The Form Of The Expression For The Function F(t) = 6-3(t + 2) Tells You A Point On The Graph And The

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Understanding the form of a mathematical function is fundamental in graphing, analyzing, and interpreting data. The function F(t) = 6 - 3(t + 2) is a linear function expressed in a form that reveals key information about its graph, including points, slope, and intercepts. In this comprehensive article, we will explore how to interpret this function's expression, what it tells us about its graph, and how to utilize this understanding for various mathematical applications.

Deciphering the Function F(t) = 6 - 3(t + 2)

Before diving into the details, it’s essential to understand the structure of the function. The given function is a linear function, which generally takes the form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept.

The function in question is written as:

$$ F(t) = 6 - 3(t + 2) $$

which can be simplified into the slope-intercept form for easier interpretation.

Simplifying the Expression

Let's simplify the given function step-by-step:

1. Distribute the -3 across the parentheses:

$$ F(t) = 6 - 3t - 6 $$

2. Combine like terms:

$$F(t) = (6 - 6) - 3t = 0 - 3t = -3t$$

Thus, the simplified form of the function is:

$$F(t) = -3t$$

This simplified form makes it straightforward to analyze the graph because it clearly shows a linear relationship with a slope of -3 and no y-intercept (since when t=0, F(t)=0).

Understanding the Graph of F(t) = -3t

The simplified form reveals the key characteristics of the graph:

- Slope (m): -3
- Y-intercept (b): 0

What Does the Slope Tell Us?

The slope of -3 indicates that for every unit increase in t, the function decreases by 3 units. This negative slope signifies a downward trend as t increases, which is typical of many real-world phenomena such as depreciation, cooling, or decreasing quantities.

What Does the Y-Intercept Tell Us?

Since the y-intercept is 0, the graph passes through the origin (0,0). This point is fundamental because it anchors the line and provides a reference for plotting other points.

Interpreting the Point on the Graph

The original form, F(t) = 6 - 3(t + 2), explicitly encodes a point that the graph passes through, which can be identified by examining the expression.

How To Find This Point

- The expression inside the parentheses, (t + 2), suggests a shift in the graph along the t-axis.
- When t = -2, the expression inside the parentheses becomes zero:

$$F(-2) = 6 - 3(-2 + 2) = 6 - 3(0) = 6$$

Thus, the point (-2, 6) lies on the graph of the function.

Significance of This Point

This point confirms the function's position and can be used as a reference for graphing or understanding the function's behavior. It also helps verify the correctness of the simplified form.

Graphing the Function

Having simplified the function, graphing becomes more straightforward.

Step-by-Step Graphing Process

1. Plot the y-intercept: Since the y-intercept is 0, mark the point (0, 0).

2. Use the slope to find other points:
- From (0, 0), move down 3 units and right 1 unit (due to slope -3).
- This gives the point (1, -3).
- Alternatively, from (0, 0), move left 1 unit and up 3 units to find (-1, 3).

3. Plot additional points based on the slope to ensure accuracy.

4. Draw the line passing through these points.

Special Point Confirmation

- The point (-2, 6) can be plotted directly, confirming the line's position and slope.

Applications and Significance of the Function's Form

Understanding the form of $F(t) = 6 - 3(t + 2)$ has practical implications across various fields.

1. Real-World Modeling

Such functions often model scenarios where a quantity decreases at a constant rate, such as:

- Depreciation of assets
- Cooling of objects over time
- Decrease in population under certain conditions

2. Algebraic Manipulation and Problem Solving

Knowing how to manipulate the function into different forms enables solving complex problems, such as:

- Finding specific values

- Determining intercepts
- Calculating the rate of change

3. Graphical Analysis and Data Visualization

Graphing the function provides a visual understanding of the relationship, trends, and key points, which is crucial in data analysis.

Analyzing Variations in the Function's Expression

The original form:

$$ F(t) = 6 - 3(t + 2) $$

contains shifts and transformations that can be analyzed.

Horizontal Shift

- The term (t + 2) indicates a shift to the left by 2 units.
- This shift affects the graph's position along the t-axis.

Vertical Shift

- The constant 6 affects the intercept, although after simplification, the y-intercept is 0.
- In the original form, the intercept is at F(t) = 6 when t = -2.

Reflection and Slope

- The negative slope indicates a reflection across the x-axis compared to a positive slope line.

Additional Mathematical Insights

1. Domain and Range

- Domain: All real numbers, since the function is linear.
- Range: All real numbers, as the line extends infinitely in both directions.

2. Intercepts

- t-intercept: Set F(t) = 0:

$$ 0 = -3t \Rightarrow t = 0 $$

- F(t)-intercept: Set t = 0:

$$ F(0) = 0 $$

- Point (-2, 6): Derived from the original expression, confirming the function passes through this point.

3. Slope-Intercept Form Advantages

Expressing the function in slope-intercept form simplifies interpretations and calculations, crucial for quick analysis.

Conclusion: The Significance of the Function’s Form

The expression F(t) = 6 - 3(t + 2) is more than just an algebraic formula; it encapsulates the essence of the line it represents. By simplifying it to F(t) = -3t, we uncover the fundamental characteristics — the slope and intercepts — that define its graph. The explicit point (-2, 6) embedded within the original expression provides a concrete coordinate that helps visualize and plot the line accurately.

Understanding how to interpret and manipulate such functions is essential in mathematics, physics, economics, and engineering, where modeling real-world phenomena rely heavily on linear relationships. Recognizing the transformations, shifts, and key points within the function enables precise analysis and effective problem-solving.

Summary in key points:

- The original function can be simplified to F(t) = -3t.
- The function passes through the point (-2, 6).
- The slope of the line is -3, indicating a downward trend.
- The y-intercept is at (0, 0) after simplification.
- The expression reveals shifts and transformations that help in graphing and analysis.
- Such linear functions are instrumental in modeling and understanding real-world data.

By mastering the interpretation of the function's form, students and professionals can better analyze data, draw accurate graphs, and apply mathematical concepts effectively in various disciplines.

Frequently Asked Questions

What is the simplified form of the function F(t) = 6 - 3(t + 2)?

The simplified form is F(t) = 6 - 3t - 6, which simplifies further to F(t) = -3t.

What does the point (t, F(t)) = (t, -3t) tell us about the graph of the function?

It indicates that the graph is a straight line with a slope of -3 passing through the point where t is the input and F(t) is the output.

How can we interpret the point derived from the function F(t) = 6 - 3(t + 2)?

The point illustrates a specific value of t and the corresponding value of F(t), helping visualize the function’s behavior at that point.

What is the significance of the point (t, F(t)) in understanding the function's graph?

It provides a specific coordinate on the graph, allowing us to plot and analyze the function's shape and slope at that point.

How does knowing a point on the graph help in graphing the function F(t) = 6 - 3(t + 2)?

Knowing a point helps establish the position of the graph, and combined with the slope, it aids in accurately sketching the line.

How can you find the y-intercept of the function F(t) = 6 - 3(t + 2)?

Set t = 0 in the original function: F(0) = 6 - 3(0 + 2) = 6 - 6 = 0, so the y-intercept is at (0, 0).

What does the expression F(t) = 6 - 3(t + 2) reveal about the relationship between t and F(t)?

It shows a linear relationship where F(t) decreases by 3 units for each increase of 1 in t, with a specific adjustment based on the shift inside the parentheses.

Additional Resources

Understanding the Expression of the Function F(t) = 6 - 3(t + 2): An In-Depth Exploration

When delving into the world of algebra and functions, the way an expression is written reveals a wealth of information about its behavior, characteristics, and the points it passes through on a graph. Among the myriad of functions encountered in mathematics, linear functions like F(t) = 6 - 3(t + 2) serve as foundational building blocks. This particular expression, at first glance, might seem straightforward, but unpacking its form offers deep insights into the nature of the graph it describes.

In this comprehensive review, we will dissect the expression F(t) = 6 - 3(t + 2), explore what it tells us about a point on the graph, and understand the significance of its form in the broader context of algebraic functions. Think of this as an expert’s guide to reading and interpreting the structure of mathematical expressions—akin to analyzing a well-designed product for its features, usability, and underlying mechanics.

Understanding the Basic Components of the Expression

Before diving into the specifics, it’s essential to break down the expression into its fundamental parts. The function:

$$ F(t) = 6 - 3(t + 2) $$

is a linear function expressed in a form that combines constants, coefficients, and a linear term within parentheses.

Key components include:

- Constant term: 6
- Coefficient of the linear term: -3
- Linear expression inside parentheses: (t + 2)

Each piece plays a crucial role in shaping the graph of the function.

Rewriting the Expression: The Slope-Intercept Form

What does the given form tell us?

Mathematically, the expression is similar to the slope-intercept form:

$$ y = mx + b $$

where:

- m is the slope, indicating the steepness and direction of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

Transforming $F(t) = 6 - 3(t + 2)$ into slope-intercept form:

1. Distribute the coefficient:

$$
F(t) = 6 - 3t - 6

\]

2. Combine like terms:

\[
F(t) = (6 - 6) - 3t = 0 - 3t
\]

3. Simplify:

\[
F(t) = -3t
\]

Result:

\[\boxed{F(t) = -3t} \]

This simplified form reveals the core characteristics of the function:

- Slope (m): -3
- Y-intercept (b): 0

Implication: The graph of this function is a straight line passing through the origin with a slope of -3, implying it declines three units vertically for every one unit increase horizontally.

What Does This Tell You About a Point on the Graph?

Understanding the form of the function allows us to determine specific points that lie on its graph.

Key points derived from \(F(t) = -3t \):

- For \(t = 0 \):

\[
F(0) = -3 \times 0 = 0
\]

- For \(t = 1 \):

\[
F(1) = -3 \times 1 = -3
\]

- For \(t = -1 \):

\[

$$
F(-1) = -3 \times -1 = 3
$$

- For $t = 2$:

$$
F(2) = -3 \times 2 = -6
$$

List of points:

t	$F(t)$	Point on the graph
0	0	(0, 0)
1	-3	(1, -3)
-1	3	(-1, 3)
2	-6	(2, -6)

These points demonstrate the linear nature of the function and how the slope influences the direction of the line.

Interpreting the Form for Graphing and Analysis

The simplified form, $F(t) = -3t$, provides a concise way to understand how the graph behaves. However, the original expression also offers valuable insights into the transformations applied to the basic linear function.

Transformations in the Original Expression

The initial form can be viewed as a transformation of the basic line $y = -3t$:

$$ F(t) = 6 - 3(t + 2) $$

Step-by-step interpretation:

1. Horizontal shift: The inner parentheses $(t + 2)$ suggest a shift along the t-axis (horizontal). Replacing t with $t + 2$ shifts the graph left by 2 units.

2. Vertical shift: The constant $+6$ outside the parentheses indicates a vertical shift upward by 6 units.

3. Coefficient (-3): Determines the slope, as before.

Visualizing the transformations:

- Start with the basic line $y = -3t$.
- Shift it left by 2 units.

- Shift it up by 6 units.

Result: The graph of the original function is a line with slope -3 passing through the point obtained by applying these shifts.

Finding the Point on the Original Graph

To locate a specific point on the graph of the original function:

- Choose a value for t: for example, $t = 0$:

$$
F(0) = 6 - 3(0 + 2) = 6 - 3 \times 2 = 6 - 6 = 0
$$

- Coordinate: (0, 0) — consistent with the simplified form.

Similarly, for $t = -2$:

$$
F(-2) = 6 - 3(-2 + 2) = 6 - 3(0) = 6
$$

- Coordinate: (-2, 6)

This demonstrates that shifting the graph left by 2 units (from t to $t + 2$) and up by 6 units results in the new position.

Significance of the Expression in Practical Contexts

The form of the function is not just an abstract mathematical construct; it has real-world interpretations and applications.

Example 1: Physics - Velocity and Displacement

Suppose t represents time (in seconds), and $F(t)$ represents displacement (in meters). The expression:

$$ F(t) = 6 - 3(t + 2) $$

could model a situation where an object starts at a certain position and moves with a constant velocity.

- Interpretation of the original form:

- The $(t + 2)$ indicates a delayed start by 2 seconds.
- The coefficient -3 indicates the object is moving in the negative direction at 3 meters per second.

- The constant 6 (or after transformations, the initial position) indicates where the object was at $t = -2$.

Example 2: Economics - Cost Functions

In economics, such functions can model costs or revenues that change linearly with quantity or time, with shifts representing changes in starting points or fixed costs.

Key Takeaways and Highlights

- The form of the expression reveals the slope and intercept:

- The original form $6 - 3(t + 2)$ indicates a slope of -3.
- The y-intercept can be found by evaluating at $t = 0$.

- Transformations are embedded in the structure:

- $(t + 2)$ signifies a shift left by 2 units.
- The constant $+6$ signifies a shift upward by 6 units.

- Point identification:

- By substituting specific t values, you can find corresponding points on the graph.
- The simplified form makes plotting and analysis straightforward.

- Graph characteristics:

- The line passes through points like (0, 0), (-2, 6).
- The slope indicates a decreasing function with a rise/run ratio of -3.

Conclusion: The Power of Expression Forms

The detailed analysis of the expression F(t) = 6 - 3(t + 2) underscores the importance of understanding the structure and form of mathematical functions. Recognizing the transformations, slopes, and intercepts embedded in the expression enables us to visualize, analyze, and apply the function effectively.

Much like evaluating a high-quality product based on its features and design, examining the form of a mathematical expression provides clarity and insight. It allows students, educators, and professionals to grasp the underlying mechanics of the function, anticipate its behavior, and use it confidently in practical situations.

In essence, mastering how to interpret the form of such functions transforms a mere algebraic expression into a powerful tool for understanding the relationships and dynamics of the system it models.

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