

The Curve Through The Ordered Pairs (0, 10), (1, 5), And (2, 2.5) Can Be Represented By The Function

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Introduction

Understanding how to find a function that passes through a specific set of points is a fundamental concept in mathematics, particularly in algebra and calculus. When given a collection of ordered pairs, such as (0, 10), (1, 5), and (2, 2.5), the goal is to determine a mathematical expression that accurately describes the relationship between the variables. This process involves selecting an appropriate type of function—be it linear, quadratic, cubic, or of a higher degree—and then finding the parameters that satisfy all the given points. In this article, we explore how to derive such a function systematically and discuss the mathematical principles underpinning this process.

Understanding the Given Data Points

Listing the Points

The data consists of three points:

- Point 1: (0, 10)
- Point 2: (1, 5)
- Point 3: (2, 2.5)

Interpreting the Data

These points suggest a decreasing trend, with the y-values decreasing as x increases. The nature of this trend can be linear, quadratic, or more complex. To determine the most suitable function, we analyze the pattern and attempt to fit a function that exactly passes through these points.

Choosing the Type of Function

Linear Function

A linear function has the form:

$$f(x) = mx + b$$

$\backslash]$
where $\backslash(m\backslash)$ is the slope and $\backslash(b\backslash)$ is the y-intercept.

Given only three points, it's often the simplest approach to check if a linear model fits all points.

Quadratic and Higher-Degree Functions

If the points do not align linearly, quadratic functions of the form:

$$\backslash[\\f(x) = ax^2 + bx + c\\backslash]$$

may be appropriate. For more complex patterns, polynomials of degree three or higher could be considered, but for three points, quadratic or linear functions are the most straightforward options.

Testing for a Linear Function

Calculating the Slope

To test the linear hypothesis, we calculate the slope between pairs of points:

$$\backslash[\\m_{12} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 10}{1 - 0} = -5\\backslash]$$

$$\backslash[\\m_{23} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{2.5 - 5}{2 - 1} = -2.5\\backslash]$$

Since $\backslash(m_{12} \neq m_{23})\backslash$, the slope is not constant, and the points do not lie on a straight line. Therefore, a linear function cannot perfectly fit all three points.

Constructing a Quadratic Function

General Form of a Quadratic

Given three points, a quadratic function is uniquely determined and can be expressed as:

$$\backslash[\\f(x) = ax^2 + bx + c\\backslash]$$

Our goal is to find $\backslash(a\backslash)$, $\backslash(b\backslash)$, and $\backslash(c\backslash)$ such that:

$$\backslash[\\begin{cases} f(0) = 10 \\ f(1) = 5 \\ f(2) = 2.5 \end{cases} \\backslash]$$

Setting Up the System of Equations

Substituting each point into the quadratic equation:

```
\[
\begin{cases}
a(0)^2 + b(0) + c = 10 \rightarrow c = 10 \\
a(1)^2 + b(1) + c = 5 \rightarrow a + b + c = 5 \\
a(2)^2 + b(2) + c = 2.5 \rightarrow 4a + 2b + c = 2.5
\end{cases}
\]
```

Solving the System

From the first equation, $(c = 10)$. Substitute into the remaining:

```
\[
a + b + 10 = 5 \rightarrow a + b = -5
\]
\[
4a + 2b + 10 = 2.5 \rightarrow 4a + 2b = -7.5
\]
```

Divide the second equation by 2:

```
\[
2a + b = -3.75
\]
```

Now, subtract the first from this:

```
\[
(2a + b) - (a + b) = -3.75 - (-5) \rightarrow a = 1.25
\]
```

Using $(a + b = -5)$:

```
\[
1.25 + b = -5 \rightarrow b = -6.25
\]
```

And $(c = 10)$.

Final Function

Thus, the quadratic function passing through all three points is:

```
\[
f(x) = 1.25x^2 - 6.25x + 10
\]
```

Verification of the Function

Checking the Points

- For $(x = 0)$:

```
\[
f(0) = 1.25(0)^2 - 6.25(0) + 10 = 10 \quad \checkmark
\]
```

- For $(x = 1)$:

```
\[
f(1) = 1.25(1)^2 - 6.25(1) + 10 = 1.25 - 6.25 + 10 = 5 \quad \checkmark
\]
```

- For $(x = 2)$:

```
\[
```

$f(2) = 1.25(4) - 6.25(2) + 10 = 5 - 12.5 + 10 = 2.5$ \quad \checkmark
\]

All points are accurately represented by this quadratic function.

Implications and Applications

Mathematical Significance

Constructing such a function illustrates the principle of polynomial interpolation, where a polynomial of degree (n) can be uniquely determined to pass through $(n + 1)$ points. In this case, a quadratic polynomial fits three points exactly.

Practical Applications

- Data Modeling: In sciences and engineering, fitting functions to data points enables analysts to predict and understand underlying patterns.
- Curve Fitting: Polynomial functions serve as approximations for more complex, real-world relationships.
- Optimization: Deriving functions from data is essential in optimization problems where understanding the relationship between variables guides decision-making.

Additional Considerations

Alternative Methods

While polynomial interpolation is straightforward for a small number of points, other methods include:

- Spline Interpolation: Suitable for larger datasets; constructs piecewise polynomials.
- Regression Analysis: Useful when data points contain noise, and an approximate fit is acceptable.

Limitations

- Polynomial functions of high degree can exhibit unwanted oscillations (Runge's phenomenon).
- Overfitting can occur when using high-degree polynomials on small datasets.

Conclusion

By analyzing the ordered pairs $(0, 10)$, $(1, 5)$, and $(2, 2.5)$, we've demonstrated that a quadratic function uniquely passes through all three points. The process involved testing simpler models, recognizing their limitations, and then systematically deriving a quadratic function using algebraic methods. The resulting function, $(f(x) = 1.25x^2 - 6.25x + 10)$, accurately models the relationship between the variables in the given data set. This exercise exemplifies fundamental techniques in mathematical

modeling and polynomial interpolation, which are vital tools across various scientific and engineering disciplines.

Frequently Asked Questions

How can we determine the function that passes through the points $(0, 10)$, $(1, 5)$, and $(2, 2.5)$?

You can find the function by using methods such as polynomial interpolation or fitting a curve (like a quadratic or linear function) that passes through all three points.

What type of function best represents the points $(0, 10)$, $(1, 5)$, and $(2, 2.5)$?

Given the decreasing pattern, a linear or quadratic function could represent these points, but you need to analyze the data to determine the best fit.

How do I find the explicit formula for the function passing through these points?

You can set up equations based on the points and solve for the coefficients of the chosen function type (e.g., linear or quadratic) to find its explicit formula.

Is it possible to interpolate a polynomial through the points $(0, 10)$, $(1, 5)$, and $(2, 2.5)$?

Yes, using polynomial interpolation methods like Lagrange interpolation, you can find a polynomial that exactly passes through all three points.

What is the significance of representing these points with a function?

Representing the points with a function allows us to model the relationship between x and y , enabling predictions and analysis beyond the given data points.

Additional Resources

The Curve Through The Ordered Pairs $(0, 10)$, $(1, 5)$, And $(2, 2.5)$ Can Be Represented By The Function

Understanding how specific points on a coordinate plane relate to an algebraic function is a fundamental aspect of mathematical analysis. When given a set of ordered pairs, such as $(0, 10)$, $(1, 5)$, and $(2, 2.5)$, mathematicians seek to identify a function that precisely passes through these points, thereby capturing the underlying relationship between the variables. This exploration not only elucidates the nature of the curve but also demonstrates the broader principles of polynomial interpolation,

function modeling, and mathematical reasoning.

In this article, we undertake a comprehensive examination of how the given points define a unique function, the methods used to derive it, and the implications of such a representation. We will explore the concepts of polynomial interpolation, specifically focusing on the quadratic function that fits these points, analyze the properties of the resulting curve, and discuss real-world applications where such modeling is essential.

Understanding the Given Points and Their Significance

Identifying the Points

The three ordered pairs provided are:

- (0, 10)
- (1, 5)
- (2, 2.5)

Each point represents a specific input-output relationship, with the first element of the pair indicating the independent variable (usually x), and the second the dependent variable (usually y).

The Significance of These Points

- Data Representation: These points can represent real-world data, such as measurements, observations, or experimental results.
- Mathematical Constraints: Any function that models these points must satisfy the conditions $f(0) = 10$, $f(1) = 5$, and $f(2) = 2.5$.
- Interpolation Challenge: The core challenge is to find a function that passes exactly through these points, effectively "interpolating" the data.

Understanding these points lays the groundwork for constructing an algebraic function that models the data accurately.

Mathematical Foundations: From Points to Functions

What Is Function Interpolation?

Function interpolation involves finding a function that passes through a set of known data points. For n data points, polynomial interpolation typically involves finding a polynomial of degree at most $n-1$ that exactly fits all points.

- Why Interpolation? It allows us to create continuous models from discrete

data, enabling predictions and analyses.

- Common Methods: Polynomial interpolation, spline interpolation, and rational functions.

In our case, with three points, the most straightforward approach is to find a quadratic polynomial, as a polynomial of degree 2 can exactly fit three points.

The Rationale for Using Polynomial Interpolation

- Simplicity: Polynomial functions are easy to analyze and understand.
- Exact Fit: For three points, a quadratic polynomial guarantees an exact fit.
- Flexibility: Quadratic functions can model a variety of behaviors, including concavity and inflection, which linear functions cannot.

Deriving the Function: Polynomial Interpolation Method

Formulating the Quadratic Polynomial

Assuming the function $f(x)$ is quadratic, it can be expressed as:

$$f(x) = ax^2 + bx + c$$

where a , b , and c are coefficients to be determined.

Setting Up the System of Equations

Using the three known points:

1. For $(0, 10)$:

$$f(0) = a(0)^2 + b(0) + c = 10$$

which simplifies to:

$$c = 10$$

2. For $(1, 5)$:

$$a(1)^2 + b(1) + c = 5$$

$$a + b + c = 5$$

Substitute $c = 10$:

$$a + b + 10 = 5$$

$$a + b = -5$$

3. For $(2, 2.5)$:

$$a(2)^2 + b(2) + c = 2.5$$

$$4a + 2b + 10 = 2.5$$

$$4a + 2b = -7.5$$

Solving the System for Coefficients

From the previous equations:

$$a + b = -5 \quad (\text{Equation 1})$$

$$- \ (4a + 2b = -7.5 \) \text{ (Equation 2)}$$

Divide Equation 2 by 2:

$$\ [2a + b = -3.75 \]$$

Now, subtract Equation 1 from this:

$$\ [(2a + b) - (a + b) = -3.75 - (-5) \]$$

$$\ [2a + b - a - b = 1.25 \]$$

$$\ [a = 1.25 \]$$

Substitute $a = 1.25$ into Equation 1:

$$\ [1.25 + b = -5 \]$$

$$\ [b = -6.25 \]$$

Recall $c = 10$.

Final quadratic function:

$$\ [f(x) = 1.25x^2 - 6.25x + 10 \]$$

Analysis of the Derived Function and Its Properties

Verification of the Fit

- At $x = 0$: $\ (f(0) = 1.25(0)^2 - 6.25(0) + 10 = 10 \)$ (matches the point)
- At $x = 1$: $\ (f(1) = 1.25(1) - 6.25(1) + 10 = 1.25 - 6.25 + 10 = 5 \)$ (matches)
- At $x = 2$: $\ (f(2) = 1.25(4) - 6.25(2) + 10 = 5 - 12.5 + 10 = 2.5 \)$ (matches)

The function perfectly interpolates all three points, confirming its validity.

Graphical Interpretation

Plotting this quadratic reveals a parabola opening upwards, with a vertex indicating the minimum point. The curve smoothly passes through the specified points, illustrating the nature of polynomial interpolation.

Behavior and Characteristics

- Concavity: Since $a = 1.25 > 0$, the parabola opens upward.
- Vertex: The x-coordinate of the vertex is given by $\ (-b/(2a) = -(-6.25)/(2 \times 1.25) = 6.25/2.5 = 2.5 \)$. The y-coordinate can be computed by plugging $x = 2.5$ into the function.
- Minimum Point: The vertex at $x = 2.5$ indicates the lowest point on the curve.

Broader Implications and Applications

Mathematical Significance

The process demonstrates the core principles of polynomial interpolation, showcasing how discrete data points can be used to derive continuous functions. Such techniques are foundational in numerical analysis, data science, and computational mathematics.

Real-World Applications

- Physics: Modeling projectile trajectories or other physical phenomena based on discrete measurements.
- Economics: Estimating trends from limited data points, such as profit over time.
- Engineering: Designing curves for automotive or aerospace components based on specific key points.
- Biology: Modeling growth curves or response functions from experimental data.

Limitations and Considerations

While quadratic interpolation is elegant and straightforward for three points, it has limitations:

- Overfitting for larger datasets: Higher-degree polynomials can oscillate wildly between points.
- Runge's Phenomenon: Higher-degree polynomial interpolation can lead to large oscillations near the interval's edges.
- Alternative methods: Spline interpolation or piecewise functions may be preferable for larger datasets.

Conclusion: The Power of Polynomial Interpolation

The derivation of the quadratic function $f(x) = 1.25x^2 - 6.25x + 10$ exemplifies how a simple mathematical process can yield a precise and elegant representation of discrete data points. Through systematic algebraic techniques, one can transform a set of coordinates into a meaningful function that encapsulates the relationship between variables.

This approach underscores the importance of polynomial interpolation in various scientific and engineering disciplines, enabling practitioners to model complex phenomena, make predictions, and analyze data with mathematical rigor. As data becomes increasingly central to decision-making, understanding how to construct such functions remains an essential skill in the modern analytical toolkit.

In conclusion, the curve passing through the points (0, 10), (1, 5), and (2, 2.5) is not merely a mathematical curiosity but a testament to the power of algebraic methods in translating discrete observations into continuous

understanding. Whether for academic purposes or real-world applications, the ability to represent data with functions continues to be a cornerstone of scientific inquiry and technological advancement.

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