The Nth Term Of A Sequence Is 3n2 - N + 10 Where N Is A Positive Integer. Which Term In The Sequence

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Understanding sequences and their terms is fundamental in mathematics, especially in algebra and number theory. When given a specific formula for the nth term of a sequence, such as $a_n = 3n^2 - n + 10$, it becomes essential to interpret what the sequence looks like, how to find particular terms, and what the sequence tells us about the pattern or behavior of the numbers involved. This article explores the detailed process of analyzing this sequence, determining specific terms, and understanding the implications of the formula.

Understanding the Sequence and Its Formula

What is the Sequence?

The sequence in question is defined by the formula:

$$a_n = 3n^2 - n + 10$$

where n is a positive integer (i.e., $n = 1, 2, 3, \dots$). Each term of the sequence is generated by substituting the value of n into the formula.

Characteristics of the Sequence

- Quadratic Nature: Since the formula involves n^2, the sequence is quadratic, implying that the differences between consecutive terms will not be constant but will change in a predictable way.
- Growth Rate: The dominant term $3n^2$ indicates that as n increases, the terms grow approximately proportionally to n^2.
- Initial Terms: Calculating the first few terms can help visualize the sequence:

n	$a_n = 3n^2 - n + 10$	Calculated Value
1	$3(1)^2 - 1 + 10$	$3 - 1 + 10 = 12$
2	$3(2)^2 - 2 + 10$	$3(4) - 2 + 10 = 12 - 2 + 10 = 20$
3	$3(3)^2 - 3 + 10$	$3(9) - 3 + 10 = 27 - 3 + 10 = 34$
4	$3(4)^2 - 4 + 10$	$3(16) - 4 + 10 = 48 - 4 + 10 = 54$
5	$3(5)^2 - 5 + 10$	$3(25) - 5 + 10 = 75 - 5 + 10 = 80$

From these initial terms, the sequence begins as: 12, 20, 34, 54, 80, and so on.

How To Find Which Term In The Sequence Corresponds To A Given

Value

Setting Up the Equation

Suppose you are asked: "Which term in the sequence is equal to a specific number?" or "At what value of n does the sequence reach a particular value T?"

Given the general term:

$$ a_n = 3n^2 - n + 10 $$

If you want to find the value of n for a specific term T, set:

$$ 3n^2 - n + 10 = T $$

This is a quadratic equation in n:

$$ 3n^2 - n + (10 - T) = 0 $$

Solving the Quadratic Equation

To find n, solve:

$$ 3n^2 - n + (10 - T) = 0 $$

Using the quadratic formula:

$$ n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:

- $a = 3$
- $b = -1$
- $c = 10 - T$

Substituting:

$$ n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times (10 - T)}}{2 \times 3} $$

Simplify:

$$ n = \frac{1 \pm \sqrt{1 - 12(10 - T)}}{6} $$

$$ n = \frac{1 \pm \sqrt{1 - 120 + 12T}}{6} $$

$$ n = \frac{1 \pm \sqrt{12T - 119}}{6} $$

Interpreting the Solution

- For n to be a valid term index (a positive integer), the discriminant:

$$D = 12T - 119$$

must be a perfect square (since n must be real and the square root must be an integer).

- Additionally, the value of n must be positive, so:

$$n = \frac{1 \pm \sqrt{D}}{6}$$

must be positive and an integer.

- Usually, we consider the positive root for n:

$$n = \frac{1 + \sqrt{D}}{6}$$

- If this yields a positive integer n, then the term at position n in the sequence is T.

Example:

Suppose we want to find out which term in the sequence is 54.

Set $T = 54$:

$$D = 12 \times 54 - 119 = 648 - 119 = 529$$

Since $529 = 23^2$, a perfect square.

Calculate:

$$n = \frac{1 + 23}{6} = \frac{24}{6} = 4$$

Thus, the 4th term in the sequence is 54, matching the earlier calculation.

Finding the Term Number Given a Specific Term Value

Step-by-Step Procedure

1. Identify the value of the term you want to locate in the sequence, say T.

2. Calculate the discriminant:

$$D = 12T - 119$$

3. Check if D is a perfect square:

- If not, then the term T does not exist in the sequence (assuming the sequence only contains integer terms).

- If yes, proceed.

4. Calculate n:

$$n = \frac{1 + \sqrt{D}}{6}$$

- Verify if n is a positive integer.

5. Conclusion:

- If n is a positive integer, then the term T appears at position n in the sequence.

Analyzing the Sequence's Behavior and Pattern

Growth Rate and Asymptotic Behavior

Since the dominant term is $3n^2$, the sequence's growth is quadratic. As n increases:

$$a_n \sim 3n^2$$

This indicates the sequence grows approximately three times faster than the standard quadratic sequence n^2.

Differences Between Terms

Calculating the first differences:

$$a_{n+1} - a_n = [3(n+1)^2 - (n+1) + 10] - [3n^2 - n + 10]$$

Simplify:

$$= 3(n^2 + 2n + 1) - n - 1 + 10 - 3n^2 + n - 10$$
$$= 3n^2 + 6n + 3 - n - 1 + 10 - 3n^2 + n - 10$$
$$= (3n^2 - 3n^2) + (6n - n + n) + (3 - 1 + 10 - 10)$$
$$= 0 + (6n) + (2) = 6n + 2$$

Thus, the difference between consecutive terms:

$$a_{n+1} - a_n = 6n + 2$$

which increases linearly with n. This confirms the quadratic nature of the sequence.

Practical Applications and Significance

Use in Pattern Recognition and Mathematical Modelling

- Recognizing quadratic sequences assists in modeling real-world phenomena where growth accelerates over time, such as certain population models, physics problems involving acceleration, or financial projections with quadratic trends.

Educational Value

- Analyzing such sequences helps students deepen their understanding of algebra, quadratic equations, and the relationship between formulas and sequence behaviors.

Sequence Inversion and Problem Solving

- The process of finding \(n \) given \(T \) demonstrates the importance of solving quadratic equations and understanding their solutions in context.

Summary and Key Takeaways

- The sequence

Frequently Asked Questions

What is the general formula for the nth term of the sequence?

The nth term of the sequence is given by 3n^2 - n + 10.

How do you find the 5th term of the sequence?

Substitute n=5 into the formula: 3(5)^2 - 5 + 10 = 3(25) - 5 + 10 = 75 - 5 + 10 = 80. So, the 5th term is 80.

Which term in the sequence is the largest among the first 10 terms?

Since the sequence is quadratic with a positive leading coefficient, the terms increase as n increases. Therefore, the 10th term is the largest among the first 10 terms.

How do you determine the n-th term for any positive integer n?

Replace n with the specific positive integer in the formula 3n^2 - n + 10 to find the term.

What is the 1st term of the sequence?

Substitute n=1: 3(1)^2 - 1 + 10 = 3 - 1 + 10 = 12. So, the first term is 12.

Can the sequence be classified as an arithmetic or geometric sequence?

No, because the nth term involves n^2, indicating it is a quadratic sequence, not arithmetic or geometric.

How would you find the term in the sequence where the value exceeds 200?

Set 3n^2 - n + 10 > 200 and solve for n: 3n^2 - n + 10 > 200. Simplify to 3n^2 - n - 190 > 0 and find n using quadratic inequalities.

Is the sequence increasing or decreasing for positive integers?

Since the leading coefficient of n^2 is positive, the sequence is increasing as n increases.

How can you verify the correctness of the nth term formula?

By calculating several terms for specific values of n and comparing them to the formula's output to ensure consistency.

Additional Resources

The Nth Term Of A Sequence Is $3n^2 - n + 10$ Where N Is A Positive Integer. Which Term In The Sequence?

Understanding sequences and their properties is fundamental in mathematics, especially in algebra and number theory. When given a formula for the nth term of a sequence, a natural question arises: which term in the sequence corresponds to a particular value? Conversely, given a position n, what is the value of the term? In this article, we delve into a specific sequence defined by the formula:

$T(n) = 3n^2 - n + 10$

where n is a positive integer (n = 1, 2, 3, ...). We will explore how to interpret this formula, analyze the sequence's behavior, determine particular terms, and address common questions such as identifying the position of a specific term.

Understanding the Sequence Formula

The Formula: $T(n) = 3n^2 - n + 10$

At its core, this formula is a quadratic expression in n, comprising three components:

- $3n^2$: A quadratic term that dominates as n increases, indicating that the sequence grows rapidly.
- $-n$: A linear term that slightly adjusts the growth rate.
- $+10$: A constant term that shifts the entire sequence upwards.

This combination results in a sequence where each term increases as n increases, with the growth rate accelerating due to the quadratic component.

Analyzing the Sequence Behavior

Initial Terms and Pattern Recognition

Calculating the first few terms provides insight into the sequence's nature:

n	$T(n) = 3n^2 - n + 10$	Calculation
1	$3(1)^2 - 1 + 10$	$3 - 1 + 10 = 12$
2	$3(2)^2 - 2 + 10$	$3(4) - 2 + 10 = 12 - 2 + 10 = 22$
3	$3(3)^2 - 3 + 10$	$3(9) - 3 + 10 = 27 - 3 + 10 = 34$
4	$3(4)^2 - 4 + 10$	$3(16) - 4 + 10 = 48 - 4 + 10 = 54$
5	$3(5)^2 - 5 + 10$	$3(25) - 5 + 10 = 75 - 5 + 10 = 80$

The sequence begins as: 12, 22, 34, 54, 80, and so on. Notice that the difference between consecutive terms is increasing:

- $22 - 12 = 10$
- $34 - 22 = 12$
- $54 - 34 = 20$
- $80 - 54 = 26$

While the differences don't form a simple pattern, they grow larger, consistent with quadratic growth.

Determining the Term at a Given Position

Suppose a reader wants to know what the 10th term is. The process involves substituting $n = 10$ into the formula:

$$T(10) = 3(10)^2 - 10 + 10$$

Calculations:

- $10^2 = 100$
- $3 \times 100 = 300$
- $300 - 10 + 10 = 300$

Therefore, the 10th term in the sequence is 300.

This straightforward substitution method is applicable for any positive integer n.

Finding the Term Number Corresponding to a Given Value

A more challenging problem is: Given a term value, which term does it correspond to? For example, if the sequence contains the value 80, at what position n does this occur?

This requires solving the quadratic equation:

$$3n^2 - n + 10 = \text{value}$$

Suppose the value is 80:

$$3n^2 - n + 10 = 80$$

Rearranged:

$3n^2 - n + 10 - 80 = 0$

which simplifies to:

$3n^2 - n - 70 = 0$

This is a quadratic in n:

$3n^2 - n - 70 = 0$

Solving the Quadratic Equation

Using the quadratic formula:

$n = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

where:

- $a = 3$
- $b = -1$
- $c = -70$

Calculations:

- Discriminant $D = b^2 - 4ac = (-1)^2 - 4 \times 3 \times (-70) = 1 + 840 = 841$

- $\sqrt{D} = \sqrt{841} = 29$

Now, find the roots:

$n = [1 \pm 29] / (2 \times 3)$

- $n = (1 + 29) / 6 = 30 / 6 = 5$
- $n = (1 - 29) / 6 = -28 / 6 \approx -4.666...$

Since n must be a positive integer, the meaningful solution is $n = 5$.

Thus, the term with value 80 appears at $n = 5$.

General Approach for Inverse Problems

The above example demonstrates how to find the position of a term given its value:

1. Set up the quadratic equation: $T(n) = \text{value}$.
2. Rearrange to standard quadratic form.
3. Calculate the discriminant to ensure real solutions.
4. Apply the quadratic formula.
5. Identify positive integer solutions: only positive integer n are valid since the sequence is defined for positive integers.

This approach is crucial for problems involving inverse sequences or reverse lookup.

Application: Real-World Contexts and Problem-Solving

Sequences defined by quadratic formulas like $T(n) = 3n^2 - n + 10$ appear in various contexts:

- Physics: Modeling motion where displacement relates quadratically to time.
- Economics: Calculating cumulative investments or costs with quadratic growth.
- Computer Science: Analyzing algorithms with quadratic time complexity.

Understanding how to manipulate such sequences enables problem-solvers to predict future values, find specific terms, or determine the position of terms with particular values.

Practical Examples and Exercises

To solidify understanding, consider these exercises:

1. Calculate the 15th term of the sequence.
2. Find the term corresponding to the value 150.
3. Determine if the value 200 appears in the sequence, and if so, find its position.
4. Identify the position of the first term greater than 100 in the sequence.

Conclusion: The Power of Quadratic Sequences

The sequence defined by $T(n) = 3n^2 - n + 10$ exemplifies how quadratic formulas generate sequences with increasing complexity and growth. By mastering techniques such as substitution, solving quadratic equations, and analyzing differences, students and professionals alike can decode the behavior of such sequences.

Whether predicting future terms, analyzing existing data, or solving inverse problems, understanding the structure and properties of quadratic sequences unlocks a range of mathematical and real-world applications. As with many mathematical tools, practice and familiarity are key to leveraging their full potential.

Final Thoughts

In sum, the question "which term in the sequence corresponds to a particular value?" can be answered systematically by:

- Substituting known n to find $T(n)$,
- Solving quadratic equations when given $T(n)$,
- And understanding the growth pattern through initial terms and differences.

Sequences like $T(n) = 3n^2 - n + 10$ serve as excellent examples to develop problem-solving skills and deepen comprehension of quadratic functions in mathematics.

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