The Function F Is Defined By F(x) = Esinx. Find The First Two Derivatives Of F(x) And Hence Find The

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Understanding derivatives is fundamental in calculus, as they provide critical insights into the behavior of functions, including their rates of change, maxima, minima, and points of inflection. In this article, we will explore a specific function, $F(x) = e^{\sin x}$, and develop a comprehensive understanding of its derivatives. We will start by computing the first derivative, then proceed to the second derivative, and finally discuss how these derivatives can be applied to analyze the function's behavior.

Understanding the Function $F(x) = e^{\sin x}$

Before diving into derivatives, it is essential to understand the structure of the function:

- The function involves an exponential component, e^u, where $u = \sin x$.
- Since $\sin x$ is a standard trigonometric function, the overall composition $e^{\sin x}$ requires the use of the chain rule for differentiation.

This composition makes the function a classic example of applying the chain rule in calculus, as it combines an exponential function with a trigonometric function.

Calculating the First Derivative $F'(x)$

The first derivative of $F(x) = e^{\sin x}$ measures how the function changes as x varies. To compute $F'(x)$, we will use the chain rule, which states that:

$$
\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)
$$

In our case:

- $f(u) = e^{u}$, so $f'(u) = e^{u}$
- $g(x) = \sin x$, so $g'(x) = \cos x$

Applying the chain rule:

$$
F'(x) = \frac{d}{dx} e^{\sin x} = e^{\sin x} \cdot \cos x
$$

Therefore, the first derivative is:

$$
\boxed{
F'(x) = e^{\sin x} \cos x
}
$$

This derivative indicates how the function $F(x)$ changes with respect to x, involving both the exponential of $\sin x$ and the cosine function.

Calculating the Second Derivative $F''(x)$

The second derivative provides information about the concavity and points of inflection of the function. To find $F''(x)$, differentiate $F'(x)$:

$$
F'(x) = e^{\sin x} \cos x
$$

Since this is a product of two functions, $u(x) = e^{\sin x}$ and $v(x) = \cos x$, we will use the product rule:

$$
\frac{d}{dx} [u(x) v(x)] = u'(x) v(x) + u(x) v'(x)
$$

Step 1: Compute $u'(x)$

- $u(x) = e^{\sin x}$
- $u'(x) = e^{\sin x} \cdot \cos x$ (from the chain rule, as previously shown)

Step 2: Compute $v'(x)$

- $v(x) = \cos x$
- $v'(x) = - \sin x$

Step 3: Apply the product rule

$$
F''(x) = u'(x) v(x) + u(x) v'(x)
$$

$$

Substituting the derivatives:

$$
F''(x) = e^{\sin x} \cos x \cdot \cos x + e^{\sin x} \cdot (- \sin x)
$$

Simplify:

$$
F''(x) = e^{\sin x} (\cos^2 x - \sin x)
$$

Final expression for the second derivative:

$$
\boxed{
F''(x) = e^{\sin x} (\cos^2 x - \sin x)
}
$$

This expression captures the curvature of $F(x)$ at any point x, providing insights into the concavity and potential inflection points.

Analyzing the Derivatives and Their Applications

Having derived the first and second derivatives, we can now explore how these are used in analyzing the behavior of the function $F(x) = e^{\sin x}$.

Critical Points and Increasing/Decreasing Intervals

- Critical points occur where $F'(x) = 0$:

$$
e^{\sin x} \cos x = 0
$$

Since $e^{\sin x} \neq 0$ for all real x, the equation reduces to:

$$
\cos x = 0
$$

- $\cos x = 0$ at:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

- Increase and decrease:

- $F(x)$ is increasing where $F'(x) > 0$:

$$
e^{\sin x} \cos x > 0
$$

- Since $e^{\sin x} > 0$ always, the sign depends on $\cos x$:

$$
\cos x > 0 \Rightarrow F(x) \text{ is increasing}
$$
$$
\cos x < 0 \Rightarrow F(x) \text{ is decreasing}
$$

- Intervals:

- $\cos x > 0$ in intervals:

$$
(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi)
$$

- $\cos x < 0$ in intervals:

$$
(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi)
$$

h3>Concavity and Points of Inflection

- Concavity:

- The sign of $F''(x)$ determines concavity:

$$
F''(x) = e^{\sin x} (\cos^2 x - \sin x)
$$

- Since $e^{\sin x} > 0$, concavity depends on:

$$
\cos^2 x - \sin x
$$

- Points of inflection occur where $F''(x) = 0$:

$$
\cos^2 x - \sin x = 0
$$

which simplifies to:

$$
\cos^2 x = \sin x
$$

Using the Pythagorean identity:

$$
\cos^2 x = 1 - \sin^2 x
$$

Substitute into the equation:

$$
1 - \sin^2 x = \sin x
$$

Rearranged:

$$
\sin^2 x + \sin x - 1 = 0
$$

Let $s = \sin x$, then:

$$
s^2 + s - 1 = 0
$$

Solve quadratic:

$$
s = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2} = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}
$$

- Since $\sin x$ is bounded between -1 and 1:

$$
s = \frac{-1 + \sqrt{5}}{2} \approx 0.618 \quad (\text{valid})
$$
$$
s = \frac{-1 - \sqrt{5}}{2} \approx -1.618 \quad (\text{not valid, outside bounds})
$$

Thus, the only valid solution:

$$
\sin x = \frac{-1 + \sqrt{5}}{2}
$$

- To find the corresponding x:

$$
x = \arcsin \left(\frac{-1 + \sqrt{5}}{2} \right) + 2\pi n \quad \text{or} \quad \pi - \arcsin \left(\frac{-1 + \sqrt{5}}{2} \right) + 2\pi n
$$

for integers n.

Summary of Key Results

- First derivative:

$$
F'(x) = e^{\sin x} \cos x
$$

- Second derivative:

$$
F''(x) = e^{\sin x} (\cos^2 x - \sin x)
$$

- Critical points: occur where $\cos x = 0$, i.e., at $x = \frac{\pi}{2} + n\pi$.

- Points of inflection: occur where \(\sin x = \frac{-1 + \sqrt{5

Frequently Asked Questions

What is the first derivative of the function $F(x) = e^{\sin x}$?

The first derivative $F'(x)$ is $e^{\sin x}$ multiplied by $\cos x$, i.e., $F'(x) = e^{\sin x} \cdot \cos x$.

How do we find the second derivative of $F(x) = e^{\sin x}$?

To find the second derivative, differentiate $F'(x) = e^{\sin x} \cdot \cos x$ using the product rule, resulting in $F''(x) = e^{\sin x} (\cos^2 x - \sin x)$.

What is the significance of the derivatives of F(x) = e^{sin x}?

The derivatives reveal the rate of change and concavity of the function, helping analyze its increasing/decreasing behavior and points of inflection.

Can you provide the explicit expressions for the first and second derivatives of F(x)?

Yes. The first derivative is F'(x) = e^{sin x} · cos x, and the second derivative is F''(x) = e^{sin x} (cos^2 x - sin x).

How does the chain rule apply to differentiate F(x) = e^{sin x}?

The chain rule states that F'(x) = e^{sin x} · derivative of sin x, which is cos x, leading to F'(x) = e^{sin x} · cos x.

What is the role of the product rule in finding the second derivative of F(x)?

The product rule is used to differentiate the product e^{sin x} · cos x, resulting in the second derivative F''(x) = e^{sin x} (cos^2 x - sin x).

Are there any special points where the derivatives of F(x) are zero?

Yes, F'(x) = 0 when cos x = 0, i.e., at x = π/2 + kπ, and F''(x) can be zero when cos^2 x = sin x.

What can be inferred about the concavity of F(x) based on its second derivative?

The sign of F''(x) determines concavity: F''(x) > 0 indicates concave up, while F''(x) < 0 indicates concave down, depending on the values of x.

How can the derivatives of F(x) assist in graphing the function?

They help identify increasing/decreasing intervals, points of inflection, and the overall shape, making it easier to accurately sketch the graph of F(x).

Additional Resources

Comprehensive Analysis of the Function F(x) = e^{\sin x} and Its Derivatives

Understanding the behavior of functions and their derivatives is a fundamental aspect of calculus, offering insights into the function's rate of change, concavity, and points of inflection. In this detailed review, we will examine the function $F(x) = e^{\sin x}$, derive its first two derivatives meticulously, and explore related mathematical properties. This comprehensive exploration aims to deepen understanding of exponential and trigonometric function interactions and how derivatives reveal the function's behavior.

Introduction to the Function $F(x) = e^{\sin x}$

The function $F(x) = e^{\sin x}$ is an example of a composite function involving exponential and trigonometric components. Recognizing the structure of this function is essential for applying the chain rule effectively and understanding its properties.

- Exponential Function e^{u}: Known for its smoothness and constant rate of growth, the exponential function exhibits continuous derivatives of all orders.
- Sine Function $\sin x$: A periodic function with period 2π, oscillating between -1 and 1, influencing the behavior of $F(x)$ accordingly.

The composition $e^{\sin x}$ inherits the smoothness and periodicity traits, but its derivatives reveal more nuanced behaviors, especially when considering maxima, minima, and inflection points.

Deriving the First Derivative $F'(x)$

Calculating the first derivative involves applying the chain rule because $F(x)$ is a composition of the exponential and sine functions.

Step-by-Step Derivation

Given:
$$
F(x) = e^{\sin x}
$$

The derivative $F'(x)$ is obtained by differentiating the outer function e^{u} with respect to $u = \sin x$, then multiplying by the derivative of u.

1. Identify the outer and inner functions:
- Outer function: e^{u}
- Inner function: $u = \sin x$

2. Apply the chain rule:
$$

$$
F'(x) = \frac{d}{dx} e^{\sin x} = e^{\sin x} \cdot \frac{d}{dx} (\sin x)
$$

3. Differentiate $\sin x$:

$$
\frac{d}{dx} (\sin x) = \cos x
$$

4. Combine results:

$$
F'(x) = e^{\sin x} \cdot \cos x
$$

Result:

$$
\boxed{
F'(x) = e^{\sin x} \cos x
}
$$

This derivative indicates how the function $F(x)$ increases or decreases depending on the values of $\sin x$ and $\cos x$. Notably:

- When $\cos x = 0$, $F'(x) = 0$, corresponding to potential maxima, minima, or inflection points.
- The sign of $F'(x)$ is governed by $\cos x$, since $e^{\sin x}$ is always positive.

Deriving the Second Derivative $F''(x)$

The second derivative provides insights into the concavity and inflection points of the function. Computing $F''(x)$ involves differentiating $F'(x) = e^{\sin x} \cos x$ using the product rule.

Step-by-Step Derivation

Given:

$$
F'(x) = e^{\sin x} \cos x
$$

Applying the product rule:

$$
F''(x) = \frac{d}{dx} \left(e^{\sin x} \cdot \cos x \right) = \frac{d}{dx} e^{\sin x} \cdot \cos x + e^{\sin x} \cdot \frac{d}{dx} (\cos x)
$$

1. Differentiate $e^{\sin x}$:

$$
\frac{d}{dx} e^{\sin x} = e^{\sin x} \cdot \cos x
$$
(from the chain rule, as previously established)

2. Differentiate $\cos x$:
$$
\frac{d}{dx} (\cos x) = - \sin x
$$

3. Substitute into the product rule:
$$
F''(x) = e^{\sin x} \cos x \cdot \cos x + e^{\sin x} \cdot (- \sin x)
$$

4. Simplify:
$$
F''(x) = e^{\sin x} \left(\cos^2 x - \sin x \right)
$$

Final expression:
$$
\boxed{
F''(x) = e^{\sin x} \left(\cos^2 x - \sin x \right)
}
$$

This second derivative combines the exponential of $\sin x$ with a quadratic form of $\cos x$ and $\sin x$, revealing the interplay between the function's curvature and oscillatory components.

Mathematical Properties and Behavior Analysis

Understanding the derivatives allows us to analyze where the function is increasing or decreasing, as well as its concavity and points of inflection.

Critical Points and Extrema

- Where $F'(x) = 0$:
$$
e^{\sin x} \cos x = 0
$$
Since $e^{\sin x} \neq 0$ for all real x, the zeros occur when:
$$
\cos x = 0
$$

- $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$, where $k \in \mathbb{Z}$.

- Nature of critical points:
- To classify these points, examine $F''(x)$ at $x = \frac{\pi}{2} + k\pi$.

Concavity and Inflection Points

- The second derivative:
$$
F''(x) = e^{\sin x} (\cos^2 x - \sin x)
$$

- At $\cos x = 0$:
$$
\cos^2 x = 0
$$
So,
$$
F''(x) = e^{\sin x} (0 - \sin x) = - e^{\sin x} \sin x
$$

- Evaluating $\sin x$ at $x = \frac{\pi}{2} + k\pi$:
- When $x = \frac{\pi}{2} + 2k\pi$:
$$
\sin x = 1
$$
$$
F''(x) = - e^{1} \times 1 = - e
$$
Negative second derivative indicates local maxima at these points.

- When $x = \frac{3\pi}{2} + 2k\pi$:
$$
\sin x = -1
$$
$$
F''(x) = - e^{-1} \times (-1) = e^{-1} > 0
$$
Positive second derivative indicates local minima at these points.

Graphical Behavior and Periodicity

The structure of $F(x) = e^{\sin x}$ and its derivatives reveals a wave-like pattern with amplitude modulation:

- Periodicity:
- Both $\sin x$ and $\cos x$ are periodic with period 2π.

- Consequently, $F(x)$, $F'(x)$, and $F''(x)$ exhibit periodic behaviors with the same period.

- Amplitude:
- The exponential component $e^{\sin x}$ oscillates between e^{-1} and e^{1}, leading to bounded oscillations in the derivatives.

- Behavior at critical points:
- Maxima of $F(x)$ occur where $\sin x = 1$, with derivatives indicating maxima.
- Minima occur where $\sin x = -1$.

Applications and Further Implications

The derivatives of $F(x) = e^{\sin x}$ have practical implications in various scientific and engineering domains:

- Signal Processing:
- The oscillatory nature resembles modulated signals, where understanding derivatives helps in analyzing frequency and phase shifts.

- Physics:
- In wave mechanics and oscillatory systems, derivatives inform about acceleration, stability, and inflection points.

- Mathematical Modeling:
- Exponential-tr

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