

The Point Given Below Is On The Terminal Side Of An Angle In Standard Position. Find The Exact Value

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Understanding the concepts of angles, especially in the context of the coordinate plane, is fundamental in trigonometry. When dealing with angles in standard position, the terminal side's location plays a crucial role in determining the exact values of trigonometric functions. This article aims to provide a comprehensive guide to understanding how to find the exact value of a trigonometric function given a point on the terminal side of an angle in standard position.

What Is an Angle in Standard Position?

Before diving into the specifics of finding exact values, it's essential to understand what an angle in standard position entails.

Definition of an Angle in Standard Position

An angle in standard position is an angle whose vertex is at the origin $(0,0)$ of the coordinate plane, and its initial side lies along the positive x-axis. The terminal side is the ray that results after rotating the initial side by a certain measure, either clockwise or counterclockwise.

Visual Representation

- Vertex: Located at the origin.
- Initial Side: Along the positive x-axis.
- Terminal Side: The ray after rotation, which can be in any quadrant depending on the angle's measure.

Given Point on the Terminal Side: Understanding Coordinates

When a point (x, y) lies on the terminal side of an angle in standard position, it provides critical information to determine the angle's trigonometric values.

Significance of the Coordinates

- The coordinates directly relate to the angle's sine, cosine, tangent, and other trigonometric functions.
- The distance from the origin to the point, known as the radius (r), helps normalize these values.

Calculating the Radius (r)

The radius (also called the hypotenuse in the context of the right triangle) is calculated using the Pythagorean theorem:

$$r = \sqrt{x^2 + y^2}$$

This value is essential in deriving the exact trigonometric ratios.

Key Trigonometric Ratios and Their Calculation

Given a point (x, y) on the terminal side, the basic trigonometric functions are:

- Sine ($\sin \theta$)
- Cosine ($\cos \theta$)
- Tangent ($\tan \theta$)
- Cosecant ($\csc \theta$)
- Secant ($\sec \theta$)
- Cotangent ($\cot \theta$)

Formulas for Each Function

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

Note: These formulas assume $(r \neq 0)$, and the signs of x and y determine

the quadrant of the angle.

Step-by-Step Procedure to Find the Exact Value

To find the exact value of a trigonometric function given a point on the terminal side, follow these steps:

Step 1: Identify the Coordinates

- Extract the x and y values from the given point.

Step 2: Calculate the Radius (r)

- Use the Pythagorean theorem:

$$r = \sqrt{x^2 + y^2}$$

Step 3: Determine the Trigonometric Function

- Plug the x, y, and r into the relevant formula to compute the exact value.

Step 4: Simplify the Expression

- Simplify the resulting fraction or radical to its simplest form, which gives the exact value.

Practical Examples

Let's consider some practical examples to illustrate the process clearly.

Example 1: Point in Quadrant I

Suppose the point is (3, 4).

Step 1: Coordinates:

- $x=3$
- $y=4$

Step 2: Calculate r:

$$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 3: Find $(\sin \theta)$, $(\cos \theta)$, and $(\tan \theta)$:

$$\begin{aligned} & \left[\right. \\ & \sin \theta = \frac{4}{5} \\ & \left. \right] \\ & \left[\right. \\ & \cos \theta = \frac{3}{5} \\ & \left. \right] \\ & \left[\right. \\ & \tan \theta = \frac{4}{3} \\ & \left. \right] \end{aligned}$$

Step 4: Exact values are $(\sin \theta = \frac{4}{5})$, $(\cos \theta = \frac{3}{5})$, and $(\tan \theta = \frac{4}{3})$.

Example 2: Point in Quadrant II

Suppose the point is $(-6, 8)$.

Step 1: Coordinates:

- $(x = -6)$
- $(y = 8)$

Step 2: Calculate r :

$$\begin{aligned} & \left[\right. \\ & r = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \\ & \left. \right] \end{aligned}$$

Step 3: Find $(\sin \theta)$, $(\cos \theta)$, and $(\tan \theta)$:

$$\begin{aligned} & \left[\right. \\ & \sin \theta = \frac{8}{10} = \frac{4}{5} \\ & \left. \right] \\ & \left[\right. \\ & \cos \theta = \frac{-6}{10} = -\frac{3}{5} \\ & \left. \right] \\ & \left[\right. \\ & \tan \theta = \frac{8}{-6} = -\frac{4}{3} \\ & \left. \right] \end{aligned}$$

Step 4: The exact values reflect the signs dictated by the quadrant: sine is positive, cosine negative, tangent negative.

Example 3: Point in Quadrant III

Suppose the point is $(-7, -24)$.

Step 1: Coordinates:

- $(x = -7)$

$$- \ (y = -24)$$

Step 2: Calculate r :

$$r = \sqrt{(-7)^2 + (-24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

Step 3: Find the ratios:

$$\sin \theta = \frac{-24}{25}$$

$$\cos \theta = \frac{-7}{25}$$

$$\tan \theta = \frac{-24}{-7} = \frac{24}{7}$$

Step 4: The signs indicate the angle is in the third quadrant where both sine and cosine are negative, and tangent is positive.

Special Cases and Considerations

When working with points on the terminal side, consider the following:

1. Points on Axes

- If the point is on the x-axis ($y=0$), then:
 - $\sin \theta = 0$
 - $\cos \theta = \pm 1$ depending on the x-coordinate
 - $\tan \theta = 0$
- If the point is on the y-axis ($x=0$), then:
 - $\cos \theta = 0$
 - $\sin \theta = \pm 1$ depending on the y-coordinate
 - $\tan \theta$ is undefined

2. Quadrant Signs

- Quadrant I: Both x and y are positive.
- Quadrant II: x is negative, y is positive.
- Quadrant III: Both x and y are negative.
- Quadrant IV: x is positive, y is negative.

3. Angle Measurement

- The angle θ can be found using inverse trigonometric functions if needed.

- For example:

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\[
\theta = \arcsin \left( \frac{y}{r} \right)
\]
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Understanding the Significance of Exact Values in Trigonometry

Exact values of trigonometric functions are essential in precise calculations, especially in fields such as engineering, physics, and computer graphics. They also provide insight into the properties of angles and their relationships.

Common Exact Values

Some well-known exact values for special angles are:

Angle (θ)	$\sin \theta$	$\cos \theta$	$\tan \theta$
0° or 0 rad	0	1	0
30° or $\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45° or $\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	

Frequently Asked Questions

What does it mean when a point lies on the terminal side of an angle in standard position?

It means the point is located on the ray that forms the terminal side of the angle when the angle is measured from the positive x-axis in standard position.

How do you find the exact value of a trigonometric function given a point on the terminal side of an angle?

You identify the coordinates of the point (x, y) , then use the definitions of

trigonometric functions, such as $\text{sine} = y/r$, $\text{cosine} = x/r$, and $\text{tangent} = y/x$, where r is the distance from the origin to the point.

What is the significance of the radius r in finding the exact value of a trigonometric function?

The radius r , calculated as $\sqrt{x^2 + y^2}$, represents the hypotenuse of the right triangle formed and is essential for computing the exact values of sine, cosine, and other functions.

If a point on the terminal side of an angle in standard position is given as $(3, 4)$, what are the exact values of $\sin \theta$ and $\cos \theta$?

First, find $r = \sqrt{3^2 + 4^2} = 5$. Then, $\sin \theta = 4/5$ and $\cos \theta = 3/5$.

How can understanding the point on the terminal side help in solving trigonometric equations?

Knowing the coordinates allows you to directly compute the exact values of sine, cosine, etc., which are useful in solving equations involving these functions with precision.

What are common methods to verify the exact value of a trigonometric function given a point on the terminal side?

Common methods include calculating the radius r and applying the definitions of the trigonometric functions, or using known special triangles and their ratios if the point corresponds to a standard angle.

Additional Resources

Understanding the Terminal Side of an Angle in Standard Position: An Expert Breakdown

When delving into the world of trigonometry, one of the foundational concepts that students and professionals alike encounter is the angle in standard position and its associated terminal side. These ideas serve as the backbone for understanding how angles relate to the unit circle, coordinate systems, and trigonometric functions. This article provides an in-depth, comprehensive exploration of the topic, aiming to clarify complex points and guide you through the process of finding exact values when the terminal side of an angle is given.

What Is an Angle in Standard Position?

Before diving into the specifics of the terminal side, it's essential to understand what constitutes an angle in standard position.

Definition and Significance

An angle in standard position is a geometric concept where:

- The initial side of the angle is positioned along the positive x-axis.
- The vertex of the angle is at the origin point $(0,0)$ in the coordinate plane.
- The measure of the angle is typically denoted in degrees or radians, and it is formed by rotating the initial side counterclockwise (for positive angles) or clockwise (for negative angles).

This positioning provides a standardized reference frame, allowing consistent analysis across different problems and applications.

Visual Representation

Imagine the coordinate plane:

- The initial side is a ray starting at the origin, extending along the positive x-axis.
- The terminal side is the ray where the angle's measure ends after rotation.

The angle itself is the measure of the rotation from the initial side to the terminal side, usually indicated with a curved arrow around the origin.

The Terminal Side of an Angle: Its Role and Importance

The terminal side is the final position of the ray after rotating from the initial side.

Understanding the Terminal Side

- It determines the exact position of the angle in the coordinate plane.

- It can lie in any of the four quadrants or on the axes, depending on the angle's measure.
- The location of the terminal side directly affects the values of the sine, cosine, tangent, and other trigonometric functions.

Quadrant Positions and Their Significance

Angles and their terminal sides can be positioned in:

- Quadrant I (0° to 90° / 0 to $\pi/2$ radians): Both x and y are positive.
- Quadrant II (90° to 180° / $\pi/2$ to π radians): x is negative, y is positive.
- Quadrant III (180° to 270° / π to $3\pi/2$ radians): Both x and y are negative.
- Quadrant IV (270° to 360° / $3\pi/2$ to 2π radians): x is positive, y is negative.
- On the axes: The terminal side may lie exactly on the x -axis or y -axis, representing special angles like 0° , 90° , 180° , etc.

Finding the Exact Value of the Trigonometric Function: Step-by-Step Approach

Suppose the problem states: "The point on the terminal side of an angle in standard position is given, and you are asked to find the exact value of a trigonometric function."

The process involves several steps:

Step 1: Identify the Point on the Terminal Side

- Usually provided as an ordered pair $((x, y))$.
- This point lies on the terminal side of the angle, which passes through it.
- The point's coordinates are crucial because they directly relate to the trigonometric functions.

Step 2: Calculate the Radius (r)

- The radius (r) is the distance from the origin to the point $((x, y))$:

$$r = \sqrt{x^2 + y^2}$$

- The value of r helps normalize the coordinates when dealing with the unit circle.

Step 3: Use the Definitions of Trigonometric Functions

With the point (x, y) and the radius r , the basic functions are:

- Sine: $\sin \theta = \frac{y}{r}$
- Cosine: $\cos \theta = \frac{x}{r}$
- Tangent: $\tan \theta = \frac{y}{x}$, provided $x \neq 0$

Step 4: Simplify to Find Exact Values

- Express the functions in simplest radical form.
- Use known Pythagorean identities to simplify if necessary.
- Consider the quadrant to determine the sign (positive or negative) of the function.

Application Example: Finding the Exact Value

Let's walk through an example that illustrates these steps in action.

Given Data

Suppose the point on the terminal side is $(-3, 4)$.

Step 1: Determine r

Calculate the distance from the origin:

$$r = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 2: Find $(\sin \theta)$, $(\cos \theta)$, and $(\tan \theta)$

- $(\sin \theta = \frac{y}{r} = \frac{4}{5})$
- $(\cos \theta = \frac{x}{r} = \frac{-3}{5})$
- $(\tan \theta = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3})$

Step 3: Identify the Quadrant

- Since (x) is negative and (y) is positive, the point lies in Quadrant II.
- In Quadrant II:
 - $(\sin \theta)$ is positive (consistent with $(+4/5)$)
 - $(\cos \theta)$ is negative (consistent with $(-3/5)$)
 - $(\tan \theta)$ is negative (consistent with $(-4/3)$)

Thus, the signs align with quadrant II.

Step 4: Exact Values of the Trigonometric Functions

- $(\boxed{\sin \theta = \frac{4}{5}})$
- $(\boxed{\cos \theta = -\frac{3}{5}})$
- $(\boxed{\tan \theta = -\frac{4}{3}})$

Dealing with Special and Common Angles

In many cases, the point corresponds to well-known angles like 30° , 45° , 60° , etc. Recognizing these angles simplifies finding exact values.

Unit Circle and Special Angles

Angle	Coordinates on Unit Circle	Exact Values of Functions
$0^\circ / 0 \text{ rad}$	$(1, 0)$	$(\sin 0=0), (\cos 0=1), (\tan 0=0)$
$30^\circ / \pi/6$	$(\sqrt{3}/2, 1/2)$	$(\sin 30^\circ=1/2), (\cos 30^\circ=\sqrt{3}/2), (\tan 30^\circ=1/\sqrt{3})$
$45^\circ / \pi/4$	$(\sqrt{2}/2, \sqrt{2}/2)$	$(\sin 45^\circ=\sqrt{2}/2), (\cos 45^\circ=\sqrt{2}/2), (\tan 45^\circ=1)$
$60^\circ / \pi/3$	$(1/2, \sqrt{3}/2)$	$(\sin 60^\circ=\sqrt{3}/2), (\cos$

$$\cos 60^\circ = 1/2, \quad \tan 60^\circ = \sqrt{3}$$

Recognizing and Applying These Values

When the point matches these coordinates or can be scaled from them, calculating the exact value becomes straightforward.

Special Cases and Considerations

While the process described covers most scenarios, there are certain special cases to be aware of:

Angles on the Axes

- When the terminal side lies exactly on the axes (e.g., $(r, 0)$ or $(0, r)$), some functions are undefined or zero.
- For example, at $\theta = 0^\circ$, $\sin \theta = 0$, but $\cos \theta = 1$.
- When $x = 0$, $\tan \theta$ is undefined.

Negative and Coterminal Angles

- Angles differing by full rotations (2π)

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