

The Base Of The Solid Is Square, One Of Whose Sides Is The Interval $[0, 2]$ Along The X-axis. The

The Base Of The Solid Is Square, One Of Whose Sides Is The Interval $[0, 2]$ Along The X-axis. The construction and analysis of three-dimensional solids with specified bases are fundamental topics in calculus and solid geometry. This article explores the characteristics, volume calculation, and various methods for analyzing such a solid, focusing on the base that is a square with one side along the interval $[0, 2]$ on the x-axis. Whether you are a student studying calculus, a teacher preparing instructional material, or a math enthusiast, understanding these concepts will deepen your grasp of spatial reasoning and integral calculus.

Understanding the Geometric Setup

The Base of the Solid

The base of the solid is a square, positioned in the xy-plane. One side of this square lies along the interval $[0, 2]$ on the x-axis, meaning:

- The x-coordinates of the square's vertices span from 0 to 2.
- The other sides are perpendicular to this side, extending along the y-axis.
- The square's side length along the x-axis is 2 units.

Assuming the square is aligned with the coordinate axes, its vertices can be represented as:

- $(0, 0)$
- $(2, 0)$
- $(2, a)$
- $(0, a)$

where 'a' is the length of the side along the y-axis. The value of 'a' depends on the problem's specifics—whether the square is fixed or variable.

Positioning the Square in the Coordinate Plane

For simplicity, let's consider the square to be positioned with its vertices at:

- $(0, 0)$
- $(2, 0)$
- $(2, a)$
- $(0, a)$

This setup allows us to analyze the solid formed when this base is extended into three dimensions, either through revolution, extrusion, or other operations.

Types of Solids Based on the Square Base

The nature of the solid depends on how the square base is extended or manipulated. Common types include:

1. Rectangular Prism (Cuboid)

- Created by extruding the square along a third dimension (z-axis).
- Example: Extending the square from $z=0$ to $z=h$ results in a rectangular prism.

2. Solid of Revolution

- Generated by revolving the square or a portion of it around an axis.
- Common axes include the x-axis or y-axis.
- Results in shapes like cylinders or more complex solids depending on the profile.

3. Extruded or Revolved Shapes with Variable Cross-Sections

- The cross-sectional shape might vary along the axis of extrusion or revolution.
- Useful in modeling more complex geometries.

Mathematical Formulation of the Volume

Calculating the volume of such a solid involves integrating over the specified domain. The approach depends on the type of solid.

Volume of a Rectangular Prism

If the solid is a simple rectangular prism with height h :

- Volume, $V = \text{base area} \times \text{height} = (a^2) h$

Volume of a Solid of Revolution

Suppose the square is revolved around the x-axis:

- The resulting shape is a cylindrical shell or a more complex solid if the profile varies.
- Using the disk or washer method, the volume can be computed as:

$$V = \int_{x=0}^a \pi [r(x)]^2 dx$$

where $r(x)$ is the radius of the cross-section at position x .

Example:

If the height along y is constant at (a) , and revolve around the x-axis:

$$V = \int_0^2 \pi a^2 dx = \pi a^2 \times 2 = 2 \pi a^2$$

Generalized Volume Calculation

When the shape or profile varies, the volume integral adapts accordingly. For example:

- If the height $(a(y))$ varies with y, and the base extends from $x=0$ to $x=2$, $y=0$ to $y=a$:

$$V = \int_{y=0}^a \left[\int_{x=0}^2 f(x, y) dx \right] dy$$

- Here, $(f(x, y))$ defines the height or the profile function of the solid.

Applications and Examples

To solidify understanding, consider practical examples and applications.

Example 1: Cylindrical Solid from Revolution

- Base: square with vertices at $(0, 0)$, $(2, 0)$, $(2, a)$, $(0, a)$.
- Operation: revolve around the x-axis.
- Result: a cylindrical shape with height (a) and radius 2.

Volume Calculation:

$$V = \pi \times (\text{radius})^2 \times \text{height} = \pi \times 2^2 \times a = 4\pi a$$

Example 2: Rectangular Prism

- Extend the square along the z-axis from 0 to h.
- Volume:

$$V = \text{area of base} \times \text{height} = (2 \times a) \times h$$

Advanced Topics and Variations

The initial setup can be extended to more complex scenarios.

Variable Base Lengths

- The side length of the square base may vary along the x-axis or y-axis.
- For example, if the side length is a function $a(x)$, the volume integral becomes:

$$V = \int_{x=0}^2 a(x)^2 dx$$

Non-rectangular Bases

- The base can be a square deformed into other quadrilaterals or polygons.
- Calculations involve setting appropriate limits and functions to describe the shape.

Revolving Around Different Axes

- Revolving around the y-axis or other axes creates different geometries.
- The choice of axis impacts the integral setup and resulting volume.

Summary and Key Takeaways

- The base of the solid is a square with one side along $[0, 2]$ on the x-axis.
- The nature of the solid depends on how the base is extended or revolved.
- Calculating volume involves setting up appropriate integrals using methods like slicing, washers, or shells.
- Variations include changing the profile function or the axis of revolution.

Key Points to Remember:

- The base's dimensions determine the overall size and shape.
- Solid of revolution calculations often use the disk or washer method.
- Extrusions lead to rectangular prisms with volume easily calculated.
- Be mindful of the coordinate system and orientation for accurate integral setup.

Conclusion

Understanding the solid with a square base along the interval $[0, 2]$ on the x-axis offers a foundational perspective in solid geometry and calculus. By analyzing different ways of extending this

base—through extrusion, revolution, or variable profiles—you can model a wide array of three-dimensional objects. Mastery of the integral calculus techniques used to compute their volumes is essential for students and professionals working in mathematics, engineering, architecture, and related fields. As you explore further, consider complex base shapes, multiple axes of revolution, and variable profiles to deepen your understanding of solid geometry's rich landscape.

Frequently Asked Questions

What is the shape of the base of the solid described?

The base of the solid is a square with one side along the interval $[0, 2]$ on the x-axis.

How can the volume of the solid be calculated given the base and its height?

The volume can be found by integrating the height function over the base interval $[0, 2]$ and multiplying by the width of the square.

What assumptions are made about the height of the solid in this problem?

It is typically assumed that the height is a function of x , defined over the interval $[0, 2]$, which determines the shape of the solid.

How does the shape of the base influence the shape of the resulting solid?

Since the base is a square with one side along $[0, 2]$, the solid's cross-sectional shape depends on the height function over this interval, affecting its overall volume and form.

What are common methods to compute the volume of a solid with a square base and variable height?

Common methods include setting up and evaluating a definite integral of the height function over the base interval, often applying techniques from calculus such as integration by parts or substitution.

Additional Resources

The base of the solid is square, one of whose sides is the interval $[0, 2]$ along the x-axis. This statement sets the stage for a deeper exploration into geometric solids, their properties, and the methods used to analyze them. In this article, we will dissect the significance of such a description, understand the geometric implications, and explore the mathematical techniques involved in analyzing these types of solids.

Introduction

The description "the base of the solid is square, one of whose sides is the interval $[0, 2]$ along the x-axis" immediately informs us about the shape and positioning of the solid's foundation. Such statements are common in calculus, geometry, and engineering contexts, where the precise description of a solid's base is essential for calculating volume, surface area, or other properties.

Understanding this phrase requires a grasp of basic geometric concepts, coordinate systems, and the methods used to analyze three-dimensional objects. This article aims to provide a comprehensive guide to interpreting this description and applying it to the analysis of the solid.

Deconstructing the Description

The Base is Square

A square is a four-sided polygon with all sides equal in length and four right angles. When we specify that the base of a solid is square, we are describing the two-dimensional shape that forms the "bottom" of the object in a three-dimensional space.

Implication:

- The base lies in a plane, typically the xy-plane unless otherwise specified.
- The shape of the base is a perfect square, facilitating calculations due to symmetry and regularity.

One of Its Sides is the Interval $[0, 2]$ Along the X-Axis

This part specifies the position and extent of one side of the square along the x-axis:

- The side runs from $x=0$ to $x=2$, making its length 2 units.
- The side lies along the x-axis, which suggests that this side is aligned with the coordinate axis and may be positioned at $y=0$, depending on the orientation.

Combining the Two Parts

Putting these together, we understand that:

- The base is a square lying in a plane, with one side explicitly aligned along the x-axis from 0 to 2.
- The remaining sides are determined by the square's properties, with the other sides parallel and perpendicular to this known side.

Visualizing the Solid

Step 1: Establishing Coordinates

Assuming the square lies in the xy-plane for simplicity:

- The side on the x-axis spans from $(0,0)$ to $(2,0)$.
- Since the base is a square, the other sides are perpendicular and equal in length.

Step 2: Determining the Coordinates of the Square

Let's denote:

- The side along the x-axis: from (0, 0) to (2, 0).
- The side parallel to the x-axis at the other end: from (0, b) to (2, b), where b is the length of the side along the y-axis.

Since the base is a square, the length of the side along y must also be 2:

- The vertices are:
- A: (0, 0)
- B: (2, 0)
- C: (2, 2)
- D: (0, 2)

This square has vertices at these points, with the side along the x-axis being from A to B.

Step 3: Extending into Three Dimensions

The base lies in the xy-plane ($z=0$). To analyze the solid, we need to know:

- The height of the solid (how tall it is along the z-axis).
- The shape of the top surface (if any), or whether the solid extends uniformly upward.

If the problem states nothing further, common assumptions include:

- The solid extends vertically (along z) from the base.
- The top could be flat, curved, or defined differently depending on context.

Mathematical Analysis of the Solid

Once the base is understood, the next step involves analyzing the solid's properties, such as:

1. Volume Calculation

Method:

- Double Integration: Integrate over the base area, then extend along the height.
- Using Cross-Sections: If the cross-sectional shape varies, analyze slices.

Example:

Suppose the solid extends vertically with height h , which could be a constant or a function of x and y .

Volume formula:

$$V = \iint_D h(x, y) \, dx \, dy$$

where D is the domain of the base square, i.e., $(0 \leq x \leq 2)$, $(0 \leq y \leq 2)$.

2. Surface Area Calculation

Depending on the shape of the top and sides, surface area can be computed by summing areas of each surface component.

3. Centroid and Center of Mass

Using integration, the centroid coordinates:

$$\begin{aligned}\bar{x} &= \frac{1}{V} \iiint_V x \, dV, \quad \bar{y} = \frac{1}{V} \iiint_V y \, dV, \quad \bar{z} \\ &= \frac{1}{V} \iiint_V z \, dV\end{aligned}$$

are determined based on the volume distribution.

Common Variations and Extensions

Varying the Height Function

Instead of a uniform height, the height could be a function $h(x, y)$, giving a more complex shape.

Examples:

- $h(x, y) = 3 - x$, creating a sloped surface.
- $h(x, y) = \sin(\pi x/2) \times \sin(\pi y/2)$, resulting in a curved top.

Changing the Orientation

The base could be oriented differently:

- In the yz -plane, with the side along $[0, 2]$ in the y -direction.
- Inclined or skewed, introducing more complex coordinate transformations.

Solid of Revolution

Rotating the base square around an axis (e.g., the x -axis) creates a three-dimensional shape like a cylinder or a more complex solid.

Practical Applications and Real-World Relevance

Understanding the base of a solid is fundamental in numerous fields:

- Engineering: Designing components with specific volume and surface constraints.
- Architecture: Calculating materials needed for construction.
- Physics: Analyzing mass distribution and center of gravity.
- Mathematics and Education: Developing intuition about three-dimensional objects and integral calculus.

Summary and Key Takeaways

- The phrase "the base of the solid is square, one of whose sides is the interval $[0, 2]$ along the x -axis" describes a fundamental geometric property that influences the entire analysis of the solid.
- Recognizing the orientation and dimensions of the base simplifies the process of calculating volume, surface area, and other properties.

- Assumptions about the height, top surface, and orientation are necessary for precise calculations, but the core understanding begins with the base shape and its positioning.
- Variations in the height function or orientation lead to more complex but solvable problems using calculus and coordinate transformations.

Final Thoughts

Describing a solid with such precise language emphasizes the importance of clear geometric understanding in problem-solving. Whether dealing with simple prisms or complex solids, starting with the base's shape and position provides a solid foundation—pun intended—for further analysis. As you encounter similar descriptions in textbooks, exams, or real-world applications, remember that breaking down the description into coordinate geometry, visualization, and integration techniques will often lead to straightforward solutions.

By mastering these fundamental concepts, you'll be well-equipped to analyze and understand a wide variety of three-dimensional shapes, laying the groundwork for advanced mathematical and engineering pursuits.

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