The Function F(x)=-x^(2)-4x+12 Increases On The Interval [DROP DOWN 1] And Decreases On The Interval

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Understanding the behavior of quadratic functions is fundamental in algebra and calculus, as they often model real-world phenomena such as projectile motion, profit maximization, and more. The given function, $F(x) = -x^2 - 4x + 12$, is a quadratic function with a negative leading coefficient, indicating it opens downward, and consequently, it has a maximum point (vertex). Analyzing where this function increases and decreases involves finding the vertex and understanding the function's slope over various intervals.

In this article, we will explore in depth the increasing and decreasing intervals of $F(x)$, how to determine them, and the significance of these intervals in understanding the function's behavior.

Understanding the Quadratic Function $F(x)=-x^{2}-4x+12$

General Form of a Quadratic Function

A quadratic function can be expressed in the standard form:

$$
f(x) = ax^2 + bx + c
$$

where:
- a, b, and c are constants,
- $a \neq 0$.

For the given function:
$$
F(x) = -x^2 - 4x + 12
$$
we identify:
- $a = -1$,
- $b = -4$,
- $c = 12$.

The negative value of a indicates the parabola opens downward.

Vertex of the Parabola

The vertex is the highest or lowest point on the parabola, depending on the direction it opens. Since $a = -1 < 0$, the parabola opens downward, and the vertex corresponds to the maximum point.

The vertex's x-coordinate is given by:

$$
x_{v} = -\frac{b}{2a}
$$

Calculating:

$$
x_{v} = -\frac{-4}{2 \times -1} = -\frac{-4}{-2} = -\frac{4}{-2} = 2
$$

The y-coordinate of the vertex is:

$$
F(2) = - (2)^2 - 4(2) + 12 = -4 - 8 + 12 = 0
$$

Therefore, the vertex is at $(2, 0)$.

Determining Intervals of Increase and Decrease

First Derivative and Its Significance

To analyze where the function increases or decreases, we examine its first derivative:

$$
F'(x) = \frac{d}{dx} (-x^2 - 4x + 12) = -2x - 4
$$

The derivative indicates the slope of the tangent line at any point x. When:
- $F'(x) > 0$, the function is increasing,
- $F'(x) < 0$, the function is decreasing.

Find critical points where $F'(x) = 0$:

$$

-2x - 4 = 0 \Rightarrow -2x = 4 \Rightarrow x = -2
\]

This critical point divides the domain into two intervals:

- \(\boxed{(-\infty, -2)}\),
- \(\boxed{(-2, \infty)}\).

Sign of the Derivative in Each Interval

- For \(x < -2\):
\[
F'(x) = -2x - 4
\]
Pick \(x = -3\):
\[
F'(-3) = -2(-3) - 4 = 6 - 4 = 2 > 0
\]
So, \(F'(x) > 0\) on \((-\infty, -2)\), meaning \(F(x)\) is increasing there.

- For \(x > -2\):
\[
F'(-1) = -2(-1) - 4 = 2 - 4 = -2 < 0
\]
So, \(F'(x) < 0\) on \((-2, \infty)\), meaning \(F(x)\) is decreasing there.

Summary:
- \(F(x)\) increases on \((-\infty, -2)\),
- \(F(x)\) decreases on \((-2, \infty)\).

Complete Description of the Intervals

Intervals of Increase

The function \(F(x)\) increases on the interval:

\[
\boxed{(-\infty, -2)}
\]

This means, as \(x\) moves from negative infinity up to \(-2\), the value of \(F(x)\) is rising.

Intervals of Decrease

The function decreases on:

$$
\boxed{(-2, \infty)}
$$

From -2 onward, the function's value declines.

Graphical Representation and Intuitive Understanding

Plotting the Function

Plotting $F(x) = -x^2 - 4x + 12$ would reveal a downward-opening parabola with its vertex at $(2, 0)$. However, the critical point at $x = -2$ indicates the transition point from increasing to decreasing.

Behavior of the Graph

- As $x \to -\infty$, $F(x) \to -\infty$. The parabola rises from negative infinity, increases until $x = -2$, reaching a local maximum at the vertex, then declines towards negative infinity as $x \to \infty$.

Significance of the Intervals in Real-World Contexts

Applications of Increasing and Decreasing Intervals

Understanding where a quadratic function increases or decreases helps in various applications:

- **Optimization problems:** Identifying maximum profit or minimum cost by locating the vertex.
- **Physics:** Modeling projectile motion where the height increases up to the apex and then decreases.
- **Economics:** Determining points of diminishing returns or maximum revenue.

Implications for Analysis and Decision Making

Knowing the intervals guides decision-making processes, such as:

- When to expect growth or decline,
- How to interpret the behavior of the system modeled by the quadratic function.

Conclusion

The quadratic function $F(x) = -x^2 - 4x + 12$ exhibits a clear pattern of increasing and decreasing behavior based on its derivative. It increases on $(-\infty, -2)$ and decreases on $(-2, \infty)$, with the critical point at $x = -2$ marking the transition. Recognizing these intervals enables better understanding of the function's overall shape and its real-world applications.

In summary:
- Increases on $\boxed{(-\infty, -2)}$,
- Decreases on $\boxed{(-2, \infty)}$.

This analysis provides a comprehensive understanding of the function's behavior, essential for both theoretical studies and practical applications involving quadratic functions.

Frequently Asked Questions

For the function F(x) = -x^2 - 4x + 12, on which interval does it increase?

The function increases on the interval (-∞, -2).

Where does the function F(x) = -x^2 - 4x + 12 reach its maximum value?

It reaches its maximum at x = -2, which is the vertex of the parabola.

On which interval does the function decrease?

The function decreases on the interval (-2, ∞).

How do you determine the increasing and decreasing intervals for this quadratic function?

By finding the vertex at x = -2 and analyzing the sign of the derivative, the function increases for $x < -2$ and decreases for $x > -2$.

What is the vertex of the parabola for F(x) = -x^2 - 4x + 12?

The vertex is at (-2, 16).

Is the parabola opening upwards or downwards, and how does that affect the increasing and decreasing intervals?

Since the coefficient of x^2 is negative, the parabola opens downward, meaning it increases until the vertex and then decreases afterward.

What is the significance of the interval [DROP DOWN 1] in the context of this function?

The interval [DROP DOWN 1] refers to the interval where the function is increasing; it should be filled with $(-\infty, -2)$.

How can you verify the increasing and decreasing intervals without graphing?

By calculating the derivative, $F'(x) = -2x - 4$, and analyzing its sign: $F'(x) > 0$ when $x < -2$ (increasing), and $F'(x) < 0$ when $x > -2$ (decreasing).

Additional Resources

The Function $F(x) = -x^2 - 4x + 12$ Increases On The Interval [DROP DOWN 1] And Decreases On The Interval

In the realm of quadratic functions, understanding where a function increases or decreases provides vital insights into its overall behavior. The function $F(x) = -x^2 - 4x + 12$ exemplifies this, with its parabola opening downward, signifying a maximum point at its vertex. Analyzing the intervals of increase and decrease not only enhances our grasp of the function’s shape but also illustrates fundamental concepts in calculus and algebra. This article delves into the detailed examination of the function's increasing and decreasing intervals, elucidating the mathematical reasoning behind these characteristics, and shedding light on the broader implications for curve analysis.

Understanding the Nature of the Function

Before pinpointing the specific intervals where the function increases or decreases, it’s essential to understand the core properties of the quadratic function $F(x) = -x^2 - 4x + 12$.

Quadratic Functions and Parabolas

Quadratic functions are polynomial functions of degree two, generally expressed in the form $y = ax^2 + bx + c$, where a, b, and c are constants. The graph of a quadratic function is a parabola, which can open upward (if $a > 0$) or downward (if $a < 0$).

In the case of $F(x) = -x^2 - 4x + 12$:

- The coefficient 'a' is -1, indicating the parabola opens downward.
- The parabola has a maximum point at its vertex, which is the highest point on the graph.

Significance of the Coefficient 'a'

The negative coefficient of x^2 (-1) determines the parabola’s orientation. Since the parabola opens downward, it:

- Ascends to a maximum point at the vertex.
- Descends on either side of the vertex.

This orientation directly influences where the function increases or decreases, which we will analyze through calculus.

Finding the Vertex: The Turning Point of the Parabola

The vertex of a parabola given by $y = ax^2 + bx + c$ can be calculated using the vertex formula:

$x = -b / (2a)$

Applying this to F(x):

- $a = -1$
- $b = -4$

Calculating:

$x = -(-4) / (2 -1) = 4 / (-2) = -2$

This x-value marks the vertex's horizontal coordinate, and plugging it back into the function yields the y-coordinate:

$F(-2) = -(-2)^2 - 4(-2) + 12 = -4 + 8 + 12 = 16$

Therefore, the vertex is at the point (-2, 16).

Implication: The maximum value of the function occurs at $x = -2$, with $F(-2) = 16$.

Determining Intervals of Increase and Decrease

The key to understanding where the function increases or decreases lies in analyzing the slope of the function, which is represented by its derivative in calculus.

Calculating the Derivative

Given $F(x) = -x^2 - 4x + 12$, the derivative $F'(x)$ is:

$F'(x) = d/dx [-x^2 - 4x + 12] = -2x - 4$

The sign of $F'(x)$ indicates whether the function is increasing or decreasing:

- If $F'(x) > 0$, the function increases.
- If $F'(x) < 0$, the function decreases.

Finding Critical Points

Critical points occur where $F'(x) = 0$:

$-2x - 4 = 0$

Solving:

$-2x = 4$

$x = -2$

This critical point coincides with the vertex, confirming that the maximum occurs at $x = -2$.

Interval Analysis Based on the Derivative

The derivative $F'(x) = -2x - 4$ divides the real line into two regions:

1. For $x < -2$:

- Pick a test point, e.g., $x = -3$:

$F'(-3) = -2(-3) - 4 = 6 - 4 = 2 > 0$

- Since the derivative is positive, $F(x)$ is increasing on $(-\infty, -2)$.

2. For $x > -2$:

- Pick a test point, e.g., $x = 0$:

$F'(0) = -2(0) - 4 = -4 < 0$

- Since the derivative is negative, $F(x)$ is decreasing on $(-2, \infty)$.

Conclusion:

- $F(x)$ increases on the interval $(-\infty, -2)$.
- $F(x)$ decreases on the interval $(-2, \infty)$.

Summary of the Function's Behavior

Combining all findings, we can succinctly state:

- The function $F(x) = -x^2 - 4x + 12$ increases when x is less than -2.
- It reaches its maximum at $x = -2$.
- Beyond $x = -2$, the function decreases as x increases.

Therefore, the complete statement is:

The Function $F(x) = -x^2 - 4x + 12$ Increases On The Interval $(-\infty, -2)$ And Decreases On The Interval $(-2, \infty)$.

Broader Implications and Applications

Understanding the increasing and decreasing intervals of quadratic functions extends beyond pure mathematics. It plays a crucial role in various fields:

- Economics: Maximizing profit or minimizing costs.
- Physics: Analyzing projectile motion.
- Engineering: Optimizing structural designs.

- Data Analysis: Identifying trends and turning points.

Knowing where a function increases or decreases helps in predicting behavior, making informed decisions, and optimizing outcomes.

Conclusion

The analysis of the quadratic function $F(x) = -x^2 - 4x + 12$ reveals a classic parabola opening downward, with a maximum point at $x = -2$. By examining its derivative, we establish that the function increases on the interval $(-\infty, -2)$ and decreases on $(-2, \infty)$. Recognizing these intervals provides foundational insight into the function's shape and behavior, illustrating the powerful interplay between algebraic methods and calculus in understanding mathematical functions. Whether applied in academic contexts or real-world scenarios, this knowledge underscores the importance of derivative analysis in charting the ascent and descent of curves across the mathematical landscape.

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