The Point Is On The Terminal Side Of An Angle In Standard Position. Find The Exact Values Of The Six

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Understanding the six trigonometric functions—sine, cosine, tangent, cosecant, secant, and cotangent—is fundamental in trigonometry. These functions provide essential insights into the relationships between angles and side lengths in right triangles, as well as their extensions to the coordinate plane. When dealing with angles in standard position, particularly those with points on their terminal sides, it becomes crucial to determine the exact values of these functions based on the coordinates of the point.

This comprehensive guide aims to clarify how to find the exact values of all six trigonometric functions when a point resides on the terminal side of an angle in standard position. We will explore the concepts step-by-step, including the role of the coordinate plane, the significance of the point's position, and the formulas needed to compute these values accurately.

Understanding Angles in Standard Position

Before diving into the calculations, it is essential to understand what it means for an angle to be in standard position and how points relate to that angle.

Definition of Standard Position

- An angle is said to be in standard position if:
- Its vertex is at the origin (0,0) of the coordinate plane.
- Its initial side lies along the positive x-axis.
- Its terminal side is some ray emanating from the origin, forming an angle with the positive x-axis.

Point on the Terminal Side

- When a point (x, y) lies on the terminal side of an angle in standard position:
- The coordinates (x, y) uniquely determine the size of the angle (except for coterminal angles).
- The point's position relative to the origin indicates the direction of the terminal side.

Key Concepts for Calculating Trigonometric Functions

The six trigonometric functions are based on a right triangle or, more generally, on the coordinates of the point on the terminal side.

Radii and Distance from the Origin

- The radius (or hypotenuse in the right triangle context) is the distance from the origin to the point:

$$
r = \sqrt{x^2 + y^2}
$$

\]

- This value is crucial for calculating sine, cosine, and the other functions.

Significance of Quadrants

- The signs of x and y depend on the quadrant where the point lies:
- Quadrant I: $x > 0, y > 0$
- Quadrant II: $x < 0, y > 0$
- Quadrant III: $x < 0, y < 0$
- Quadrant IV: $x > 0, y < 0$

Knowing the quadrant helps determine the signs of the trigonometric functions.

Calculating the Six Trigonometric Functions

Given a point (x, y) on the terminal side of the angle, the functions are defined as follows:

Sine ($\sin \theta$)
- $\sin \theta = \frac{y}{r}$

Cosine ($\cos \theta$)
- $\cos \theta = \frac{x}{r}$

Tangent ($\tan \theta$)
- $\tan \theta = \frac{y}{x}$, provided $x \neq 0$

Cosecant ($\csc \theta$)
- $\csc \theta = \frac{r}{y}$, provided $y \neq 0$

Secant ($\sec \theta$)
- $\sec \theta = \frac{r}{x}$, provided $x \neq 0$

Cotangent ($\cot \theta$)
- $\cot \theta = \frac{x}{y}$, provided $y \neq 0$

Step-by-Step Procedure to Find Exact Values

To find the exact values of the six functions when a point is given:

Step 1: Identify the Coordinates

- Determine the values of x and y from the given point (x, y).

Step 2: Calculate the Radius r

- Use the distance formula:

$$
r = \sqrt{x^2 + y^2}
$$

- Ensure that $r \neq 0$. If $r=0$, the point is at the origin, and the functions are undefined.

Step 3: Compute Each Function

- Use the formulas provided above to find the six functions:
- $\sin \theta = y / r$
- $\cos \theta = x / r$
- $\tan \theta = y / x$
- $\csc \theta = r / y$
- $\sec \theta = r / x$
- $\cot \theta = x / y$

Step 4: Determine the Signs

- Based on the quadrant:
- Assign the correct signs to each function.
- Remember that:
- Sine and cosecant are positive in Quadrants I and II.
- Cosine and secant are positive in Quadrants I and IV.
- Tangent and cotangent are positive in Quadrants I and III.

Examples of Finding Exact Trigonometric Values

To clarify the process, let's explore a couple of examples.

Example 1: Point in Quadrant I

- Given $(x, y) = (3, 4)$:

1. Calculate r:

$$
r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

\]

2. Compute functions:
- $\sin \theta = 4 / 5 = 0.8$
- $\cos \theta = 3 / 5 = 0.6$
- $\tan \theta = 4 / 3 \approx 1.333$
- $\csc \theta = 5 / 4 = 1.25$
- $\sec \theta = 5 / 3 \approx 1.666$
- $\cot \theta = 3 / 4 = 0.75$

3. Since the point is in Quadrant I, all functions are positive.

Example 2: Point in Quadrant III

- Given $(x, y) = (-5, -12)$:

1. Calculate r:

$$
r = \sqrt{(-5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13
$$

2. Compute functions:
- $\sin \theta = -12 / 13 \approx -0.923$
- $\cos \theta = -5 / 13 \approx -0.385$
- $\tan \theta = -12 / -5 = 12/5 = 2.4$ (positive in Quadrant III)
- $\csc \theta = 13 / -12 \approx -1.083$
- $\sec \theta = 13 / -5 \approx -2.6$
- $\cot \theta = -5 / -12 = 5 / 12 \approx 0.417$

3. Signs are consistent with Quadrant III: sine, cosine, cosecant, and secant are negative; tangent and cotangent are positive.

Special Cases and Simplifications

While the above methods work universally, some points lead to special cases:

Points on the Axes

- When $x=0$ or $y=0$, some functions become undefined:
- If $x=0$, $\cos \theta = 0$, $\sec \theta$ undefined.
- If $y=0$, $\sin \theta = 0$, $\csc \theta$ undefined.

Angles Corresponding to Standard Positions

- Angles like $\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ have points on axes and lead to undefined functions in some cases.

Simplification for Special Points

- For points where x and y are integers, the functions often simplify to rational numbers.

Practice Problems and Solutions

To master these concepts, practice is essential. Here are some problems:

Problem 1:

- Given the point \((x

Frequently Asked Questions

What is the significance of the point being on the terminal side of an angle in standard position?

The point on the terminal side of an angle in standard position helps determine the sine, cosine, tangent, and other trigonometric function values exactly, often using coordinates of that point.

How can I find the exact values of the six trigonometric functions given a point on the terminal side of an angle?

Identify the coordinates (x, y) of the point, compute the hypotenuse $r = \sqrt{(x^2 + y^2)}$, then use the definitions: sine $= y/r$, cosine $= x/r$, tangent $= y/x$, cosecant $= r/y$, secant $= r/x$, and cotangent $= x/y$.

What are the six trigonometric functions for a point on the terminal side of an angle in standard position?

The six functions are sine ($\sin \theta$), cosine ($\cos \theta$), tangent ($\tan \theta$), cosecant ($\csc \theta$), secant ($\sec \theta$), and cotangent ($\cot \theta$), which can all be calculated from the coordinates of the point.

If the point on the terminal side is (3, 4), what are the exact values of the six trigonometric functions?

First, find $r = \sqrt{(3^2 + 4^2)} = 5$. Then: $\sin \theta = 4/5$, $\cos \theta = 3/5$, $\tan \theta = 4/3$, $\csc \theta = 5/4$, $\sec \theta = 5/3$, $\cot \theta = 3/4$.

Why is it important to find the exact values of the six trigonometric functions when the point is on the terminal side of an angle?

Finding exact values allows for precise calculations in solving trigonometric equations and understanding the relationships between angles and their functions without approximations.

Additional Resources

The Point Is On The Terminal Side Of An Angle In Standard Position: Find The Exact Values Of The Six Trigonometric Functions

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Introduction

In the realm of trigonometry, understanding the relationships between angles and points on the coordinate plane is fundamental. A key concept is the position of points relative to an angle in standard position, especially when considering the six primary trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. The question of whether a point lies on the terminal side of an angle in standard position is not mere theoretical curiosity—it forms the backbone of many applications across mathematics, physics, engineering, and computer science.

This article explores the intricacies of points lying on the terminal side of an angle in standard position, with a focus on deriving the exact values of the six trigonometric functions. Through detailed analysis, illustrative examples, and rigorous mathematical reasoning, we aim to provide a comprehensive understanding suitable for students, educators, and professionals seeking clarity in this foundational topic.

Understanding Standard Position and the Terminal Side

Definition of Standard Position

An angle in standard position is one whose vertex is at the origin (0,0) of the coordinate plane, with its initial side along the positive x-axis. The terminal side of the angle is obtained by rotating the initial side counterclockwise (for positive angles) or clockwise (for negative angles) by a certain measure.

The Significance of the Terminal Side

The terminal side determines the angle's position and the point on the coordinate plane associated with that angle. When a point (x, y) lies on the terminal side of an angle θ, it essentially represents the endpoint of the radius vector forming that angle, with the length of the vector being the distance from the origin to that point.

The Role of the Point in Trigonometry

Coordinates and the Unit Circle

The unit circle—a circle centered at the origin with radius 1—is a central tool in trigonometry. For any angle θ in standard position, the point where its terminal side intersects the unit circle has coordinates (cos θ, sin θ). These values directly give the six trigonometric functions:

- sin θ = y
- cos θ = x
- tan θ = y / x (when x ≠ 0)
- csc θ = 1 / y (when y ≠ 0)
- sec θ = 1 / x (when x ≠ 0)
- cot θ = x / y (when y ≠ 0)

Understanding whether a point lies on the terminal side of an angle in the coordinate plane allows for exact calculation of these functions, especially when the point is on the unit circle.

The Core Question: When Is a Point On The Terminal Side?

Suppose we are given a point (x, y) in the coordinate plane. How can we determine whether it lies on the terminal side of an angle in standard position?

Key Criterion:

- The point (x, y) lies on the terminal side of some angle θ in standard position if and only if the point is colinear with the origin (vertex) and the terminal side, i.e., (x, y) lies along a line passing through the origin.

Implication:

- The point defines the direction of the terminal side, and the angle θ can be found using the arctangent of y/x, provided $x \neq 0$.
- The point's distance from the origin (the length of the radius) influences the magnitude but not the direction, which determines the angle.

Exact Values of the Six Trigonometric Functions for Points on the Terminal Side

To find the exact values of the six functions, the key is to understand the relationship between the point's coordinates and the circle's radius. When the point is on the unit circle, calculations are straightforward. For points not on the unit circle, the process involves normalization.

Step 1: Confirm the Point Lies on the Terminal Side

- Check if (x, y) is proportional to a point on the unit circle: $(x / r, y / r)$, where $r = \sqrt{(x^2 + y^2)}$.
- If this scaled point is on the unit circle, then the angle θ can be directly derived from these ratios.

Step 2: Calculate the Radius (r)

- $r = \sqrt{(x^2 + y^2)}$

Step 3: Derive the Six Trigonometric Functions

- $\sin\theta = y / r$
- $\cos\theta = x / r$
- $\tan\theta = y / x \; (x \neq 0)$
- $\csc\theta = r / y \; (y \neq 0)$
- $\sec\theta = r / x \; (x \neq 0)$
- $\cot\theta = x / y \; (y \neq 0)$

Examples of Exact Value Calculations

Example 1: Point on the Unit Circle

Suppose $(x, y) = (\sqrt{3}/2, 1/2)$

- $r = \sqrt{[(\sqrt{3}/2)^2 + (1/2)^2]} = \sqrt{[(3/4) + (1/4)]} = \sqrt{1} = 1$
- $\sin\theta = y / r = 1/2$
- $\cos\theta = x / r = \sqrt{3}/2$
- $\tan\theta = (1/2) / (\sqrt{3}/2) = 1/\sqrt{3}$
- $\csc\theta = 1 / (1/2) = 2$
- $\sec\theta = 1 / (\sqrt{3}/2) = 2/\sqrt{3}$
- $\cot\theta = (\sqrt{3}/2) / (1/2) = \sqrt{3}$

This matches the well-known values for $\theta = 30°$ or $\pi/6$ radians.

Example 2: Point Not on the Unit Circle

Suppose $(x, y) = (3, 4)$

- r = ? $(3^2 + 4^2) = 5$
- sin θ = 4/5
- cos θ = 3/5
- tan θ = 4/3
- csc θ = 5/4
- sec θ = 5/3
- cot θ = 3/4

The angle θ in this case has a tangent of 4/3, corresponding to a 53.13° or approximately 0.927 radians.

Special Cases and Limitations

When x or y is Zero

- If $x = 0$ and $y \neq 0$, then θ is either 90° or 270°, with the point lying on the y-axis.
- If $y = 0$ and $x \neq 0$, then θ is either 0° or 180°, with the point on the x-axis.
- These cases require careful handling, especially when computing cotangent or cosecant functions.

When Both x and y are Zero

- The point (0, 0) is at the origin, which does not define an angle—thus, trigonometric functions are undefined at this point.

Implications for Trigonometric Function Values

Understanding whether a point lies on the terminal side of an angle allows for the exact determination of all six primary functions, which is crucial in solving complex trigonometric equations, analyzing wave functions, and modeling periodic phenomena.

Applications and Significance

- Mathematical Modeling: Precise function values are essential in physics and engineering for modeling oscillations, wave motion, and signal processing.
- Analytical Geometry: Recognizing points on terminal sides facilitates the solution of geometric problems involving angles and distances.
- Calculus: Derivatives and integrals involving trigonometric functions rely on these exact values for accuracy.
- Computer Graphics: Rotation, shading, and transformations depend on precise angular positions and their functions.

Conclusion

The question of whether a point lies on the terminal side of an angle in standard position is more than a geometric curiosity; it is a gateway to the precise calculation of fundamental trigonometric functions. By understanding the relationships between coordinates, the radius, and the angle, mathematicians and practitioners can derive exact values that underpin countless applications across scientific disciplines.

In summary, the key steps involve confirming the point's position relative to the origin, calculating the radius, and then applying the ratios to find the six trigonometric functions. Mastery of this process enhances one's ability to analyze angles and their associated functions with clarity and rigor, forming a cornerstone of advanced mathematical understanding.

End of Article

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