

The Can Of Soup Has A Surface Area Of 286.3 Square Centimeters. What Is The Height Of The Can? Round

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Understanding the dimensions of a can of soup is an intriguing mathematical problem that combines geometry, algebra, and practical estimations. When given the total surface area of a cylindrical can, such as 286.3 square centimeters, determining its height involves applying formulas related to the surface area of cylinders. This article explores how to calculate the height of a can based on its surface area, providing a comprehensive guide for students, educators, and curious minds alike.

Introduction to Cylinder Geometry and Surface Area

Before diving into calculations, it's essential to understand the basic geometry of a cylinder and how surface area is determined.

What Is a Cylinder?

A cylinder is a three-dimensional geometric shape with:

- Two parallel circular bases of equal size
- A curved surface connecting the bases

In the context of a soup can, the shape is typically a right circular cylinder, which means the sides are perpendicular to the bases.

Surface Area of a Cylinder

The surface area (SA) of a cylinder includes the areas of the two circular bases and the curved surface area (lateral surface).

The formula for the total surface area is:

$$SA = 2\pi r^2 + 2\pi r h$$

Where:

- r = radius of the base
- h = height of the cylinder

- $\pi \approx 3.1416$

This formula sums the area of the top and bottom bases ($2\pi r^2$) and the lateral surface area ($2\pi r h$).

Given Data and Unknowns

In the problem:

- Total surface area, $\text{SA} = 286.3 \text{ cm}^2$
- Unknown: height h (rounded to the nearest unit)
- Also, the radius r of the can is not directly given.

To find the height, we need to make assumptions or obtain the radius.

Assumption: The Radius Is Known or Estimated

Since the problem does not specify the radius, typical soup cans have standard dimensions. For example, a standard can might have a radius around 3 cm, but for precision, let's consider the general case.

Alternatively, if the problem provides or implies the radius, we can proceed with calculations. For the sake of completeness, let's explore the calculation process assuming a known radius.

Calculating the Height of the Can

To find the height h , rearrange the surface area formula:

$$\text{SA} = 2\pi r^2 + 2\pi r h$$

Solve for h :

$$h = \frac{\text{SA} - 2\pi r^2}{2\pi r}$$

This formula indicates that, given a radius r , we can compute h .

Let's proceed with a practical example assuming a standard radius.

Example Calculation with a Radius of 3 cm

Suppose the radius $r = 3 \text{ cm}$. Then,

1. Calculate the area of the two bases:

$$2\pi r^2 = 2 \times 3.1416 \times 3^2 = 2 \times 3.1416 \times 9 = 2 \times 28.2744 = 56.5488 \text{ cm}^2$$

2. Subtract this from the total surface area:

$$286.3 - 56.5488 = 229.7512 \text{ cm}^2$$

3. Compute the height:

$$h = \frac{229.7512}{2 \times 3.1416 \times 3} = \frac{229.7512}{18.8496} \approx 12.2 \text{ cm}$$

So, with a radius of 3 cm, the can's height is approximately 12.2 cm.

Adjusting for Different Radii

If the radius varies, the height will change accordingly. For instance:

Radius (cm)	Calculated Height (cm)
2 cm	around 17.6 cm
3 cm	around 12.2 cm
4 cm	around 8.3 cm

This illustrates the inverse relationship between radius and height for a fixed surface area.

Understanding the Relationship Between Radius and Height

The key insight is that the surface area formula couples radius and height. As the radius increases, the height decreases for a fixed surface area, and vice versa.

Graphical Interpretation

Plotting the relationship between radius and height based on the surface area formula reveals a hyperbolic curve.

- For larger radii, the height becomes shorter.
- For smaller radii, the height increases.

This understanding can help in designing cans with specific dimensions or estimating sizes based on given surface areas.

Practical Applications and Real-World Considerations

While the mathematical calculations provide precise estimates, real-world can manufacturing considers practical constraints:

- Standard sizes for cans are typically based on manufacturing standards.
- The radius and height are chosen to optimize volume and ease of stacking.
- The volume of the can is also a critical factor, calculated by:

$$V = \pi r^2 h$$

which determines how much soup the can holds.

Example: Calculating the Volume of the Can

Using the previous example with $(r = 3 \text{ cm})$ and $(h \approx 12.2 \text{ cm})$:

$$V = 3.1416 \times 3^2 \times 12.2 \approx 3.1416 \times 9 \times 12.2 \approx 3.1416 \times 109.8 \approx 344.8 \text{ cm}^3$$

This volume indicates the capacity of the can, which is relevant for product packaging and marketing.

Conclusion and Summary

Determining the height of a cylindrical can based on its surface area involves understanding the geometry of cylinders and applying the surface area formula. Given a total surface area of 286.3 square centimeters, calculating the height requires knowledge or estimation of the radius. Using the formula:

$$h = \frac{\text{SA} - 2\pi r^2}{2\pi r}$$

you can compute the height for any known radius. For common sizes, assuming a radius around 3 cm yields a height of approximately 12.2 cm, which aligns with typical dimensions of soup cans.

This exercise emphasizes the importance of geometric formulas in real-world applications, from packaging design to manufacturing quality control. It also highlights the relationship between a can's dimensions and how changing one measurement affects others, fostering a deeper understanding of three-dimensional shapes and their properties.

Remember: When solving similar problems, clearly identify known quantities, employ the correct formulas, and consider practical constraints for the most accurate and applicable results.

Frequently Asked Questions

Given a soup can with a surface area of 286.3 cm², how do you find its height if the radius is known?

To find the height, first use the surface area formula for a cylinder: $SA = 2\pi r(h + r)$. Rearrange to solve for h : $h = (SA / (2\pi r)) - r$. Plug in the known surface area and radius to calculate the height.

If the radius of the soup can is 5 cm and the surface area is 286.3 cm², what is the height of the can?

Using the formula $SA = 2\pi r(h + r)$: $286.3 = 2\pi \cdot 5 (h + 5)$. Simplify to find h : $286.3 = 10\pi(h + 5)$. Then, $h + 5 = 286.3 / (10\pi) \approx 286.3 / 31.416 \approx 9.11$. So, $h \approx 9.11 - 5 = 4.11$ cm.

How do you approximate the height of a cylindrical soup can with a surface area of 286.3 cm² if the radius is not given?

Without the radius, you need additional information. If you assume a radius based on typical can dimensions or have data about volume, you can substitute into the surface area formula and solve for height. Otherwise, the radius must be known to find the height.

What is the formula to determine the height of a cylinder when the surface area and radius are known?

The formula is $h = (SA / (2\pi r)) - r$, where SA is the surface area and r is the radius. This rearranged formula allows you to calculate the height directly.

Why is it important to know the radius when calculating the height of a cylinder with a given surface area?

Because the surface area formula for a cylinder depends on both the radius and height ($SA = 2\pi r(h + r)$). Knowing the radius allows you to solve for the height accurately; without it, you cannot determine the height uniquely.

Additional Resources

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In the realm of everyday objects, canned soup stands out as a ubiquitous item found in nearly every household pantry. Beyond its convenience and nutritional value, the design and dimensions of a soup can serve as an intriguing subject for mathematical exploration. When faced with the problem of determining the height of a can given its surface area, we step into the fascinating intersection of geometry, algebra, and real-world application. Specifically, this problem invites us to analyze how the surface area of a cylindrical object relates to its dimensions, and how to reverse-engineer these

measurements to find an unknown parameter—in this case, the height of the can. This article aims to dissect this problem comprehensively, providing clarity on the concepts involved, the step-by-step solution process, and the broader implications of such calculations.

Understanding the Geometric Model: The Cylinder

The Cylinder as a Geometric Shape

Cans, especially those used for soup, are typically modeled as cylinders for simplicity and manufacturing efficiency. A cylinder is a three-dimensional shape characterized by:

- Two parallel circular bases of equal size
- A curved surface connecting these bases

Understanding the geometry of a cylinder is essential for solving the problem, as the surface area depends on its radius and height.

Components of the Surface Area

The total surface area (SA) of a cylinder includes:

- The area of the two circular bases: $2\pi r^2$
- The lateral surface area (the side): $2\pi r h$

Where:

- r is the radius of the base
- h is the height of the cylinder
- π is a constant approximately equal to 3.1416

Thus, the total surface area formula for a cylinder is:

$$SA = 2\pi r^2 + 2\pi r h$$

Applying the Surface Area Data

Given the surface area, the goal is to find the height of the can, assuming the surface area is 286.3 square centimeters. To proceed, we must recognize what information is known and what assumptions are necessary.

Assumptions and Known Data

- Surface Area (SA): 286.3 cm²
- Shape: Perfect cylinder
- Unknowns: Radius (r) and height (h)
- Additional assumptions: For this problem, unless specified, we may assume that the can has a standard or typical radius, or that the problem is designed with a specific radius in mind.

Clarifying the Goal

Since the problem asks specifically for the height and provides only the total surface area, it suggests that either:

- The radius is known or can be reasonably estimated, or
- The problem is structured such that the radius and height are related in a way that allows us to solve for one, given the other.

In many practical contexts, the radius of a standard soup can is known or can be estimated from common dimensions. For example, a typical soup can has a radius around 3.5 cm to 4 cm.

Solving the Problem: Step-by-Step Approach

Step 1: Establish Known and Unknown Variables

Let's denote:

- r as the radius of the can (unknown or estimated)
- h as the height of the can (what we want to find)

Given:

$$SA = 2\pi r^2 + 2\pi r h = 286.3$$

Our goal:

$$h = ?$$

Step 2: Rearrange the Surface Area Formula

To isolate h , rewrite the formula:

$$2\pi r h = SA - 2\pi r^2$$

$$h = \frac{SA - 2\pi r^2}{2\pi r}$$

This expression allows us to compute h once r is known or estimated.

Step 3: Estimating the Radius

Suppose we estimate the radius based on typical can dimensions:

- Standard can radius: approximately 3.5 cm

Using this value:

$$r = 3.5, \text{ cm}$$

Step 4: Calculate the Height

Calculate the two parts:

- $(2 \pi r^2)$:

$$(2 \times 3.1416 \times (3.5)^2 = 2 \times 3.1416 \times 12.25 \approx 2 \times 38.48 \approx 76.96, \text{cm}^2)$$

- Subtract from total surface area:

$$(286.3 - 76.96 = 209.34, \text{cm}^2)$$

- Denominator:

$$(2 \pi r = 2 \times 3.1416 \times 3.5 \approx 2 \times 3.1416 \times 3.5 \approx 2 \times 10.9956 \approx 21.991, \text{cm})$$

- Compute height:

$$(h = \frac{209.34}{21.991} \approx 9.52, \text{cm})$$

Result: The estimated height of the can is approximately 9.52 cm.

Refining the Calculation: Sensitivity to Radius

Given the estimation, it's important to understand how different radius assumptions influence the height calculation. For example, if the radius is slightly larger or smaller, how does that change the height?

- If $(r = 4, \text{cm})$:

$$(2 \pi r^2 = 2 \times 3.1416 \times 16 = 2 \times 50.265 = 100.53)$$

- Remaining surface area:

$$(286.3 - 100.53 = 185.77)$$

- Denominator:

$$(2 \pi r = 2 \times 3.1416 \times 4 = 2 \times 12.566 = 25.1327)$$

- Height:

$$(h = 185.77 / 25.1327 \approx 7.4, \text{cm})$$

- If $(r = 3, \text{cm})$:

$$(2 \pi r^2 = 2 \times 3.1416 \times 9 = 2 \times 28.274 = 56.55)$$

- Remaining surface area:

$$(286.3 - 56.55 = 229.75)$$

- Denominator:

$$(2 \pi r = 2 \times 3.1416 \times 3 = 2 \times 9.4248 = 18.8496)$$

- Height:

$$(h = 229.75 / 18.8496 \approx 12.2, \text{ cm})$$

This sensitivity analysis illustrates that the radius significantly affects the calculated height, emphasizing the importance of having accurate or estimated dimensions.

Broader Implications and Practical Applications

Relevance in Manufacturing and Quality Control

Understanding the relationship between surface area and dimensions is vital for manufacturers aiming to produce cans with specific volume capacities, aesthetic qualities, and material efficiency. Accurate calculations ensure consistency, cost-effectiveness, and compliance with packaging standards.

Educational Significance

Problems like these serve as excellent teaching tools in geometry and algebra, demonstrating how mathematical formulas directly relate to real-world objects. They also enhance problem-solving skills, critical thinking, and the ability to work with estimations and approximations.

Design and Material Optimization

By manipulating the dimensions of a can—radius and height—manufacturers can optimize material usage while maintaining volume and strength. Calculations of surface area are crucial in determining the amount of material needed, impacting both sustainability and cost.

Limitations and Considerations

Assumptions in the Model

- The model assumes perfect cylindrical shapes, whereas real cans may have slight deviations due to manufacturing tolerances.
- Estimations of radius influence the accuracy of the height calculation.
- The problem does not specify whether the can is open or closed, but since the surface area includes both bases, it's assumed to be a closed cylinder.

Potential for Additional Data

If the volume of the can were provided, more precise calculations could be made to determine the radius and height simultaneously, using both volume and surface area formulas.

Conclusion: Synthesis of Mathematical Insight and Practical Understanding

In conclusion, determining the height of a soup can with a known surface area involves understanding the geometric properties of cylinders, applying the surface area formula, and making reasonable estimations or assumptions about the radius. The calculations demonstrate that, with an estimated radius of approximately 3.5 cm, the can's height is around 9.52 cm. This analytical process underscores the importance of mathematical modeling in everyday objects, revealing how simple formulas can unlock insights into design, manufacturing, and quality control. It also highlights the interconnectedness of geometry and algebra in solving real-world problems, fostering a deeper appreciation for the role of mathematics beyond the classroom.

Whether for educational purposes, industrial design, or consumer awareness, understanding how to manipulate and interpret surface area data equips individuals with valuable skills applicable across various domains. As objects like soup cans continue to be part of daily life, the ability to analyze their dimensions mathematically remains both relevant and intellectually rewarding.

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