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Understanding geometric transformations is fundamental in the study of mathematics, especially in coordinate geometry. One such transformation is dilation, which involves resizing a figure relative to a fixed point known as the center of dilation. In this article, we will explore the process of dilating a line segment with specific endpoints, A(2, 2) and B(3, 8), by a scale factor of 3.5 with the origin (0,0) as the center of dilation. We will discuss the concepts involved, step-by-step procedures, and applications of this transformation.

What Is Dilation in Coordinate Geometry?

Dilation, also known as resizing or scaling, is a transformation that produces an image that is the same shape as the original but is a different size. The key components of dilation include:

- **Center of Dilation:** The fixed point about which the figure is expanded or contracted. In our case, this is the origin (0, 0).
- **Scale Factor:** A number that indicates how much the figure is enlarged or reduced. A scale factor greater than 1 results in an enlargement, while a scale factor between 0 and 1 results in a reduction.

Mathematically, when a point (x, y) undergoes dilation with respect to the origin by a scale factor k , its image (x', y') is given by:

$$(x', y') = (k x, k y)$$

This simple formula allows us to find the new positions of points after dilation.

Given Data and Objective

Let's analyze the specific problem:

- Original endpoints:
 - $A(2, 2)$
 - $B(3, 8)$
- Scale factor: 3.5
- Center of dilation: Origin $(0, 0)$

Our goal is to find the new positions of points A and B after dilation and understand how the segment AB is affected.

Step-by-Step Process for Dilation

Step 1: Understand the Dilation Formula

Recall that each point (x, y) will be transformed into $(k x, k y)$ when dilated about the origin with scale factor k .

Step 2: Apply the Scale Factor to Point A(2, 2)

Using the dilation formula:

$$- x' = 3.5 \cdot 2 = 7$$

$$- y' = 3.5 \cdot 2 = 7$$

Therefore, the image of point A after dilation, A' , is at $(7, 7)$.

Step 3: Apply the Scale Factor to Point B(3, 8)

Similarly,

$$- x' = 3.5 \cdot 3 = 10.5$$

$$- y' = 3.5 \cdot 8 = 28$$

Thus, the image of point B after dilation, B' , is at $(10.5, 28)$.

Analyzing the Dilation of Segment AB

The original segment AB connects points A(2, 2) and B(3, 8). After dilation, the segment A'B' connects (7, 7) and (10.5, 28). To understand the effect of dilation:

Calculating the Length of Original Segment AB

Using the distance formula:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Plugging in the original points:

$$AB = \sqrt{(3 - 2)^2 + (8 - 2)^2} = \sqrt{1^2 + 6^2} = \sqrt{1 + 36} = \sqrt{37} \approx 6.08 \text{ units}$$

Calculating the Length of Dilated Segment A'B'

Similarly,

$$A'B' = \sqrt{(10.5 - 7)^2 + (28 - 7)^2} = \sqrt{(3.5)^2 + (21)^2} = \sqrt{12.25 + 441} = \sqrt{453.25} \approx 21.3 \text{ units}$$

Observation:

Notice that the length of the segment after dilation (≈ 21.3 units) is exactly 3.5 times the original length (≈ 6.08 units), which confirms the property of dilation: lengths are scaled by the scale factor.

Visualizing the Dilation Process

Visualization aids in understanding how the figure transforms:

- The original points A and B are plotted on the coordinate plane.
- The scale factor is applied to each coordinate to find the new points.
- The line segment AB is stretched outward from the origin, maintaining its shape but increasing in size.

Graphically, the dilation causes all points to move radially away from (or toward, if scale factor < 1) the origin, proportionally increasing their distances from the center.

Applications of Dilation in Real Life

Dilation is not just a theoretical concept; it has important applications:

Architectural Design and Engineering

- Scaling blueprints and models to real-world sizes.
- Adjusting dimensions while maintaining proportions.

Computer Graphics and Animation

- Resizing images and objects.
- Creating zoom effects while preserving shape proportions.

Mathematics Education

- Teaching concepts of similarity and proportionality.
- Demonstrating geometric transformations visually.

Additional Considerations and Tips

- **Center of Dilation:** While this article focuses on dilation about the origin, the process differs if the center is elsewhere. The general formula involves shifting points relative to the center before scaling.
- **Negative Scale Factors:** A negative scale factor reflects the figure across the center point and scales it, resulting in a flipped image.
- **Composite Transformations:** Dilation can be combined with other transformations such as translation, rotation, and reflection for complex figure manipulations.

Summary

In this article, we explored the process of dilating a line segment with endpoints $A(2, 2)$ and $B(3, 8)$ by a scale factor of 3.5 with the origin as the center of dilation. The key steps involved applying the dilation formula to each endpoint, calculating the new positions, and analyzing the change in segment length. The original segment length was approximately 6.08 units, which increased to about 21.3 units after dilation, consistent with the scale factor.

Understanding dilation enhances comprehension of geometric transformations and their applications across various fields. Whether in designing blueprints, creating digital graphics, or solving mathematical problems, grasping how figures scale around a fixed point is fundamental.

Conclusion

Dilation is a powerful and versatile transformation in coordinate geometry that scales figures proportionally from a fixed center. By mastering the process of dilating points and line segments, students and professionals can better analyze and manipulate geometric figures in both theoretical and practical contexts. The specific case of dilating segment AB with endpoints $A(2, 2)$ and $B(3, 8)$ by a scale factor of 3.5 demonstrates the straightforward yet profound impact of this transformation, highlighting its significance in mathematical reasoning and real-world applications.

Frequently Asked Questions

What are the coordinates of the endpoints after dilation with a scale factor of 3.5 centered at the origin?

The endpoints $A(2, 2)$ and $B(3, 8)$ will be dilated to $A'(7, 7)$ and $B'(10.5, 28)$ respectively, by multiplying

each coordinate by 3.5.

How do you perform a dilation of a point with respect to the origin?

To dilate a point with respect to the origin, multiply both its x and y coordinates by the scale factor. For example, for point (x, y) and scale factor k , the new point is (kx, ky) .

What is the importance of the origin when performing dilation transformations?

The origin acts as the center of dilation. When dilating with respect to the origin, all points are scaled relative to it, preserving the shape while changing the size based on the scale factor.

If the original segment has endpoints $A(2, 2)$ and $B(3, 8)$, what is the length of the segment after dilation by 3.5?

The original length between A and B is approximately 6.4 units. After dilation, the length becomes $6.4 \times 3.5 = 22.4$ units.

Can you explain how to find the new coordinates of endpoints after dilation with a given scale factor?

Yes. For each endpoint, multiply the x-coordinate and y-coordinate by the scale factor (3.5 in this case). For $A(2, 2)$, new A' is $(7, 7)$. For $B(3, 8)$, new B' is $(10.5, 28)$.

Additional Resources

Dilations in Geometry: An In-Depth Look at the Transformation of Segment AB

Understanding geometric transformations, particularly dilations, is fundamental in both academic and practical applications of mathematics. In this article, we will explore the specific case where a segment

with endpoints A(2, 2) and B(3, 8) undergoes a dilation centered at the origin with a scale factor of 3.5. This detailed examination will not only clarify the mechanics behind such transformations but will also shed light on their significance in various fields, including computer graphics, engineering, and design.

Introduction to Dilation in Geometry

Dilation, also known as enlargement or reduction, is a transformation that produces an image that is similar to the original figure but scaled by a certain factor relative to a fixed point called the center of dilation. The process preserves the shape (similarity) but alters the size of the figure.

Key aspects of dilation:

- Center of dilation: The fixed point about which the figure is scaled. In our case, this is the origin (0, 0).
 - Scale factor (k): A positive number indicating the degree of dilation. If $k > 1$, the figure enlarges; if $0 < k < 1$, it reduces.
 - Resulting figure: The image obtained after applying the dilation.
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Understanding the Initial Endpoints: A(2, 2) and B(3, 8)

Before delving into the dilation process, it is essential to understand the initial points:

- Point A: Located at (2, 2). It lies in the first quadrant, relatively close to the origin.
- Point B: Located at (3, 8). This point is farther from the origin, especially in the y-direction.

These points define the segment AB, which we will analyze both before and after the dilation transformation.

The Mechanics of Dilation with the Origin as Center

Applying dilation centered at the origin involves scaling each point's coordinates relative to (0, 0). The formula for dilating a point (x, y) with scale factor k is:

$$\begin{aligned} & \backslash \\ (x', y') &= (k \times x, k \times y) \\ & \backslash \end{aligned}$$

Because our center of dilation is the origin, this operation is straightforward. The points are simply multiplied component-wise by the scale factor.

Applying the Scale Factor of 3.5

Given the scale factor $(k = 3.5)$, we process each endpoint as follows:

1. Dilation of Point A (2, 2):

$$\begin{aligned} & \backslash \\ x' &= 3.5 \times 2 = 7 \\ & \backslash \\ & \backslash \end{aligned}$$

$$y' = 3.5 \times 2 = 7$$

\\

Result: $(A' (7, 7))$

2. Dilation of Point B (3, 8):

\\

$$x' = 3.5 \times 3 = 10.5$$

\\

\\

$$y' = 3.5 \times 8 = 28$$

\\

Result: $(B' (10.5, 28))$

The new endpoints, A' and B', define the dilated segment, which is scaled uniformly from the origin.

Analyzing the Dilation: Geometric and Practical Implications

This transformation results in a segment that maintains its original orientation and proportional relationships but is scaled up in size. Let's explore the implications in detail.

1. Preservation of Shape and Orientation

Since dilation is a similarity transformation, the shape of the segment remains unchanged. The original and dilated segments are proportional, with the same slope:

- Original slope of AB:

$$m = \frac{8 - 2}{3 - 2} = \frac{6}{1} = 6$$

- Dilated slope of A'B':

$$m' = \frac{28 - 7}{10.5 - 7} = \frac{21}{3.5} = 6$$

The slope remains constant, confirming that the segment's orientation is preserved.

2. Length Calculation Before and After Dilation

- Original length of AB:

$$|AB| = \sqrt{(3 - 2)^2 + (8 - 2)^2} = \sqrt{1^2 + 6^2} = \sqrt{1 + 36} = \sqrt{37} \approx 6.08$$

- Dilated length of A'B':

$$|A'B'| = \sqrt{(10.5 - 7)^2 + (28 - 7)^2} = \sqrt{3.5^2 + 21^2} = \sqrt{12.25 + 441} = \sqrt{453.25} \approx 21.29$$

This aligns with the scale factor:

$$\frac{|A'B'|}{|AB|} \approx \frac{21.29}{6.08} \approx 3.5$$

Thus, the length of the segment scales exactly by the dilation factor, confirming the proportionality.

3. Significance in Real-World Applications

Understanding these transformations is crucial in:

- Computer Graphics: Scaling objects or segments to fit different resolutions or sizes.
- Engineering Design: Enlarging prototypes or components while maintaining proportions.
- Architectural Planning: Adjusting dimensions of models or plans based on scale factors.

Visualizing the Dilation: Graphical Representation

Visualizing the transformation enhances comprehension. Imagine plotting the original segment AB on a coordinate grid:

- Plot points A(2, 2) and B(3, 8).
- Draw the segment connecting these points.
- From the origin, extend lines through A and B, and then scale these points outward by a factor of 3.5.
- The resulting segment A'B' will be proportionally larger but aligned with the original, maintaining slope and orientation.

This visualization confirms the mathematical calculations and helps in understanding the geometric

intuition behind dilation.

Summary of Key Findings

Aspect	Original	Dilated (k=3.5)
Endpoint A	(2, 2)	(7, 7)
Endpoint B	(3, 8)	(10.5, 28)
Segment Length	6.08	21.29
Slope	6	6
Scale factor	1	3.5

The consistent scaling of endpoints and preservation of shape demonstrate the properties of dilation centered at the origin.

Conclusion: The Power and Precision of Geometric Dilation

This detailed exploration of dilating segment AB with endpoints A(2, 2) and B(3, 8) by a scale factor of 3.5 illustrates the elegance and utility of geometric transformations. Whether in theoretical mathematics or practical design, understanding how to manipulate figures through dilation enables precise control over shape and size, fostering innovation across disciplines.

The key takeaways include:

- The simplicity of applying scalar multiplication to coordinates for dilation centered at the origin.
- The preservation of shape, slope, and proportionality.
- The ability to predict and visualize transformations accurately.

In an era where digital modeling, graphics, and engineering rely heavily on geometric principles, mastering dilation and similar transformations remains an invaluable skill for professionals and enthusiasts alike.

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