# The Current Through A 0.5-f Capacitor Is 6(1-e-t)a. Determine The Voltage And Power At T=2 S. Assume

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Understanding the behavior of capacitors in electrical circuits is fundamental for engineers and students alike. When analyzing a capacitor's response to a given current, it is crucial to understand how voltage and power are related to current over time. This article explores the problem of determining the voltage and power across a 0.5-f capacitor at a specific time, given a prescribed current function. We will walk through the fundamental concepts, detailed calculations, and implications of these results, providing a comprehensive guide for both theoretical understanding and practical applications.

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## Introduction to Capacitors and Their Behavior in Circuits

Capacitors are passive electronic components that store electrical energy in an electric field. They are widely used in various applications, including filtering, energy storage, and signal processing. The fundamental relationship governing a capacitor is:

$$ i(t) = C \frac{d v(t)}{dt} $$

where:
- $i(t)$ is the instantaneous current through the capacitor,
- $v(t)$ is the voltage across the capacitor,
- $C$ is the capacitance.

Understanding this relationship allows us to determine the voltage across the capacitor when the current is known, and vice versa.

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## Problem Statement and Given Data

The problem provides the current through a 0.5-f capacitor as a function of time:

$$
i(t) = 6(1 - e^{-t})\, \text{A}
$$

where:
- $C = 0.5\, \text{F}$,
- $t$ is in seconds.

The task is to:

1. Determine the voltage $v(t)$ across the capacitor at $t = 2\, \text{seconds}$.
2. Calculate the instantaneous power $p(t)$ at $t = 2\, \text{seconds}$.

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## Fundamental Equations and Approach

To find the voltage and power at a specific time, we need to:

- Integrate the current over time to find the voltage,
- Use the relationship between current and voltage in a capacitor,
- Calculate power as the product of instantaneous voltage and current.

The key equations are:

$$
v(t) = \frac{1}{C} \int_{0}^{t} i(\tau) d\tau + v(0)
$$

Assuming the initial voltage $v(0) = 0$ (the capacitor initially uncharged), this simplifies the analysis.

The power at any time is:

$$
p(t) = v(t) \times i(t)
$$

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## Calculating the Voltage $v(t)$ at $t=2\,

# \text{seconds} \)

## Step 1: Integrate the Current to Find Voltage

Given the initial voltage \( v(0) = 0 \), the voltage across the capacitor is:

\[
v(t) = \frac{1}{C} \int_{0}^{t} i(\tau) d\tau
\]

Substitute \( i(\tau) = 6(1 - e^{-\tau}) \):

\[
v(t) = \frac{1}{0.5} \int_{0}^{t} 6(1 - e^{-\tau}) d\tau
\]

Simplify the constant:

\[
v(t) = 2 \times 6 \int_{0}^{t} (1 - e^{-\tau}) d\tau = 12 \int_{0}^{t} (1 - e^{-\tau}) d\tau
\]

## Step 2: Calculate the Integral

Compute:

\[
\int_{0}^{t} (1 - e^{-\tau}) d\tau = \int_{0}^{t} 1 d\tau - \int_{0}^{t} e^{-\tau} d\tau
\]

which yields:

\[
t - \left[ -e^{-\tau} \right]_0^t = t - \left( -e^{-t} + e^{0} \right) = t - ( -e^{-t} + 1 ) = t + e^{-t} - 1
\]

Therefore:

\[
v(t) = 12 \left( t + e^{-t} - 1 \right)
\]

### Step 3: Find $v(2)$

Plug in $t=2$:

$$
v(2) = 12 \left( 2 + e^{-2} - 1 \right) = 12 (1 + e^{-2})
$$

Recall that $e^{-2} \approx 0.1353$:

$$
v(2) \approx 12 (1 + 0.1353) = 12 \times 1.1353 \approx 13.6236\, \text{V}
$$

Result:
$$
\boxed{
v(2) \approx 13.62\, \text{V}
}
$$

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# Calculating the Power $p(t)$ at $t=2\, \text{seconds}$

### Step 1: Recall the current at $t=2$

$$
i(2) = 6(1 - e^{-2}) \approx 6 \times (1 - 0.1353) = 6 \times 0.8647 \approx 5.188\, \text{A}
$$

### Step 2: Use the voltage and current to find power

Power is:

$$
p(t) = v(t) \times i(t)
$$

At $t=2$:

$$

p(2) = 13.62\, \text{V} \times 5.188\, \text{A} \approx 70.66\, \text{W}
\]

Result:
\[
\boxed{
p(2) \approx 70.66\, \text{W}
}
\]

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# Discussion of Results

The calculations reveal that at \( t=2\, \text{seconds} \):

- The voltage across the capacitor is approximately 13.62 volts.
- The instantaneous power dissipated or stored is approximately 70.66 watts.

These values reflect the capacitor's charging process, where the current initially is higher and decreases exponentially, while the voltage builds up over time.

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# Additional Insights and Practical Considerations

## 1. Behavior Over Time

- As \( t \to \infty \), \( e^{-t} \to 0 \), and the voltage approaches:

\[
v(\infty) = 12 ( \infty + 0 - 1 ) \to \infty
\]

but since the current approaches 6 A asymptotically, the voltage will tend to a steady state. In fact, the integral indicates that the voltage stabilizes at a finite value, considering the initial conditions and the nature of the current.

## 2. Power Considerations

- The power calculated is instantaneous; in real circuits, power dissipation

would also depend on resistive elements.
- Since the analysis assumes an ideal capacitor with no resistance, the power here represents energy stored in the electric field.

### 3. Application in Circuit Design

- Understanding the voltage and power at specific times helps in designing circuits that require precise timing and energy management.
- For example, in pulse circuits, knowing how quickly a capacitor charges is critical for timing applications.

### 4. Limitations and Assumptions

- The initial voltage $v(0)$ was assumed to be zero.
- The calculations assume ideal components, neglecting parasitic resistances and inductances, which would affect real-world behavior.

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## Conclusion

This detailed analysis demonstrates how to determine the voltage across a capacitor and the instantaneous power at a specific time, given a known current function. By integrating the current over time and applying fundamental capacitor equations, engineers can predict the behavior of capacitive components in various circuits. The approach outlined here—integrating current to find voltage and multiplying voltage by current for power—is fundamental in electrical engineering and essential for designing and analyzing dynamic systems involving capacitors.

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## Summary of Key Results

- Voltage at $t=2\, \text{seconds}$: approximately 13.62 volts
- Power at $t=2\, \text{seconds}$: approximately 70.66 watts

Understanding these calculations provides a foundation for analyzing more complex circuits and responses involving capacitors and other reactive components.

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Meta Description:
Learn how to determine the voltage and power across a 0.5-f capacitor at a

specific time given a current function. This comprehensive guide covers integral calculations, practical implications, and key concepts in circuit analysis.

# Frequently Asked Questions

## What is the initial voltage across the 0.5 F capacitor when the current is $6(1 - e^{-t})$ A?

At $t = 0$, the current is $6(1 - e^{0}) = 6(1 - 1) = 0$ A. Since $I = C\, dV/dt$, the initial voltage can be found by integrating the current over time, but typically, if initial voltage is not specified, it is assumed to be zero.

## How do you determine the voltage across the capacitor at a specific time t = 2 seconds?

Using the relation $I = C\, dV/dt$, integrate the current over time: $V(t) = (1/C) \int I(t)\, dt$. For $I(t) = 6(1 - e^{-t})$, integrate from 0 to 2 seconds to find $V(2)$.

## What is the voltage across the capacitor at t = 2 seconds for the given current?

$V(2) = (1/0.5) \int_0^{2} 6(1 - e^{-t})\, dt = 2\, [6t + 6e^{-t}]_0^{2} = 2\, [62 + 6e^{-2} - 0 - 6e^{0}] = 2\, [12 + 6/e^2 - 6] = 2\, [6 + 6/e^2] \approx 2\, (6 + 6/7.389) \approx 2\, (6 + 0.81) \approx 2\ 6.81 \approx 13.62$ V.

## How do you calculate the power dissipated or stored in the capacitor at t = 2 seconds?

Power in a capacitor is $P = V\, I$. At $t = 2$ s, first find $V(2)$ as above, then multiply by the current at $t = 2$ seconds: $I(2) = 6(1 - e^{-2}) \approx 6(1 - 0.135) \approx 6\ 0.865 \approx 5.19$ A. Therefore, $P \approx 13.62\text{ V}\ 5.19\text{ A} \approx 70.66$ W.

## What assumptions are made in calculating the voltage and power in this problem?

Assumptions include that the initial voltage across the capacitor is zero, the system is ideal with no parasitic elements, and the current function is accurate and continuous for $t \geq 0$.

## Why is the exponential term $e^{-t}$ significant in

### the current expression, and how does it affect voltage and power calculations?

The $e^{-t}$ term indicates the transient response of the capacitor's current, reflecting how it approaches steady-state. It influences the voltage and power calculations by determining how quickly the capacitor charges and how energy is stored over time.

## Additional Resources

The Current Through A 0.5-f Capacitor Is 6(1-e-t) A. Determine The Voltage And Power At T=2 s. Assume

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Introduction

Capacitors are fundamental components in electrical and electronic circuits, serving as energy storage devices that influence the dynamic behavior of circuits. Understanding the interplay between current, voltage, and power in capacitive elements is essential for designing efficient and reliable electronic systems. This article investigates a specific case: a capacitor with capacitance $C = 0.5\, \text{F}$ through which a time-dependent current $i(t) = 6(1 - e^{-t})\, \text{A}$ flows. The primary goal is to determine the voltage across the capacitor and the instantaneous power at $t=2\, \text{s}$.

This analysis is not only academically instructive but also practically relevant, especially in scenarios involving transient responses in power electronics, signal processing, and energy storage applications.

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Theoretical Background

Capacitor Basics

A capacitor's fundamental behavior relates the current $i(t)$ through it to the voltage $v(t)$ across its terminals:

$$
i(t) = C \frac{d v(t)}{dt}
$$

where:

- $C$ is the capacitance (Farads),
- $v(t)$ is the voltage (Volts),
- $i(t)$ is the current (Amperes).

The inverse relation allows us to find $v(t)$ given $i(t)$:

$$
v(t) = \frac{1}{C} \int i(t) \, dt + v(0)
$$

where $v(0)$ is the initial voltage across the capacitor at $t=0$.

Power in a Capacitive Circuit

The instantaneous power $p(t)$ delivered to the capacitor is given by:

$$
p(t) = v(t) \times i(t)
$$

Since a capacitor ideally only stores energy (and does not dissipate it), the power can be positive or negative, indicating energy being stored or released.

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Problem Setup and Assumptions

Given data:

- Capacitance: $C = 0.5\, \text{F}$
- Current: $i(t) = 6(1 - e^{-t})\, \text{A}$
- Time of interest: $t = 2\, \text{s}$

Assumptions:

1. The initial voltage across the capacitor at $t=0$ is zero: $v(0) = 0\, \text{V}$. This is a common starting point unless otherwise specified.
2. The circuit is ideal; no resistive or other parasitic elements are considered.
3. The current function is valid for all $t \geq 0$.

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Determining the Voltage $v(t)$

Step 1: Integrate the Current

Using the relation:

$$
v(t) = \frac{1}{C} \int_{0}^{t} i(\tau) \, d\tau + v(0)
$$

Given $v(0) = 0$, the voltage simplifies to:

$$
v(t) = \frac{1}{0.5} \int_{0}^{t} 6(1 - e^{-\tau}) \, d\tau
$$

which simplifies further to:

$$
v(t) = 2 \times \int_{0}^{t} 6(1 - e^{-\tau}) \, d\tau
$$

Step 2: Calculate the Integral

Break the integral into parts:

$$
v(t) = 12 \int_{0}^{t} (1 - e^{-\tau}) \, d\tau
$$

The integral:

$$
\int_{0}^{t} 1 \, d\tau = t
$$

$$
\int_{0}^{t} e^{-\tau} \, d\tau = [ - e^{-\tau} ]_0^{t} = 1 - e^{-t}
$$

Putting it all together:

$$
v(t) = 12 [ t - (1 - e^{-t}) ] = 12 ( t - 1 + e^{-t} )
$$

Final Expression for Voltage:

$$
\boxed{
v(t) = 12 \left( t - 1 + e^{-t} \right) \quad \text{Volts}
}
$$

---

Calculating Voltage and Power at $t=2\, \text{s}$

Step 1: Compute $v(2)$

Substitute $t=2$:

\[

$$
v(2) = 12 \left( 2 - 1 + e^{-2} \right) = 12 (1 + e^{-2})
$$

Calculate $e^{-2}$:

$$
e^{-2} \approx 0.1353
$$

Therefore:

$$
v(2) \approx 12 (1 + 0.1353) = 12 \times 1.1353 \approx 13.6236\, \text{V}
$$

Voltage at $t=2\, \text{s}$:

$$
\boxed{
v(2) \approx 13.62\, \text{V}
}
$$

Step 2: Calculate $i(2)$

Verify the current at $t=2$:

$$
i(2) = 6 (1 - e^{-2}) = 6 (1 - 0.1353) = 6 \times 0.8647 \approx 5.188\, \text{A}
$$

Step 3: Calculate Power $p(2)$

$$
p(2) = v(2) \times i(2) \approx 13.62 \times 5.188 \approx 70.66\, \text{W}
$$

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Analysis and Implications

Energy Storage and Power Dynamics

The calculated voltage and current at $t=2\, \text{s}$ indicate that the capacitor has accumulated energy, with the voltage across it reaching approximately 13.62 V. The instantaneous power of roughly 70.66 W signifies active energy transfer into the capacitor at this moment.

Transient Behavior

The exponential term $e^{-t}$ describes the transient response of the capacitor to the applied current. The initial current at $t=0$ is:

$$
i(0) = 6 (1 - 1) = 0\, \text{A}
$$

which ramps up to a steady-state value of 6 A as $t \rightarrow \infty$. Correspondingly, the voltage across the capacitor increases from zero, approaching a steady value:

$$
\lim_{t \to \infty} v(t) = 12 ( t - 1 + 0 ) \rightarrow \infty
$$

but practically, since the current approaches 6 A, the voltage's growth is unbounded with continuous application of this current, indicating a scenario where energy storage continues indefinitely unless other circuit elements limit it.

Practical Considerations

In real-world applications, resistive elements and other circuit components would limit the voltage and power. The idealized model here highlights the importance of considering all circuit parameters when designing for safety and efficiency.

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Conclusion

This investigation elucidates the dynamic behavior of a capacitor subjected to a time-dependent current. By integrating the current to find the voltage, and subsequently calculating the power, we gain a comprehensive understanding of the energy transfer process at a specific moment ($t=2\, \text{s}$). The key findings are:

- The voltage across the 0.5 F capacitor at $t=2$ is approximately 13.62 V.
- The instantaneous power delivered to the capacitor at this time is approximately 70.66 W.
- The transient response shows a gradual increase in stored energy, driven by the exponential component of the current.

Understanding such transient behaviors is vital for the design and analysis of circuits involving energy storage, filtering, and dynamic response management. Future studies might incorporate resistive elements, non-idealities, or feedback mechanisms to better mirror real-world conditions.

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References

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Note: This analysis assumes ideal components and initial conditions. Real-world scenarios should consider parasitic effects, resistive losses, and circuit constraints for comprehensive modeling.

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