

The Area Of The Rectangular Surfaces Of The Prism Is 720 Sq Cm XX = 20 Cm And XY : XZ : YZ = 5 : 3 :

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Understanding the surface area of a rectangular prism involves analyzing its dimensions, the relationships between its edges, and how these contribute to the total surface area. In this comprehensive guide, we will explore how to determine the rectangular surfaces' total area of a prism when certain parameters are given: the total surface area, a specific edge length, and the ratio between its edges. This article aims to delve deeply into the geometric principles, calculations, and problem-solving techniques related to this scenario, providing clarity and insight for students, educators, and enthusiasts alike.

Introduction to Rectangular Prisms

A rectangular prism, also known as a cuboid, is a three-dimensional geometric shape with six rectangular faces. It is characterized by three dimensions: length (X), width (Y), and height (Z). The properties of a rectangular prism include:

- Opposite faces are equal and rectangular.
- All faces are rectangles.
- It has 12 edges, with three sets of parallel edges of lengths X, Y, and Z.

Understanding the surface area of a rectangular prism is essential in various applications, including packaging, construction, and manufacturing.

Given Data and Problem Statement

In this problem, we are provided with specific information:

- The total surface area (SA) of the rectangular prism is 720 square centimeters.
- One edge, XX, (which could be interpreted as the length of a specific edge, such as the height or length) is 20 centimeters.
- The ratio of the other edges is XY : XZ : YZ = 5 : 3 : ? (Assuming the ratio is between the edges X, Y, and Z).

Our goal is to find the surface area of the rectangular surfaces and understand the relationships among the edges based on the given ratio.

Understanding the Dimensions and Ratios

Interpreting the Ratio

The ratio $XY : XZ : YZ = 5 : 3 : ?$ indicates the relative proportions of the edges. Generally, in a rectangular prism, the edges are labeled as:

- X (length)
- Y (width)
- Z (height)

Assuming the ratio applies to these dimensions, we can denote:

- $X = k \cdot 5$
- $Y = k \cdot 3$
- $Z = k \cdot r$

where k is a common scaling factor, and r is an unknown ratio component to be determined.

Clarifying the Edge Lengths

Given that $XX = 20$ cm, and assuming XX refers to a specific edge—possibly the length X or height Z —we need to determine which edge it corresponds to.

- If XX refers to the length of the edge X , then $X = 20$ cm.
- Alternatively, if XX refers to some other edge, clarification is necessary.

For the purpose of this problem, to proceed logically, we assume that XX represents the length of the edge X , i.e., $X = 20$ cm.

Calculating the Dimensions of the Prism

Step 1: Determine the value of k

Since $X = 20$ cm and $X = 5k$, then:

$$X = 5k = 20 \text{ cm}$$

$$\Rightarrow k = 20 / 5 = 4 \text{ cm}$$

Step 2: Find Y and Z

From the ratio:

- $Y = 3k = 3 \cdot 4 = 12 \text{ cm}$
- $Z = r \cdot k$

Now, we need to find the value of Z . To do that, we'll use the total surface area information.

Using the Surface Area to Find Z

The total surface area (SA) of a rectangular prism is given by:

$$SA = 2(lw + lh + wh)$$

Where:

- $l = X = 20 \text{ cm}$
- $w = Y = 12 \text{ cm}$
- $h = Z = r \cdot 4 \text{ cm}$

Given $SA = 720 \text{ cm}^2$, substitute the known values:

$$\begin{aligned} 720 &= 2(20 \cdot 12 + 20 \cdot Z + 12 \cdot Z) \\ 720 &= 2(240 + 20Z + 12Z) \\ 720 &= 2(240 + 32Z) \\ 720 &= 480 + 64Z \end{aligned}$$

Subtract 480 from both sides:

$$\begin{aligned} 720 - 480 &= 64Z \\ 240 &= 64Z \end{aligned}$$

Solve for Z :

$$Z = 240 / 64 = 15 / 4 = 3.75 \text{ cm}$$

Now, recall that $Z = r \cdot 4 \text{ cm}$, so:

$$\begin{aligned} r \cdot 4 &= 3.75 \\ r &= 3.75 / 4 = 0.9375 \end{aligned}$$

Summary of Dimensions

- X (length): 20 cm
- Y (width): 12 cm
- Z (height): 3.75 cm

The ratios are:

$$- X : Y : Z = 20 : 12 : 3.75$$

Expressed in simplest form:

Divide each by 1.25:

- $20 / 1.25 = 16$

- $12 / 1.25 = 9.6$

- $3.75 / 1.25 = 3$

Thus, the simplified ratio:

$16 : 9.6 : 3$

Alternatively, since the initial ratio was $5 : 3 : r$, we can verify the ratios:

- $X / Y = 20 / 12 = 5 / 3$, matching the ratio given.

- $Z / Y = 3.75 / 12 = 0.3125$, which is approximately $1 / 3.2$, indicating Z is roughly a third of Y, aligning with the ratio.

Calculating the Area of Rectangular Surfaces

Total Surface Area Breakdown

The total surface area includes all six faces, grouped as three pairs of identical rectangles:

1. Two faces with dimensions X and Y: Area = 2 (XY)

2. Two faces with dimensions X and Z: Area = 2 (XZ)

3. Two faces with dimensions Y and Z: Area = 2 (YZ)

Calculate each:

- $XY = 20 \times 12 = 240 \text{ cm}^2$

- $XZ = 20 \times 3.75 = 75 \text{ cm}^2$

- $YZ = 12 \times 3.75 = 45 \text{ cm}^2$

Sum of all faces:

Total Surface Area = $2 (XY + XZ + YZ) = 2 (240 + 75 + 45) = 2 \times 360 = 720 \text{ cm}^2$

This confirms the given total surface area matches our calculated dimensions.

Understanding the Surface Area of Rectangular Surfaces

Surface Areas of Individual Faces

- Face XY: 240 cm^2
- Face XZ: 75 cm^2
- Face YZ: 45 cm^2

Each pair contributes twice the area, as these faces are opposite and equal in size.

Significance of the Surface Area Components

Knowing individual face areas helps in various practical applications:

- Calculating material requirements for covering surfaces
- Estimating painting or coating costs
- Designing storage and packaging solutions

Additional Insights and Applications

Real-World Applications of Surface Area Calculations

Understanding surface areas of rectangular prisms is crucial in numerous fields:

- Packaging: Estimating material needed for wrapping boxes
- Construction: Calculating surface paint or wall coverings
- Manufacturing: Designing containers and components with precise surface specifications
- Education: Teaching geometric concepts and problem-solving strategies

Problem-Solving Techniques

- Ratio Analysis: Use ratios to determine unknown dimensions
- Algebraic Substitution: Substitute known values into formulas for straightforward calculations
- Dimensional Consistency Checks: Verify that calculated dimensions make sense with given data

Conclusion

By analyzing the given total surface area, specific edge length, and ratios, we successfully determined the dimensions of the rectangular prism: $X = 20 \text{ cm}$, $Y = 12 \text{ cm}$, $Z = 3.75 \text{ cm}$. These dimensions allowed us to compute the surfaces' individual areas and confirm the total surface area of 720 cm^2 . Understanding these relationships enhances geometric problem-solving skills and has practical implications across various industries. Whether

designing packaging, estimating materials, or exploring mathematical concepts, mastering the calculation of surface areas in rectangular prisms is an essential skill for students and professionals alike.

Summary of Key Points

- The total surface area of the prism is 720 sq cm.
- Given one edge (XX) = 20 cm, interpreted as $X = 20$ cm.
- The ratios of edges $X : Y : Z = 5 : 3 : r$, leading to the dimensions $X = 20$ cm, $Y = 12$

Frequently Asked Questions

What is the total surface area of a rectangular prism with given dimensions?

The total surface area of a rectangular prism is calculated as $2(lb + bh + hl)$, where l , b , and h are the length, breadth, and height respectively.

Given the surface area of 720 sq cm and one dimension as 20 cm, how can we find the other dimensions?

Using the surface area formula and the given dimension, substitute the known values and solve for the remaining dimensions, considering the ratios if provided.

How do the ratios $XY : XZ : YZ = 5 : 3 :$ inform the dimensions of the prism?

They represent the proportional relationships between the edges of the prism's faces, allowing us to express the edges in terms of a common variable and find their actual lengths.

What steps are involved in determining the dimensions of a prism from its surface area and edge ratios?

First, express all unknown edges in terms of a common variable using the ratios, then write the surface area formula in terms of that variable, and solve for it using the given total surface area.

If $XY = 5k$, $XZ = 3k$, and $YZ = ?$ in the ratio, how do we find YZ ?

Given the ratios $XY : XZ : YZ = 5 : 3 : y$, YZ can be expressed as $y k$, and additional information or relations are needed to find y .

Can the given surface area and ratio data determine the volume of the prism?

Yes, once the dimensions are known, the volume can be calculated as length $\times$ breadth $\times$ height.

What is the significance of the known dimension $XX = 20$ cm in solving the problem?

It provides a fixed measurement that can be used to determine the scale of the other dimensions through the ratios and surface area formula.

How do we interpret the ratio $XY : XZ : YZ$ in the context of a rectangular prism?

It indicates the proportional relationships among the edges meeting at a corner, helping to establish the actual lengths of the edges.

What is the formula for the surface area of a rectangular prism with edges a , b , and c ?

The surface area is $2(ab + bc + ac)$.

How would you approach solving for the unknown side YZ given the total surface area of 720 sq cm, one side XX = 20 cm, and ratios XY : XZ : YZ?

Express XY, XZ, and YZ in terms of a common variable using the given ratios, substitute into the surface area formula along with the known dimension, and solve for the variable to find YZ.

Additional Resources

The Area Of The Rectangular Surfaces Of The Prism Is 720 Sq Cm XX = 20 Cm And XY : XZ : YZ = 5 : 3 :

Understanding the dimensions and surface areas of three-dimensional shapes like prisms is fundamental in geometry, with applications spanning architecture, engineering, and design. Today, we delve into a problem involving a rectangular prism with specific surface area and proportional relationships between its edges. Our goal is to determine the individual dimensions of the prism based on the given data: the total surface area, one particular edge length, and the ratios of the other edges. This exploration not only highlights the elegance of geometric relationships but also demonstrates how algebraic methods can decode the properties of spatial figures.

Introduction to the Problem

Imagine a rectangular prism—a box-shaped object—whose total surface area sums up to 720 square centimeters. One of its edges, labeled XX, is known to be 20 centimeters long. Additionally, the ratios of the other three edges—XY, XZ, and YZ—are given as 5 : 3 : (missing value). To fully understand the dimensions of this prism, we need to interpret these ratios, relate them to the known edge, and use surface area formulas to find the missing lengths.

This problem encapsulates the intersection of algebra and geometry, where ratios help establish proportional relationships, and surface area formulas serve as tools to connect dimensions with measurable quantities.

Understanding the Geometric Setup

The Rectangular Prism: Dimensions and Notation

A rectangular prism is defined by three edges meeting at a vertex, which we can label as:

- XY: length along one axis
- XZ: length along the second axis
- YZ: length along the third axis

Assuming the prism's vertices are labeled accordingly, the edges are:

- XX: a specific edge, possibly representing the length of one side of the prism
- XY, XZ, YZ: other edges, with known ratios

Given that $XX = 20$ cm, and the ratios $XY : XZ : YZ = 5 : 3 : ?$, the question becomes: what are the lengths of XY, XZ, and YZ? Once these are known, the surface area can be verified or used to find other missing dimensions.

Deciphering the Ratios and Known Quantities

Clarifying the Ratios

The ratio $XY : XZ : YZ = 5 : 3 : ?$ indicates that the lengths of edges XY, XZ, and YZ are proportional to these ratios. Since XX is given as 20 cm, and assuming that:

- XY corresponds to the length along one axis
- XZ along the second
- YZ along the third

we can interpret that:

- XY is proportional to 5 units
- XZ is proportional to 3 units
- YZ is proportional to some value k

However, there's ambiguity here—does XX correspond to XY, or is it a different edge? Usually, in three-dimensional notation, edges are labeled consistently. For clarity, let's assume that:

- The dimension XY is associated with the ratio 5
- The dimension XZ with 3
- The dimension YZ with an unknown ratio k

Given $XX = 20$ cm, if XX is along the same direction as XY, then $XY = 20$ cm. Since XY is proportional to 5 units, then:

$$\boxed{XY = 5 \text{ units}}$$

Similarly, $XX = 20$ cm corresponds to the same length as XY, so:

$$\backslash[XY = 20 \text{ cm} \backslash]$$

which implies:

$$\backslash[5 \text{ units} \rightarrow 20 \text{ cm} \rightarrow 1 \text{ unit} = 4 \text{ cm} \backslash]$$

This conversion allows us to find the lengths of XZ and YZ using their ratios:

$$\backslash[XZ = 3 \text{ units} \times 4 \text{ cm/unit} = 12 \text{ cm} \backslash]$$

and

$$\backslash[YZ = k \text{ units} \times 4 \text{ cm/unit} \backslash]$$

But since the ratio XY : XZ : YZ is given as 5 : 3 : ?, and XY = 20 cm, then:

$$\backslash[XY = 20 \text{ cm} \backslash]$$

$$\backslash[XZ = \frac{3}{5} \times XY = \frac{3}{5} \times 20 = 12 \text{ cm} \backslash]$$

$$\backslash[YZ = ? \backslash]$$

Now, the missing ratio ? corresponds to YZ, which we can now express as:

$$\backslash[YZ = \frac{?}{5} \times 20 \text{ cm} \backslash]$$

Let's denote YZ = t cm; then:

$$\backslash[\frac{XY}{5} = \frac{XZ}{3} = \frac{YZ}{?} \backslash]$$

Given XY = 20 cm, XZ = 12 cm, and YZ = t cm, the ratios hold if:

$$\backslash[\frac{20}{5} = \frac{12}{3} = \frac{t}{?} \backslash]$$

which simplifies to:

$$\backslash[4 = 4 = \frac{t}{?} \backslash]$$

Thus,

$$\backslash[\frac{t}{?} = 4 \rightarrow t = 4 \times ? \backslash]$$

But without a specific value for ?, we cannot proceed further. To resolve this, perhaps the original problem intended the ratios as XY : XZ : YZ = 5 : 3 : 2, for

instance.

Assuming the ratio is 5 : 3 : 2, then:

$$\begin{aligned} & \backslash \\ & XY = 20 \text{ cm} \\ & \backslash \\ & \backslash \\ & XZ = \frac{3}{5} \times 20 = 12 \text{ cm} \\ & \backslash \\ & \backslash \\ & YZ = \frac{2}{5} \times 20 = 8 \text{ cm} \\ & \backslash \end{aligned}$$

This provides consistent dimensions:

- XY = 20 cm
- XZ = 12 cm
- YZ = 8 cm

Calculating the Surface Area

Surface Area Formula for a Rectangular Prism

The total surface area (SA) of a rectangular prism is calculated as:

$$\begin{aligned} & \backslash \\ & SA = 2 (XY \times XZ + XY \times YZ + XZ \times YZ) \\ & \backslash \end{aligned}$$

Substituting the dimensions:

$$\begin{aligned} & \backslash \\ & SA = 2 (20 \times 12 + 20 \times 8 + 12 \times 8) \\ & \backslash \end{aligned}$$

Calculations:

- $(20 \times 12 = 240)$
- $(20 \times 8 = 160)$
- $(12 \times 8 = 96)$

Sum:

$$\begin{aligned} & \backslash \\ & 240 + 160 + 96 = 496 \\ & \backslash \end{aligned}$$

Total surface area:

$$\backslash[\\ SA = 2 \times 496 = 992 \text{ sq cm} \\ \backslash]$$

But this exceeds the given total surface area of 720 sq cm. Therefore, our assumption about the ratios may need correction.

Reconciling the Given Data: Adjusting Dimensions

Given that the total surface area is 720 sq cm, and $XX = 20$ cm, the dimensions should satisfy:

$$\backslash[\\ 720 = 2 (XY \times XZ + XY \times YZ + XZ \times YZ) \\ \backslash]$$

Assuming $XY = 20$ cm, let's denote:

- $(XZ = a)$
- $(YZ = b)$

Then:

$$\backslash[\\ 720 = 2 (20a + 20b + a b) \\ \backslash[\\ 360 = 20a + 20b + a b \\ \backslash]$$

Rearranged:

$$\backslash[\\ a b + 20 a + 20 b = 360 \\ \backslash]$$

This is a quadratic relation involving (a) and (b) . Let's attempt to find integer solutions that fit the proportional ratios.

Suppose the ratios are:

$$\backslash[\\ XY : XZ : YZ = 5 : 3 : 2 \\ \backslash]$$

From earlier:

$$\backslash[\\ XY = 20 \text{ cm}$$

$$\begin{aligned} & \\ & \\ XZ &= \frac{3}{5} \times 20 = 12 \text{ cm} \\ & \\ & \\ YZ &= \frac{2}{5} \times 20 = 8 \text{ cm} \\ & \end{aligned}$$

Calculate the surface area with these:

$$\begin{aligned} & \\ SA &= 2 (20 \times 12 + 20 \times 8 + 12 \times 8) = 2 (240 + 160 + 96) = 2 \times 496 = 992 \text{ cm}^2 \\ & \end{aligned}$$

Again, too large. To match the surface area of 720 sq cm, the dimensions must be scaled down proportionally.

Set:

$$\begin{aligned} & \\ XY &= 20 \text{ cm} \\ & \\ & \\ XZ &= 12 \text{ cm} \\ & \\ & \\ YZ &= 8 \text{ cm} \\ & \end{aligned}$$

and scale all by a factor (k) :

$$\begin{aligned} & \\ XY' &= 20k \\ & \\ & \\ XZ' &= 12k \\ & \\ & \\ YZ' &= 8k \\ & \end{aligned}$$

Total surface area:

$$\begin{aligned} & \\ 720 &= 2 (20k \times 12k + 20k \times 8k + 12k \times 8k) \end{aligned}$$

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