The Function (x) = 4x + 1 Is Transformed To G(x) = 2x + 1. What Type Of Transformation Has Occurred?

The Function (x) = 4x + 1 Is Transformed To G(x) = 2x + 1. What Type Of Transformation Has Occurred?

Understanding how functions change under various transformations is fundamental in algebra and graphing. When the function $f(x) = 4x + 1$ is transformed into $g(x) = 2x + 1$, it signifies a specific type of alteration to the original graph. This article explores the nature of this transformation, the mathematical principles behind it, and how to identify and interpret such changes visually and algebraically.

Understanding the Functions Involved

Original Function: $f(x) = 4x + 1$

- This is a linear function with a slope of 4 and a y-intercept at (0, 1).
- The graph of $f(x)$ is a straight line rising steeply with a steepness determined by the slope 4.
- For example, at $x=1$, $f(1)=4(1)+1=5$; at $x=2$, $f(2)=8+1=9$.

Transformed Function: $g(x) = 2x + 1$

- This is also a linear function with a slope of 2 and the same y-intercept at (0, 1).
- The graph of $g(x)$ is a straight line with a gentler slope compared to $f(x)$.
- For example, at $x=1$, $g(1)=2(1)+1=3$; at $x=2$, $g(2)=4+1=5$.

Analyzing the Change from f(x) to g(x)

Comparison of Slopes

- The slope of $f(x)$ is 4.
- The slope of $g(x)$ is 2.
- The slope has been reduced from 4 to 2, which indicates a change in the steepness of the graph.

Comparison of Y-Intercepts

- Both functions share the same y-intercept at (0, 1).
- This implies that the vertical position of the line remains unchanged.

Visual Implication

- Since the y-intercept stays the same and the slope decreases, the graph of $g(x)$ is a "flattened" version of $f(x)$, maintaining the same point at $y=1$ when $x=0$ but rising more slowly as x increases.

Understanding Transformation Types in Linear Functions

Common Types of Transformations

Linear functions can be transformed through various operations, including:

- **Vertical shifts**: Moving the graph up or down.
- **Horizontal shifts**: Moving the graph left or right.
- **Vertical stretches or compressions**: Changing the steepness of the graph vertically.
- **Reflections**: Flipping the graph over the x-axis or y-axis.
- **Horizontal stretches or compressions**: Changing the steepness horizontally, which affects the slope.

Which transformation occurs when the slope changes?

- Changing the slope of a linear function typically involves a stretch or compression along the y-axis, often described as a vertical or horizontal transformation depending on the context.

Identifying the Specific Transformation: Slope Change

Mathematical Explanation of the Transformation

- The original function: $f(x) = 4x + 1$
- The transformed function: $g(x) = 2x + 1$

Notice that the only difference is the coefficient of x.

- The coefficient of x in $f(x)$ is 4.
- The coefficient of x in $g(x)$ is 2.

This indicates that the slope has been scaled by a factor of $1/2$.

What Does a Change in Slope Represent?

- A change in slope from 4 to 2 means the graph has become less steep.
- This can be viewed as a vertical compression by a factor of 1/2.

Algebraic Perspective on Transformation

- The function g(x) can be expressed as:

g(x) = (1/2) (4x + 1) + C (where C is some constant)

- But in this case, since both functions share the same y-intercept at (0,1), the transformation is purely a change in the slope, with no vertical shift.

Relation to Vertical Compression

- Vertical compression occurs when the output values of a function are scaled by a factor between 0 and 1.
- Here, the entire graph of f(x) is scaled vertically by a factor of 1/2 to produce g(x).

Formal Explanation of the Transformation

Vertical Compression Defined

- A vertical compression of a function y = f(x) by a factor k (where 0 < k < 1) results in a new function y = k f(x).
- The graph becomes "squeezed" toward the x-axis, reducing the slope's steepness.

Applying to Our Functions

- The original function: y = 4x + 1
- The scaled function: y = (1/2) (4x + 1) = 2x + 0.5

However, note that in our case, the y-intercept remains at 1, not 0.5, indicating that the transformation is not simply a uniform vertical compression of the entire function, but a change in the slope with the intercept remaining constant.

Refinement: Slope Scaling Without Changing the Intercept

- Since only the slope changes from 4 to 2, and the y-intercept stays at 1, the transformation can be described as:

- A vertical compression of the slope component, but with the intercept fixed.

- This is achieved by adjusting the coefficient of x in the function expression.

Visualizing the Transformation

Graphical Representation

- The original line $f(x) = 4x + 1$ is steep, crossing the y-axis at 1 and rising sharply.
- The new line $g(x) = 2x + 1$ is less steep, crossing the y-axis at the same point but increasing more gradually.

Effect of Transformation

- The graph of $g(x)$ is a horizontal compression of the graph of $f(x)$ along the x-axis, but since the slope is scaled by $1/2$, the overall effect is a steepness reduction.

- Alternatively, thinking in terms of the functions' parameters:

- The change in the coefficient of x from 4 to 2 corresponds to a vertical compression of the slope component.

Summary of the Transformation Type

Conclusion

- The transformation from $f(x) = 4x + 1$ to $g(x) = 2x + 1$ involves a vertical compression of the graph.
- Specifically, the slope of the line is scaled by a factor of $1/2$, making the line less steep.
- The y-intercept remains unchanged, indicating no vertical translation.

What Is the Main Transformation Name?

- The main transformation is a vertical compression (or scaling) of the graph by a factor of $1/2$, affecting the slope but not the y-intercept.

Additional Considerations and Applications

Why Is This Important?

- Understanding how functions change when coefficients are altered allows mathematicians and students to predict the behavior of graphs under transformations.
- These concepts are foundational in advanced topics such as affine transformations, linear algebra, and calculus.

Real-World Applications

- Transformations like these are used in engineering to model systems that scale or contract.
- Economics often models proportional changes in relationships, which can be visualized through such transformations.

Further Exploration

- How would the graph change if the intercept also changed?
- What happens if the coefficient of x is negative?
- How do combined transformations (stretch, shift, reflection) affect linear functions?

In summary, changing the function from $f(x) = 4x + 1$ to $g(x) = 2x + 1$ is primarily a vertical compression of the graph, reducing the slope from 4 to 2 while maintaining the same y-intercept. This illustrates how coefficients in linear functions directly influence the steepness and appearance of their graphs, providing a clear example of the power of algebraic transformations in visualizing and understanding mathematical relationships.

Frequently Asked Questions

What type of transformation occurs when the function $y = 4x + 1$ is transformed into $g(x) = 2x + 1$?

The transformation is a horizontal compression by a factor of 1/2.

Does changing the coefficient from 4 to 2 in the function affect the graph's slope or its shape?

Yes, it reduces the slope from 4 to 2, resulting in a less steep line, indicating a horizontal compression.

Is the transformation from $y = 4x + 1$ to $g(x) = 2x + 1$ a vertical or horizontal transformation?

It is a horizontal transformation, specifically a compression.

How does the change from 4x to 2x in the function affect the graph's steepness?

The graph becomes less steep because the slope decreases from 4 to 2.

What is the scale factor of the transformation applied to the original function?

The scale factor is 1/2, indicating a horizontal compression by half.

Does the y-intercept change when transforming from y = 4x + 1 to g(x) = 2x + 1?

No, the y-intercept remains the same at 1.

Why is the transformation called a horizontal compression rather than a vertical one?

Because only the coefficient of x (the slope) changes, affecting the horizontal shape of the graph, not the y-intercept.

If the original function is y = 4x + 1, what is the effect of replacing 4 with 2 on the graph's equation?

Replacing 4 with 2 causes the graph to be horizontally compressed, making it less steep.

Additional Resources

Transformation

Introduction

Mathematics often involves analyzing how functions change under different transformations, revealing deeper insights into their structure and behavior. When examining the transition from one linear function to another, understanding the specific type of transformation involved becomes essential for both students and seasoned mathematicians. In this article, we will explore the transformation from the function $f(x) = 4x + 1$ to $g(x) = 2x + 1$, dissecting the nature of the change, its implications, and the underlying principles that govern such modifications.

Understanding the Original and Transformed Functions

The Original Function: $f(x) = 4x + 1$

This is a linear function characterized by:

- Slope (m): 4
- Y-intercept (b): 1

Graphically, it represents a straight line crossing the y-axis at $(0, 1)$ with a steepness determined by its slope of 4. For each unit increase in x, the value of $f(x)$ increases by 4 units.

The Transformed Function: $g(x) = 2x + 1$

This is also a linear function with:

- Slope: 2
- Y-intercept: 1

The key difference is the slope, which has been reduced from 4 to 2, while the y-intercept remains unchanged at \((0, 1) \).

Deconstructing the Transformation: What Has Changed?

To understand the transformation, we need to analyze how the change from \(f(x)\) to \(g(x)\) affects the graph's features.

1. Change in Slope: From 4 to 2

The slope determines the steepness of the line:

- Original slope (4): The line rises 4 units vertically for every 1 unit horizontally.
- New slope (2): The line rises 2 units vertically for every 1 unit horizontally.

Implication: The graph becomes less steep, indicating a "flattening" or "compression" along the y-axis relative to the x-axis.

2. Y-Intercept Remains Constant

Since both functions pass through \((0, 1) \), the point where the line crosses the y-axis remains unchanged.

Identifying the Nature of the Transformation

The core question is: What kind of transformation modifies \(f(x)\) into \(g(x)\)?

Common Types of Linear Transformations

- Vertical shifts: Moving the graph up or down.
- Horizontal shifts: Moving the graph left or right.
- Vertical scaling (stretching/compressing): Changing the steepness vertically.
- Horizontal scaling (stretching/compressing): Changing the spread along the x-axis.
- Reflections: Flipping across axes.

In this case, since the y-intercept remains fixed, and only the slope changes, the transformation involves a change in the steepness of the line, which is primarily a scaling transformation.

Focused Analysis: Is It a Vertical or Horizontal Transformation?

- Since the y-intercept remains unchanged, the vertical shift is zero.
- The slope change indicates a change in steepness, which can be achieved via vertical scaling.

Mathematical Explanation: How the Slope Changes Indicate Vertical Scaling

The Concept of Vertical Scaling

Vertical scaling involves multiplying the output of a function by a constant factor. For linear functions, this impacts the slope directly.

Given a function:

$$
f(x) = m x + b
$$

Scaling the output by a factor k yields:

$$
g(x) = k \cdot f(x) = k (m x + b) = (k m) x + k b
$$

In our case, the transformation from $f(x) = 4x + 1$ to $g(x) = 2x + 1$ suggests:

$$
g(x) = \frac{1}{2} f(x)
$$

since:

$$
g(x) = \frac{1}{2} (4x + 1) = 2x + \frac{1}{2}
$$

But note that the y-intercept is not preserved exactly; it changes from 1 to 0.5. Therefore, simply scaling the entire function by 1/2 would shift the y-intercept as well.

However, in our initial functions, the y-intercept remains at 1, which means the change in slope is not achieved solely by a simple scalar multiplication of the entire function. Instead, the functions are related through a different type of transformation.

Transforming the Slope Without Changing the Intercept

To alter the slope while keeping the y-intercept constant, the transformation must involve:

- Adjusting the rate of change (slope).
- Maintaining the point $(0, 1)$.

This is characteristic of a vertical scaling combined with a translation along the x-axis, but since the y-intercept remains the same, the most straightforward explanation is that the change in the slope is a result of a vertical compression or stretch of the graph.

Formal Mathematical Perspective

From the perspective of function transformations:

$$
g(x) = a \cdot f(bx + c) + d
$$

However, since both functions are linear and pass through the same point, the transformation is more straightforward.

The transformation from $f(x)$ to $g(x)$:

$$
g(x) = \frac{1}{2} (f(x))
$$

which suggests a vertical compression by a factor of $\frac{1}{2}$.

But, if we check:

$$
f(x) = 4x + 1
$$
$$
g(x) = 2x + 1
$$

then:

$$
g(x) = \frac{1}{2} \cdot f(x) + \text{adjustment}
$$

Calculating:

$$
\frac{1}{2} \cdot (4x + 1) = 2x + 0.5
$$

which is not the same as $g(x) = 2x + 1$.

Thus, the change in the y-intercept indicates that the transformation isn't a simple scalar multiple of the entire function.

The Actual Transformation: A Reflection and Vertical Compression

Step 1: Recognize that the slopes are related

- The slope decreases from 4 to 2, which is a reduction by a factor of 1/2.
- The y-intercept stays at 1, indicating no vertical shift.

Step 2: Interpret the change as a scaling of the slope

This can be viewed as:

- Vertical compression: The graph's steepness reduces by half.
- No vertical shift: Since the y-intercept remains unchanged.

Step 3: Formal transformation description

The transformation from $f(x)$ to $g(x)$ can be modeled as:

$$
g(x) = \frac{1}{2} f(x) + \text{adjustment}
$$

but as shown earlier, simply scaling the original function by 1/2 shifts the y-intercept to 0.5, not 1.

Hence, the actual transformation involves:

- Scaling the slope by a factor of 1/2.
- Adding an adjustment to keep the y-intercept at 1.

In essence, the transformation is a change in the slope with a vertical translation to maintain the y-intercept.

Geometric Interpretation: The Transformation in Action

Visualizing the Graphs

- Original function \(f(x) = 4x + 1\): Steep line crossing the y-axis at 1, rising rapidly.
- Transformed function \(g(x) = 2x + 1\): Less steep, crossing the same y-intercept at 1, but with a gentler slope.

Transformation Summary:

- Type: Vertical compression (reducing the slope from 4 to 2).
- Effect on the graph: The line becomes flatter, maintaining the same y-intercept.

Practical Applications and Examples

Understanding this transformation is crucial in various mathematical and real-world contexts, such as:

- Scaling in physics: Adjusting the rate of change of a physical quantity.
- Data modeling: Simplifying models by reducing the slope for better interpretability.
- Computer graphics: Transforming lines and shapes through scaling.

List of Key Takeaways

- The transition from \(f(x) = 4x + 1\) to \(g(x) = 2x + 1\) involves a slope adjustment.
- Since the y-intercept remains constant, the transformation is primarily a vertical compression.
- The change reduces the steepness of the line by a factor of 1/2.
- The transformation can be viewed as multiplying the original function by 1/2 and then adjusting the intercept, or more simply, as a scaling of the slope with no vertical shift.

Conclusion

In summary, the transformation from \(f(x) = 4x + 1\) to \(g(x) = 2x + 1\) exemplifies a vertical scaling or compression that halves the slope of the

The Function X 4x 1 Is Transformed To G X 2x 1 What Type Of Transformation Has Occurred

The Function X 4x 1 Is Transformed To G X 2x 1 What Type Of Transformation Has Occurred

Related Articles

- Values-based Businesses View CSR As ________ Rather Than As ________. An Opportunity; A Way Of Doing

- The Bias In Favor Of Existing Decisions Is Captured By The Latin Expression A. Bona Fide. B. Ab Ovo.
- The Consumption Of An Additional Unit Of A Good Provides Additional Satisfaction, Which Is Called:a.

Back to Home