The Integral Expression Xcos^2(2x)dx Can Be Evaluated Using Integration By Parts Of The Form Udv=uv-

The Integral Expression Xcos^2(2x)dx Can Be Evaluated Using Integration By Parts Of The Form Udv=uv- is a statement that highlights a common technique in calculus for tackling complex integral expressions. Integration by parts is a powerful method derived from the product rule of differentiation, allowing us to transform an integral of a product into simpler components. When faced with integrals involving products of functions, such as polynomials and trigonometric functions, this technique often simplifies the evaluation process significantly.

In this article, we will explore how to evaluate the integral of the form $\int x \cos^2(2x) \, dx$ using the integration by parts method. We will discuss the theoretical foundation, step-by-step procedures, and practical tips to master this technique. Whether you are a student preparing for exams or a calculus enthusiast seeking to deepen your understanding, this comprehensive guide will provide valuable insights into the effective application of integration by parts.

Understanding the Integration by Parts Formula

The Formula and Its Components

The integration by parts formula is expressed as:

$$
\int u \, dv = uv - \int v \, du
$$

Where:
- u is a function chosen from the integrand that simplifies when differentiated.
- dv is the remaining part of the integrand, which should be easily integrable.
- du is the differential of u, obtained by differentiating u.
- v is the integral of dv.

This formula is derived from the product rule of differentiation:

$$
\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}
$$

Rearranged, it leads to the integration by parts formula.

When to Use Integration by Parts

This technique is particularly useful when:
- The integrand is a product of algebraic and transcendental functions, such

as polynomials and exponential, logarithmic, or trigonometric functions.
- Direct integration is complicated or impossible.
- Repeated application simplifies the integral step-by-step.

In our case, the integral $\int x \cos^2(2x) \, dx$ involves a polynomial x and a squared cosine function, making it an ideal candidate for integration by parts.

Step-by-Step Evaluation of $\int x \cos^2(2x) \, dx$

Step 1: Choose u and dv

A critical step is selecting which part of the integrand to assign to u and which to dv. Generally:
- Choose u as the algebraic part (here, x), because its derivative simplifies the expression.
- Assign the remaining part to dv.

Thus:
- $u = x \Rightarrow du = dx$
- $dv = \cos^2(2x) dx$

Next, we need to find $v = \int \cos^2(2x) dx$.

Step 2: Compute $v = \int \cos^2(2x) dx$

The integral of $\cos^2(2x)$ can be evaluated using the power-reduction identity:

$$
\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}
$$

Applying this with $\theta = 2x$:

$$
\cos^2(2x) = \frac{1 + \cos(4x)}{2}
$$

Then,

$$
v = \int \frac{1 + \cos(4x)}{2} dx = \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \cos(4x) dx
$$

Calculating each term:

$$
v = \frac{1}{2} x + \frac{1}{2} \times \frac{\sin(4x)}{4} + C = \frac{1}{2} x + \frac{\sin(4x)}{8} + C
$$

For the purposes of the integration by parts formula, we omit the constant.

Step 3: Apply the Integration by Parts Formula

Now, apply:

$$
\int x \cos^2(2x) dx = uv - \int v \, du
$$

Substituting the expressions:

$$
= x \left(\frac{1}{2} x + \frac{\sin(4x)}{8} \right) - \int \left(\frac{1}{2} x + \frac{\sin(4x)}{8} \right) dx
$$

Simplify the first term:

$$
= \frac{1}{2} x^2 + \frac{x \sin(4x)}{8} - \int \left(\frac{1}{2} x + \frac{\sin(4x)}{8} \right) dx
$$

Now, evaluate the remaining integral:

$$
\int \left(\frac{1}{2} x + \frac{\sin(4x)}{8} \right) dx = \frac{1}{2} \int x dx + \frac{1}{8} \int \sin(4x) dx
$$

Calculating each:

- $\int x dx = \frac{x^2}{2}$
- $\int \sin(4x) dx = - \frac{\cos(4x)}{4}$

Thus:

$$
= \frac{1}{2} \times \frac{x^2}{2} + \frac{1}{8} \times \left(- \frac{\cos(4x)}{4} \right) = \frac{x^2}{4} - \frac{\cos(4x)}{32}
$$

Putting it all together:

$$
\int x \cos^2(2x) dx = \frac{1}{2} x^2 + \frac{x \sin(4x)}{8} - \left(\frac{x^2}{4} - \frac{\cos(4x)}{32} \right)
$$

Simplify:

$$
= \frac{1}{2} x^2 + \frac{x \sin(4x)}{8} - \frac{x^2}{4} + \frac{\cos(4x)}{32}
$$

Combine like terms:

$$
= \left(\frac{1}{2} x^2 - \frac{1}{4} x^2 \right) + \frac{x \sin(4x)}{8} + \frac{\cos(4x)}{32}
$$

$$
= \frac{1}{4} x^2 + \frac{x \sin(4x)}{8} + \frac{\cos(4x)}{32} + C
$$

Final Answer:

$$
\boxed{
\int x \cos^2(2x) \, dx = \frac{1}{4} x^2 + \frac{x \sin(4x)}{8} + \frac{\cos(4x)}{32} + C
}
$$

Practical Tips for Using Integration by Parts Effectively

Choosing u and dv

- Typically, select u as the algebraic polynomial part because differentiating it simplifies the expression.
- Choose dv as the remaining part, especially if it is easily integrable.

Repeated Integration by Parts

- If the resulting integral after applying the formula is similar to the original, consider applying integration by parts repeatedly.
- For example, integrals involving powers of x often require multiple steps.

Use of Power-Reduction and Trigonometric Identities

- Simplify complex trigonometric functions using identities before integrating.
- Power-reduction formulas are particularly useful for squared cosine or sine functions.

Conclusion

The integral $\int x \cos^2(2x) dx$ exemplifies the effectiveness of the integration by parts technique, especially when dealing with products involving polynomials and trigonometric functions. By carefully selecting u and dv, simplifying intermediate steps with identities, and systematically applying the formula, we obtain a closed-form expression that captures the behavior of the original integral.

Mastering this method enhances one's problem-solving toolkit in calculus, enabling the evaluation of a wide range of integrals that may initially seem complex. Whether for academic pursuits, engineering applications, or mathematical research, the integration by parts technique remains an essential and versatile tool in the calculus arsenal.

Keywords: Integration by parts, calculus, integral evaluation, \(\int x \cos^2(2x) dx\), trigonometric integrals, power-reduction identity, step-by-step solution

Frequently Asked Questions

What is the integral of x $\cos^2(2x)$ dx and how can it be approached using integration by parts?

The integral of x $\cos^2(2x)$ dx can be evaluated using integration by parts by choosing u = x (which simplifies upon differentiation) and dv = $\cos^2(2x)$ dx. Applying the formula ∫udv = uv - ∫v du, you first find v by integrating dv and then proceed accordingly.

How do you compute the integral of x $\cos^2(2x)$ dx using the formula ∫udv = uv - ∫v du?

Set u = x so that du = dx, and set dv = $\cos^2(2x)$ dx. Find v by integrating dv, which involves using a power-reduction identity for $\cos^2(2x)$. Then, apply the integration by parts formula: $\int x \cos^2(2x)\, dx = x\cdot v - \int v\cdot dx$.

What is the importance of rewriting $\cos^2(2x)$ before applying integration by parts?

Rewriting $\cos^2(2x)$ using the power-reduction identity, such as $\cos^2(\theta) = (1 + \cos(2\theta))/2$, simplifies the integral of dv to a sum of elementary functions, making the integration process more straightforward.

Can you explain the step of finding v in the integral of x $\cos^2(2x)$ dx?

Yes. To find v, integrate dv = $\cos^2(2x)$ dx. Using the identity $\cos^2(2x) = (1 + \cos(4x))/2$, the integral becomes $(1/2)\int(1 + \cos(4x))\, dx = x/2 + (1/8)\sin(4x) + C$. So, $v = x/2 + (1/8)\sin(4x)$.

What are the key steps in applying integration by parts to evaluate ∫x $\cos^2(2x)$ dx?

The key steps are: 1) Choose u = x and dv = $\cos^2(2x)$ dx. 2) Find v by integrating dv, often using identities. 3) Apply the formula ∫udv = uv - ∫v du. 4) Simplify and evaluate the remaining integral.

How does the choice of u and dv influence the ease of evaluating the integral in this context?

Choosing $u = x$ makes du simple (dx), and selecting dv as the more complex part, such as $\cos^2(2x)\,dx$, allows us to reduce the integral to manageable parts. Proper choice minimizes the complexity and facilitates straightforward integration.

What identities are useful for integrating $\cos^2(2x)$ in this process?

The primary identity used is $\cos^2(\theta) = (1 + \cos(2\theta))/2$. Applying this to $\cos^2(2x)$ gives $\cos^2(2x) = (1 + \cos(4x))/2$, which simplifies the integration of dv.

What is the final step after applying integration by parts to the integral of $x \cos^2(2x)\,dx$?

The final step is to substitute all parts back into the integration by parts formula, simplify the resulting expression, and include the constant of integration to obtain the complete indefinite integral.

Additional Resources

Integration Techniques: Mastering the Evaluation of $\int x \cos^2(2x)\,dx$ Using Integration By Parts

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Introduction

When tackling complex integrals, mathematicians often turn to a toolkit of strategies, each suited to different types of functions. Among these, integration by parts stands out as a versatile and powerful method, especially for products of algebraic and transcendental functions. In this detailed exploration, we'll focus on evaluating the integral:

\[
\int x \cos^2(2x) \, dx
\]

using the integration by parts formula:

\[
\int u \, dv = uv - \int v \, du
\]

This integral is a classic example often encountered in calculus courses, and understanding its solution provides deep insight into the application of integration by parts, trigonometric identities, and substitution techniques.

Understanding the Integral Components

Before diving into the integration process, it's crucial to dissect the integral:

$$
\int x \cos^2(2x) \, dx
$$

- Algebraic part (x): This is a simple polynomial term, which suggests choosing it as u in the integration by parts method because it simplifies upon differentiation.
- Trigonometric part ($\cos^2(2x)$): This is a squared cosine function, which benefits from identities and substitution techniques to simplify.

The combination of an algebraic term and a trigonometric squared function makes this integral an excellent candidate for integration by parts, especially since differentiating x simplifies it, while integrating $\cos^2(2x)$ requires some trigonometric identities.

Step 1: Choosing u and dv

The first step in applying integration by parts is to choose u and dv such that the resulting integral is more manageable.

- Let $u = x$: Differentiating u simplifies it, giving $du = dx$.
- Let $dv = \cos^2(2x) \, dx$: We need to integrate this later to find v.

This choice aligns with the LIATE rule (Logarithmic, Inverse trig, Algebraic, Trig, Exponential), which suggests selecting algebraic functions as u when paired with trigonometric functions.

Step 2: Computing v — Integrating $\cos^2(2x)$

The key challenge here is to evaluate:

$$
v = \int \cos^2(2x) \, dx
$$

Applying the Power-Reduction Identity:

Using the identity:

$$
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
$$

we rewrite the integral as:

$$
v = \int \frac{1 + \cos (4x)}{2} \, dx
$$

which simplifies to:

$$
v = \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \cos(4x) \, dx
$$

Calculating these integrals:

- $\int 1 \, dx = x$
- $\int \cos(4x) \, dx = \frac{\sin(4x)}{4}$

Thus,

$$
v = \frac{1}{2} x + \frac{1}{2} \times \frac{\sin(4x)}{4} = \frac{x}{2} + \frac{\sin(4x)}{8}
$$

Step 3: Applying the Integration by Parts Formula

Now that we have:

$$
u = x, \quad du = dx
$$

$$
dv = \cos^2(2x) \, dx, \quad v = \frac{x}{2} + \frac{\sin(4x)}{8}
$$

we substitute into the integration by parts formula:

$$
\int x \cos^2(2x) \, dx = uv - \int v \, du
$$

which becomes:

$$
\int x \cos^2(2x) \, dx = x \left(\frac{x}{2} + \frac{\sin(4x)}{8} \right) - \int \left(\frac{x}{2} + \frac{\sin(4x)}{8} \right) dx
$$

Expanding the first term:

$$
= \frac{x^2}{2} + \frac{x \sin(4x)}{8} - \int \frac{x}{2} \, dx - \int \frac{\sin(4x)}{8} \, dx
$$

Step 4: Computing the Remaining Integrals

The integral now reduces to evaluating:

$$
\int \frac{x}{2} \, dx \quad \text{and} \quad \int \frac{\sin(4x)}{8} \, dx
$$

Integral 1: $\int \frac{x}{2} \, dx$

$$
= \frac{1}{2} \int x \, dx = \frac{1}{2} \times \frac{x^2}{2} = \frac{x^2}{4}
$$

Integral 2: $\int \frac{\sin(4x)}{8} \, dx$

Pulling out constants:

$$
= \frac{1}{8} \int \sin(4x) \, dx
$$

Recall:

$$
\int \sin(ax) \, dx = -\frac{\cos(ax)}{a}
$$

Applying this:

$$
= \frac{1}{8} \times \left(- \frac{\cos(4x)}{4} \right) = -
\frac{\cos(4x)}{32}
$$

Step 5: Synthesizing the Final Expression

Putting all parts together, the integral becomes:

$$
\boxed{
\int x \cos^2(2x) \, dx = \frac{x^2}{2} + \frac{x \sin(4x)}{8} -
\frac{x^2}{4} + \frac{\cos(4x)}{32} + C
}
$$

Simplify the algebraic terms:

$$
\frac{x^2}{2} - \frac{x^2}{4} = \frac{x^2}{4}
$$

Thus, the final evaluated integral is:

$$
\boxed{
\int x \cos^2(2x) \, dx = \frac{x^2}{4} + \frac{x \sin(4x)}{8} +
\frac{\cos(4x)}{32} + C
}
$$

where C is the constant of integration.

Expert Tips and Additional Insights

- Trigonometric Identities Are Key: The power-reduction identity significantly simplifies the integral of $\cos^2(2x)$. Recognizing such identities early can save time and prevent errors.
- Choosing u and dv Strategically: Selecting $u = x$ leverages the simplicity of differentiating algebraic functions, while integrating $\cos^2(2x)$ is manageable via identities.
- Handling Complex Trigonometric Integrals: When encountering powers of sine or cosine, always consider identities to reduce powers to linear combinations, simplifying the integral.
- Verification: Always differentiate your final result to confirm it produces the original integrand. This practice ensures accuracy.

Summary of the Process

1. Identify the integral structure: product of polynomial and squared trigonometric function.
2. Select $u = x$, $dv = \cos^2(2x) dx$.
3. Compute v using power-reduction identities.
4. Apply the integration by parts formula: $\int u \, dv = uv - \int v \, du$.
5. Simplify and evaluate remaining integrals, leveraging identities and substitution.
6. Combine all parts for the final answer, adding the constant of integration.

Final Remarks

Mastering the evaluation of integrals like $\int x \cos^2(2x) \, dx$ enhances not only calculus proficiency but also sharpens problem-solving skills applicable across physics, engineering, and applied mathematics. The strategic use of identities, careful selection of parts, and methodical calculation form the backbone of effective integration techniques. As with any mathematical tool, practice and familiarity turn these procedures into intuitive solutions, empowering you to tackle even more challenging integrals with confidence.

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