Even multiplicity (e.g., 2, 4, 6): $F'(x)$ does not change sign at that root.
- Odd multiplicity (e.g., 1, 3, 5): $F'(x)$ does change sign at that root.

Critical Point	Factor	Multiplicity	Sign Change at Critical Point	Effect on $F'(x)$
-8	$(x + 8)^5$	5 (odd)	Yes	Sign changes
5	$(x - 5)^6$	6 (even)	No	Sign does not change
6	$(x - 6)^4$	4 (even)	No	Sign does not change

This key information helps us understand the behavior of $F'(x)$ around these points.

3. Analyzing the Sign of $F'(x)$ Across Intervals

To determine where $F(x)$ is increasing or decreasing, analyze the sign of $F'(x)$ in each interval divided by the critical points.

3.1. Establishing the Sign Chart

The critical points divide the real line into four intervals:
- $(-\infty, -8)$
- $(-8, 5)$

- $(5, 6)$
- $(6, \infty)$

Determine the sign of each factor in each interval:

Interval	$x - 5$	$x + 8$	$x - 6$	$F'(x)$ sign	Explanation
$(-\infty, -8)$	Negative	Negative	Negative	$- \times - \times -$	Negative $\times$ Negative $\times$ Negative = Negative
$(-8, 5)$	Negative	Positive	Negative	$- \times + \times -$	Negative $\times$ Positive $\times$ Negative = Positive
$(5, 6)$	Positive	Positive	Negative	$+ \times + \times -$	Positive $\times$ Positive $\times$ Negative = Negative
$(6, \infty)$	Positive	Positive	Positive	$+ \times + \times +$	All positive = Positive

Note: Remember to consider the multiplicity's effect on the sign change at each critical point:

- At $x = -8$, multiplicity 5 (odd): sign of $F'(x)$ changes from negative to positive.
- At $x = 5$, multiplicity 6 (even): no sign change; $F'(x)$ remains positive.
- At $x = 6$, multiplicity 4 (even): no sign change; $F'(x)$ remains negative or positive depending on previous sign.

However, because the multiplicity of $(x - 5)^6$ is even, the sign of $F'(x)$ does not change at $x=5$. Similarly, for $(x - 6)^4$, the sign remains consistent across $x=6$.

3.2. Final Sign Chart Summary

Interval	Sign of $F'(x)$	Function Behavior
$(-\infty, -8)$	Negative	$F(x)$ decreasing
$(-8, 5)$	Positive	$F(x)$ increasing
$(5, 6)$	Positive	$F(x)$ increasing
$(6, \infty)$	Positive	$F(x)$ increasing

Note: The sign at $x=6$ remains positive because the multiplicity is even, so the derivative does not change sign there.

4. Interpreting the Behavior of $F(x)$

Based on the sign analysis, the function $F(x)$:

- Decreases on \((-\infty, -8)\)
- Increases on \((-8, \infty)\), with some nuances at specific points.

Let's clarify the behavior at the critical points:

- At \(x = -8\): \(F'(x)\) changes from negative to positive because of an odd multiplicity (5). This indicates a local minimum at \(x = -8\).

- At \(x=5\): The derivative's sign does not change (multiplicity 6 is even). Thus, \(F(x)\) has neither a maximum nor a minimum at \(x=5\); it is a point of inflection or a flat point.

- At \(x=6\): The derivative's sign does not change (multiplicity 4 is even). Therefore, no local extremum occurs here.

In summary:

Critical Point	Behavior of \(F(x)\)	Type of Critical Point
\(x=-8\)	Decreasing to increasing	Local minimum
\(x=5\)	Flat point, no sign change	Neither max nor min
\(x=6\)	Flat point, no sign change	Neither max nor min

5. Final Conclusion: Intervals Where \(F(x)\) Is Increasing or Decreasing

Based on the above analysis, the intervals where the original function \(F(x)\) is increasing or decreasing are:

- Decreasing: \((-\infty, -8)\)
- Increasing: \((-8, \infty)\)

Specifically:

- \(F(x)\) is decreasing on the interval \(\boxed{(-\infty, -8)}\).

- \(F(x)\) is increasing on the interval \

Frequently Asked Questions

For the function with derivative F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4, on which intervals is the function increasing?

The function is increasing where F'(x) > 0, which occurs on the intervals

(-8, 5) and (6, ∞).

Identify the critical points of the function given F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4.

The critical points are at x = -8, x = 5, and x = 6, where F'(x) = 0 or undefined.

Determine the intervals where the function is decreasing based on the derivative F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4.

The function is decreasing where F'(x) < 0, which is on the interval (-∞, -8).

Are there any points of inflection indicated by the derivative F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4?

Points of inflection require the second derivative; however, analyzing F'(x) alone does not determine inflection points.

What is the significance of the multiplicity of the roots in F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4 regarding the function's behavior?

Even multiplicities (6 and 4) suggest the function's slope touches zero and does not cross the x-axis, indicating potential points of horizontal tangent without sign change, while odd multiplicity (5) indicates a sign change at x = -8.

At which points does the derivative F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4 change sign?

The derivative changes sign at x = -8, where the factor (x + 8)^5 is involved with odd multiplicity; it does not change sign at x = 5 or 6 due to even multiplicities.

Based on the derivative, is the function increasing or decreasing on the interval (-8, 5)?

Since F'(x) > 0 on (-8, 5), the function is increasing there.

How do the multiplicities of the roots affect the

shape of the graph of the function with derivative F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4?

Even multiplicities (6 and 4) cause the graph to touch the x-axis and flatten but not cross, while the odd multiplicity (5) at x = -8 causes the graph to cross the x-axis with a change in sign.

Additional Resources

Suppose The Derivative Of A Function F is F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4: An In-Depth Analysis of Critical Points, Intervals of Increase and Decrease, and Concavity

---

Introduction

In the realm of calculus, understanding the behavior of a function based on its derivative is fundamental. The derivative not only provides the slope of the tangent line at any point but also reveals critical information about the function's increasing or decreasing nature, local extrema, and concavity. Given the derivative:

$$F'(x) = (x - 5)^6 \times (x + 8)^5 \times (x - 6)^4$$

our goal is to analyze the intervals where the original function $F(x)$ is increasing or decreasing, identify potential critical points, and understand its concavity. This investigation serves as a comprehensive example of how to interpret derivatives with multiple factors and their powers, which is essential for students and practitioners of calculus.

Understanding the Structure of the Derivative

The derivative $F'(x)$ is expressed as a product of three factors:

- $(x - 5)^6$
- $(x + 8)^5$
- $(x - 6)^4$

Each factor's degree (power) influences the behavior of $F'(x)$ at the corresponding critical points:

- Roots at $x = 5$, $x = -8$, and $x = 6$
- The multiplicity of each root determines whether $F'(x)$ crosses the x-axis or merely touches it (i.e., changes sign or stays the same).

Step 1: Identifying Critical Points

Critical points occur where $F'(x) = 0$ or where $F'(x)$ is undefined. Since $F'(x)$ is a polynomial (product of polynomials), it's defined everywhere; hence, critical points are at zeros of the derivative:

$$
x = -8, \quad x = 5, \quad x = 6
$$

The multiplicities are:

- $(x - 5)^6$ — multiplicity 6 (even)
- $(x + 8)^5$ — multiplicity 5 (odd)
- $(x - 6)^4$ — multiplicity 4 (even)

Step 2: Determining the Sign of $F'(x)$ in Each Interval

To analyze the increasing/decreasing behavior, examine the sign of $F'(x)$ in the intervals divided by the critical points:

- $(-\infty, -8)$
- $(-8, 5)$
- $(5, 6)$
- $(6, \infty)$

The sign of $F'(x)$ depends on the sign of each factor:

- $(x - 5)^6$: always non-negative; zero at $x=5$
- $(x + 8)^5$: sign depends on $x + 8$
- $(x - 6)^4$: always non-negative; zero at $x=6$

Let's analyze each interval:

Interval 1: $(-\infty, -8)$

- $x + 8 < 0$: $(x + 8)^5 < 0$ (since odd power)
- $x - 5 < 0$, but raised to an even power, so $(x - 5)^6 > 0$
- $x - 6 < 0$, but raised to an even power, so $(x - 6)^4 > 0$

Multiplying:

$$
F'(x) = (\text{positive}) \times (\text{negative}) \times (\text{positive}) = \text{negative}
$$

Conclusion: $F'(x) < 0$, so F is decreasing on $(-\infty, -8)$.

Interval 2: $(-8, 5)$

- $x + 8 > 0$: $(x + 8)^5 > 0$
- $x - 5 < 0$: $(x - 5)^6 > 0$
- $x - 6 < 0$: $(x - 6)^4 > 0$

Multiplying:

$$
\text{positive} \times \text{positive} \times \text{positive} = \text{positive}
$$

Conclusion: $F'(x) > 0$, so F is increasing on $(-8, 5)$.

Interval 3: $(5, 6)$

- $x + 8 > 0$: $(x + 8)^5 > 0$
- $x - 5 > 0$: $(x - 5)^6 > 0$
- $x - 6 < 0$: $(x - 6)^4 > 0$

Multiplying:

$$
\text{positive} \times \text{positive} \times \text{positive} = \text{positive}
$$

Conclusion: $F'(x) > 0$, so F remains increasing on $(5, 6)$.

Interval 4: $(6, \infty)$

- $x + 8 > 0$: $(x + 8)^5 > 0$
- $x - 5 > 0$: $(x - 5)^6 > 0$
- $x - 6 > 0$: $(x - 6)^4 > 0$

Multiplying:

$$
\text{positive} \times \text{positive} \times \text{positive} = \text{positive}
$$

Conclusion: $F'(x) > 0$, so F continues to increase on $(6, \infty)$.

Step 3: Behavior at Critical Points and Nature of Extrema

- At $x = -8$:

- $F'(x)$ changes from negative to positive as x passes through -8. Since the sign changes, F has a local minimum at $x = -8$.

- At $x = 5$:

- $F'(x)$ changes from positive to positive (since the multiplicity is even, the derivative touches zero but does not change sign). The derivative is zero at $x=5$ but does not change sign.

- Implication: No local max or min at $x=5$; the function has a point of inflection or a flat tangent with no extremum.

- At $x=6$:

- $F'(x)$ changes from positive to positive, again with even multiplicity. No sign change, so no extremum.

Summary of Increasing/Decreasing Intervals:

Interval	$F'(x)$ Sign	Behavior of F
$(-\infty, -8)$	Negative	Decreasing
$(-8, 5)$	Positive	Increasing
$(5, 6)$	Positive	Increasing
$(6, \infty)$	Positive	Increasing

Step 4: Analyzing Concavity and Inflection Points

While our initial derivative analysis provides increasing/decreasing behavior, a full understanding requires second derivative analysis.

Computing the Second Derivative $F''(x)$

Given:

\[

$$
F'(x) = (x - 5)^6 (x + 8)^5 (x - 6)^4
$$

Applying the product rule (or logarithmic differentiation) to find $F''(x)$ involves differentiating a product of three functions. To simplify, consider the logarithmic derivative:

$$
\frac{F'(x)}{F(x)} = \text{Sum of derivatives of logs}
$$

However, an explicit second derivative can be complex; instead, we analyze the concavity qualitatively by considering the behavior near critical points and the multiplicities.

Step 5: Inflection Points and Concavity Behavior

- At $x = -8$: Since $F'(x)$ crosses from negative to positive with an odd multiplicity (power 5), the function's concavity may change. Typically, inflection points occur where $F''(x) = 0$ and the concavity switches.

- At $x=5$ and $x=6$: The derivative does not change sign, so these are not extrema but may be points of inflection if $F''(x)$ changes sign there.

A detailed second derivative analysis would involve differentiating the product explicitly, but for the scope of this analysis, noting the multiplicity and change

Suppose The Derivative Of A Function Fis F X X 5 6 X 8 5 X 6 4 On What Interval S Is

Suppose The Derivative Of A Function Fis F X X 5 6 X 8 5 X 6 4 On What Interval S Is

Related Articles

- Please Show Your Work!***Remember, M (molarity) = Moles Of Solute / Liters Of Solution1) What Is The
- Represent The Following Sentence As An Algebraic Expression, Where "a Number" Is Theletter X. You Do
- Please Help: When Lorenzo Was Growing Up, His Family Did Not Have Access To Health Care. Now That He

Back to Home