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Understanding the Problem: Tangent Lines and Function Values

When exploring calculus and the behavior of functions, one of the fundamental concepts is the tangent line at a specific point. In this case, we are given that the tangent line to a function $f(x)$ at $x=3$ is described by the equation:

$$y = 9x + 4$$

Our goal is to determine the value of $f(3)$, which is the value of the function at $x=3$. This type of problem is common in introductory calculus courses and involves understanding the relationships between functions, their tangent lines, derivatives, and the values of the functions themselves.

Key Concepts to Understand

Before diving into the solution, let's clarify some essential concepts:

1. The Tangent Line to a Function at a Point

- The tangent line to a function $f(x)$ at $x=a$ is a straight line that "touches" the graph of the function exactly at that point.
- This line has the same slope as the function's derivative $f'(a)$ at $x=a$.
- The equation of the tangent line provides information about the function's behavior near $x=a$.

2. Derivative as the Slope of the Tangent Line

- The derivative $f'(a)$ gives the slope of the tangent line to $f(x)$ at $x=a$.
- If the tangent line's equation is known, the coefficient of x in that line is the slope $f'(a)$.

3. Function Value at a Point

- The function value $f(a)$ is the point on the graph of $f(x)$ at $x=a$.
- The tangent line at $x=a$ passes through the point $(a, f(a))$, so knowing the tangent line helps determine $f(a)$.

Step-by-Step Solution Approach

Given the tangent line:

$$y = 9x + 4$$

and the point of tangency at $x=3$, we want to find $f(3)$.

Step 1: Identify the slope of the tangent line

- The slope m of the tangent line is the coefficient of x:

$$m = 9$$

- Since the tangent line touches the graph of $f(x)$ at $x=3$, the slope of $f(x)$ at $x=3$ is:

$$f'(3) = 9$$

Step 2: Find the point of tangency

- The tangent line passes through the point $(x, y) = (3, y_t)$, where y_t is the value of $f(3)$.

- The line's equation is:

$$y = 9x + 4$$

- Substituting $x=3$:

$$ y_t = 9 \times 3 + 4 = 27 + 4 = 31 $$

- Therefore, the point of tangency is $(3, 31)$:

$$ f(3) = 31 $$

Step 3: Summarize the findings

- The value of the function at $x=3$ is 31.
- The derivative at $x=3$ (the slope of the tangent) is 9.

Interpreting the Result: What Does This Mean?

The above calculation reveals that:

- The function $f(x)$ passes through the point $(3, 31)$.
- Its slope at that point is 9, which aligns with the tangent line's slope.
- The tangent line's equation $y=9x+4$ represents the best linear approximation of $f(x)$ near $x=3$.

This information is valuable in multiple contexts:

- Approximate Function Values: Near $x=3$, the function behaves similarly to the tangent line.
- Understanding Derivatives: The slope 9 indicates the rate of change of $f(x)$ at $x=3$.
- Graph Sketching: Knowing the tangent line and point helps sketch the graph of $f(x)$.

Extending the Concept: Related Calculus Topics

Understanding tangent lines and their relationship to the original function opens doors to various calculus concepts:

1. Differentiation and Slope

- The derivative $f'(a)$ provides the slope of the tangent line at a.
- In our problem, $f'(3) = 9$.

2. Linear Approximation

- The tangent line serves as the linear approximation of $f(x)$ near $x=3$:

$$ f(x) \approx f(3) + f'(3)(x - 3) $$

- Using $f(3)=31$ and $f'(3)=9$:

$$ f(x) \approx 31 + 9(x - 3) $$

3. Mean Value Theorem (MVT)

- The MVT states that for a continuous function on $[a, b]$ differentiable on (a, b), there exists some $c \in (a, b)$ such that:

$$ f'(c) = \frac{f(b) - f(a)}{b - a} $$

- While not directly used here, this theorem underpins the relationship between derivatives and average rates of change.

4. Applications in Physics and Engineering

- Tangent lines and derivatives are foundational in analyzing motion, velocity, and acceleration.
- The slope indicates how rapidly a quantity is changing at a specific point.

Conclusion: Final Answer and Summary

In the context of the problem where the tangent line to $f(x)$ at $x=3$ is given by:

$$ y = 9x + 4 $$

the value of the function at $x=3$ is 31. This conclusion stems from recognizing that the tangent line passes through the point $(3, f(3))$, and substituting $x=3$ into the tangent line's equation yields the corresponding y-coordinate.

Final answer:

$$ \boxed{f(3) = 31} $$

Additional Tips for Solving Similar Problems

- Always identify the slope (derivative) from the tangent line's equation.
- Use the point of tangency to find the corresponding point on the function.
- Substitute the known x-value into the tangent line to find $f(a)$.
- Remember that the tangent line touches the function at exactly one point, providing valuable information about the function's behavior at that point.

Further Exploration

If you're interested in expanding your understanding, consider exploring:

- How to find the derivative $f'(x)$ if the tangent line is given at multiple points.
- How to reconstruct a function $f(x)$ given its tangent lines at various points.
- The role of the second derivative in understanding the concavity and curvature of $f(x)$.

By mastering these concepts, you'll strengthen your calculus foundation and enhance your problem-solving skills related to tangent lines and function analysis.

References:

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning.
- Thomas, George B., and Ross L. Finney. Calculus and Analytic Geometry. Pearson.
- Khan Academy. Tangent lines and derivatives. [Online resource]

Remember: The key to solving these types of problems is understanding the relationship between the tangent line and the original function, especially how the slope of the tangent line relates to the derivative and how the point of tangency gives the function's value at that point.

Frequently Asked Questions

Given the tangent line to f(x) at x=3 is y=9x+4, what is the value of f(3)?

f(3) = 9(3) + 4 = 27 + 4 = 31.

How does the tangent line equation y=9x+4 relate to the value of the function at x=3?

Since the tangent line touches the graph at x=3, its y-value at x=3 gives f(3).

What is the significance of the slope 9 in the tangent line y=9x+4 for f(x) at x=3?

The slope 9 is the derivative f'(3), indicating the slope of f(x) at x=3.

Can we determine f(3) solely from the tangent line y=9x+4?

Yes, because the tangent line passes through the point (3, f(3)), so plugging in x=3 gives f(3).

If the tangent line at x=3 is y=9x+4, what is the tangent point (x, y)?

The point is (3, f(3)), which is (3, 31).

Is the value of f(3) directly given by the tangent line equation?

Yes, the y-value of the tangent line at x=3 gives f(3), which is 31.

How would you verify the value of f(3) using the tangent line equation?

Substitute x=3 into y=9x+4, resulting in y=31, which is f(3).

Does knowing the tangent line's equation allow us to find other values of f(x)?

No, only the value at x=3; additional information about f(x) elsewhere is needed.

What information about f(x) can be inferred from the tangent line y=9x+4 at x=3?

We can determine f(3) = 31 and the slope of f(x) at x=3 is 9.

Additional Resources

Understanding the Tangent Line and Its Significance in Calculus

When exploring the fascinating world of calculus, one of the fundamental concepts that often emerges is the tangent line to a function at a particular point. This geometric idea isn't just an abstract mathematical construct; it provides critical insights into the behavior of functions, their slopes, and rates of change. In this article, we'll delve into a specific scenario: given the tangent line to a function $f(x)$ at $x=3$, defined by the equation $y = 9x + 4$, what can we determine about the function $f(x)$, specifically its value at $x=3$?

This question might seem straightforward at first glance, but unpacking it involves understanding the relationship between a function and its tangent line, the significance of the slope, and how these pieces come together to reveal the value of the function at a given point. Let's explore this step-by-step.

The Geometric and Analytical Foundations of a Tangent Line

What Is a Tangent Line?

A tangent line to a function $f(x)$ at a point $x=a$ is a straight line that "just touches" the curve at that point. Geometrically, it represents the instantaneous direction in which the function is heading at $x=a$. Analytically, it provides the best linear approximation of the function near that point.

Mathematically, the tangent line at $x=a$ can be expressed as:

$$
y = f(a) + f'(a) (x - a)
$$

where:

- \(f(a)\) is the value of the function at \(x=a\),
- \(f'(a)\) is the derivative of the function at \(x=a\), representing the slope of the tangent line at that point.

This equation is derived from the point-slope form of a line, where the slope is given by the derivative at the point.

The Derivative and Its Role

The derivative \(f'(a)\) is crucial because it tells us how the function is changing at \(a\). A positive derivative indicates the function is increasing at that point, while a negative derivative indicates it's decreasing.

In the context of our problem, the tangent line's slope is directly connected to the derivative:

\[
f'(a) = \text{slope of the tangent line at } x=a
\]

Deciphering the Given Tangent Line Equation

Analyzing the Equation \(y = 9x + 4 \)

The tangent line at \(x=3\) is provided by the equation:

\[
y = 9x + 4
\]

This is a linear equation in slope-intercept form, where:

- The slope \(m = 9\),
- The y-intercept (when \(x=0\)) is \(4\).

Since this line is tangent to \(f(x)\) at \(x=3\), it shares key properties with the original function at that point.

Implications for the Derivative \(f'(3)\)

From the general form of the tangent line:

$$
y = f(a) + f'(a)(x - a)
$$

we can compare this with the given line:

$$
y = 9x + 4
$$

at $a=3$. To do this, rewrite the tangent line in point-slope form:

$$
y = f(3) + f'(3)(x - 3)
$$

Matching this with $y=9x + 4$:

$$
f(3) + f'(3)(x - 3) = 9x + 4
$$

This comparison allows us to determine both $f(3)$ and $f'(3)$.

Determining $f(3)$ and $f'(3)$: Step-by-Step

Step 1: Express the Tangent Line in Point-Slope Form

Given that the tangent line passes through $x=3$, the point of tangency is $(3, f(3))$. The tangent line at this point is:

$$
y = f(3) + f'(3)(x - 3)
$$

Our goal is to find $f(3)$.

Step 2: Use the Given Equation to Find the Slope $f'(3)$

Since the slope of the tangent line is the derivative at $x=3$, and the given tangent line has slope 9, we have:

$$
f'(3) = 9
$$

This is a key insight: the slope of the tangent line directly gives us the derivative of the function at the point of tangency.

Step 3: Find $f(3)$ Using the Point of Tangency

The tangent line passes through the point $(3, f(3))$. Plugging $x=3$ into the tangent line:

$$
y = 9(3) + 4 = 27 + 4 = 31
$$

Since the tangent line passes through $(3, f(3))$, the y-coordinate at $x=3$ must be $f(3)$:

$$
f(3) = 31
$$

Result:

$$
\boxed{
f(3) = 31
}
$$

This is the primary quantity of interest: the value of the function at $x=3$.

Interpreting the Results: What Does This Tell Us?

Summary of Findings

- The derivative at $x=3$ is $f'(3) = 9$.

- The function's value at $x=3$ is $f(3) = 31$.

These two pieces of information not only define the tangent line but also offer insight into the function's local behavior near $x=3$.

What Can We Infer About $f(x)$ as a Whole?

While the problem only provides data at a specific point, understanding the derivative's value can guide us in approximating the function nearby:

- The function is increasing at $x=3$ since $f'(3) > 0$.
- Near $x=3$, $f(x)$ behaves roughly like:

$$
f(x) \approx 31 + 9(x - 3)
$$

which is a linear approximation.

Additional Considerations and Broader Applications

Why Is This Important?

Understanding how a tangent line relates to a function is fundamental in calculus because it:

- Provides a means to approximate functions locally (linear approximation).
- Helps in understanding the rate of change at specific points.
- Serves as the foundation for more advanced topics like differentials, optimization, and differential equations.

Real-World Analogies

Consider a car traveling along a curved road:

- The position of the car can be modeled by a function $f(x)$.
- The tangent line at a particular point models the car's immediate trajectory.
- The slope of this tangent line indicates the car's instantaneous speed.

- Knowing the position at that point ($f(3)$) and the speed ($f'(3)$) gives a snapshot of the car's current state.

Similarly, in economics, physics, and engineering, understanding the tangent line allows for precise predictions and optimizations based on local behavior.

Conclusion: Piecing It All Together

In summary, given the tangent line to a function $f(x)$ at $x=3$:

$$
y = 9x + 4
$$

we can extract significant information:

- The slope of the tangent line (9) equals the derivative $f'(3)$.
- The point of tangency lies on this line at $x=3$, where $f(3) = 31$.

This example illustrates the powerful interplay between geometric intuition and analytical rigor in calculus. It underscores how a simple line equation encapsulates both the rate of change and the specific value of a function at a point, providing a comprehensive snapshot of its local behavior.

Understanding these core concepts not only enhances problem-solving skills but also lays the groundwork for more advanced mathematical modeling and analysis in various scientific disciplines.

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