The Polynomial Y = (x - 5)(x - 2)(4x + 3) Has Been Graphed Incorrectly. Identify Theerror And How To

The Polynomial Y = (x - 5)(x - 2)(4x + 3) Has Been Graphed Incorrectly. Identify The Error And How To

Understanding how to accurately graph polynomials is a fundamental skill in algebra and calculus. Mistakes in graphing can lead to misconceptions about the behavior of functions, critical points, and intersections. In this article, we will analyze the polynomial Y = (x - 5)(x - 2)(4x + 3), identify common errors made during its graphing, and provide step-by-step guidance on how to correct these mistakes for an accurate representation.

Understanding the Polynomial: Y = (x - 5)(x - 2)(4x + 3)

Before delving into the errors and corrections, it is essential to understand the structure of the polynomial.

Factored Form and Roots

The polynomial is written in factored form:
- (x - 5)
- (x - 2)
- (4x + 3)

The roots of the polynomial are the x-values where each factor equals zero:
- x = 5
- x = 2
- x = -3/4 (since 4x + 3 = 0 ⇒ x = -3/4)

These roots are critical points where the graph intersects the x-axis.

Degree and Leading Coefficient

- The polynomial is of degree 3 (cubic), as it is the product of three linear factors.
- The leading term can be found by multiplying the highest degree terms:
- (x) (x) (4x) = 4x^3
- Therefore, the leading coefficient is 4, which is positive.

Common Errors in Graphing Polynomial Y = (x - 5)(x - 2)(4x + 3)

When plotting this polynomial, learners and even some experts can make mistakes that lead to an inaccurate graph. Here are some typical errors:

Error 1: Misidentifying the Roots

Many graphers incorrectly identify the x-intercepts, either by miscalculating or neglecting the linear factors.

Error 2: Ignoring Multiplicity and Behavior at Roots

Failing to observe how the graph behaves at each root—whether it crosses the x-axis or touches and bounces—can lead to incorrect shape representation.

Error 3: Overlooking the Leading Coefficient and End Behavior

Misjudging the end behavior of the graph, especially for cubic functions, can cause incorrect assumptions about the graph's direction as x approaches infinity or negative infinity.

Error 4: Failing to Find and Plot Critical Points

Neglecting to calculate the first derivative to find local maxima, minima, or inflection points results in an incomplete understanding of the function's shape.

Error 5: Incorrectly Calculating the Y-Values

Calculating the y-values at various x-points without careful substitution and check can introduce inaccuracies.

How to Correctly Graph the Polynomial Y = (x - 5)(x - 2)(4x + 3)

To ensure an accurate graph, follow these systematic steps:

Step 1: Find the Roots and x-Intercepts

- Set each factor equal to zero:
- $x - 5 = 0 \Rightarrow x = 5$
- $x - 2 = 0 \Rightarrow x = 2$
- $4x + 3 = 0 \Rightarrow x = -3/4$
- Plot these points on the x-axis to mark the x-intercepts.

Step 2: Determine the End Behavior

- The degree of the polynomial is 3 (odd degree).
- The leading coefficient is positive (4).
- For large positive x, $y \to +\infty$.
- For large negative x, $y \to -\infty$.
- The end behavior can be summarized as:
- As $x \to -\infty$, $y \to -\infty$
- As $x \to +\infty$, $y \to +\infty$

Step 3: Analyze the Sign of Each Factor and the Overall Polynomial

- For values of x less than -3/4, determine the sign of each factor:
- $(x - 5)$: negative
- $(x - 2)$: negative
- $(4x + 3)$: negative
- Count the number of negative factors:
- If the number of negative factors is odd, the overall product is negative.
- If even, the product is positive.

- Repeat for values between roots to understand how the graph crosses the x-axis.

Step 4: Find Critical Points (Optional but Helpful)

- Calculate the first derivative Y' to find local maxima and minima.
- Use the product rule:
$Y = (x - 5)(x - 2)(4x + 3)$

Derivative:

Y' = derivative of first factor other factors + first factor derivative of second factor third + first two factors derivative of third factor

Or, more systematically:

$Y' = (x - 2)(4x + 3) + (x - 5)(4x + 3) + (x - 5)(x - 2)4$

- Find critical points by setting $Y' = 0$ and solving for x.

Step 5: Calculate Y-Values at Key Points

- Evaluate y at the roots:
- Y(5) = 0
- Y(2) = 0
- Y(-3/4) = 0
- Choose additional x-values between roots to understand the shape, e.g., x = 0, x = 1, x = 3, etc.
- Plug these into the polynomial to find corresponding y-values.

Step 6: Sketch the Graph

- Plot the roots and key points.
- Draw the end behavior lines.
- Connect the points smoothly, respecting where the graph crosses or touches the x-axis.
- Use the critical points to identify local maxima and minima for a more accurate curve.

Visualizing the Polynomial: Step-by-Step Example

Let's walk through an example to see how to implement these steps in practice.

Calculating Roots

- Roots:
- x = 5
- x = 2
- x = -3/4

Evaluating Y at Selected x-values

- x = 0:

Y = (0 - 5)(0 - 2)(40 + 3) = (-5)(-2)(3) = (10)(3) = 30

- x = 1:

Y = (1 - 5)(1 - 2)(41 + 3) = (-4)(-1)(7) = (4)(7) = 28

- x = 3:

Y = (3 - 5)(3 - 2)(43 + 3) = (-2)(1)(15) = (-2)(15) = -30

Plot these points to understand the curve's ascent and descent.

Determining Critical Points

- Derivative:

Y' = (x - 2)(4x + 3) + (x - 5)(4x + 3) + (x - 5)(x - 2)4

Simplify step-by-step:

1. (x - 2)(4x + 3) = 4x^2 + 3x - 8x - 6 = 4x^2 - 5x - 6
2. (x - 5)(4x + 3) = 4x^2 + 3x - 20x - 15 = 4x^2 - 17x - 15
3. (x - 5)(x - 2) = x^2 - 7x + 10

Multiply by 4:

4(x^2 - 7x + 10) = 4x^2 - 28x + 40

Sum all parts:

Y' = (4x^2 - 5x - 6) + (4x^2 - 17x - 15) + (4x^2 - 28x + 40)

Combine like terms:

Y' = (4x^2 + 4x^2 + 4x^2) + (-5x - 17x - 28x) + (-6 - 15 + 40)

Y' = 12x^2 - 50x + 19

- Set Y' = 0:

12x^2 - 50x + 19 = 0

Use quadratic formula:

x = [50 ± √(50^2 - 41219)] / (212)

x = [50 ± √(2500 - 912)] / 24

x = [50 ± √1588] / 24

Approximate √1588 ≈

Frequently Asked Questions

What is the common mistake made when graphing the polynomial Y = (x - 5)(x - 2)(4x + 3)?

A common mistake is miscalculating the roots or not correctly determining the end behavior, leading to an incorrect graph.

How do the roots of the polynomial Y = (x - 5)(x - 2)(4x + 3) affect its graph?

The roots are x = 5, x = 2, and x = -3/4; these are the x-intercepts where the graph crosses the x-axis, and they should be accurately plotted.

What is the correct way to determine the end behavior of the polynomial?

Since the polynomial is degree 3 with a positive leading coefficient (from 4x), the graph should fall to the left and rise to the right, consistent with cubic functions.

How can factoring errors lead to incorrect graphing of the polynomial?

Incorrectly factoring or expanding the polynomial can lead to wrong roots or shape; ensure the factorization is correct and fully expanded if needed.

What role do multiplicities of roots play in the graph of Y = (x - 5)(x - 2)(4x + 3)?

All roots have multiplicity one, so the graph will cross the x-axis at each root, not just touch and turn.

How should you verify the accuracy of the graph of this polynomial?

Check the roots, verify the end behavior, and plot additional points to confirm the shape matches the expected cubic curve.

What is a common error when plotting the turning points of this polynomial?

A common mistake is not calculating the derivative correctly or overlooking the critical points, leading to an inaccurate depiction of maxima and minima.

How can you correct an incorrectly graphed polynomial of this form?

Identify the roots and their multiplicities, verify the end behavior, and replot key points, especially around the roots and potential turning points.

Why is it important to expand the polynomial correctly

before graphing?

Expanding ensures you understand the degree, leading coefficient, and shape, which are essential for accurate plotting and interpretation.

Additional Resources

The Polynomial Y = (x - 5)(x - 2)(4x + 3) Has Been Graphed Incorrectly. Identify The Error And How To Correct It

In the world of mathematics, especially in algebra and calculus, accurate graphing of polynomials is crucial for understanding their behavior, roots, and overall characteristics. A recent review of the polynomial Y = (x - 5)(x - 2)(4x + 3) reveals that a common misinterpretation or misapplication has led to an incorrect graph, potentially misleading students, educators, and professionals relying on visual data. This article aims to dissect the error, examine its origins, and provide a comprehensive guide on how to correctly graph this polynomial.

Understanding the Polynomial Structure

Before delving into the error, it is essential to understand the structure and components of the polynomial:

- The polynomial is expressed as the product of three factors:
- (x - 5)
- (x - 2)
- (4x + 3)

- Each factor contributes to the polynomial's roots, behavior, and degree.

- Since it is a product of three factors, the degree of the polynomial is 3, making it a cubic polynomial.

Key properties:

- Roots:
- x = 5 (from x - 5)
- x = 2 (from x - 2)
- x = -3/4 (from 4x + 3)

- Leading coefficient:
- The product of the coefficients of the highest degree terms:
- From (x - 5) and (x - 2), each has a leading coefficient of 1.
- From (4x + 3), the leading coefficient is 4.
- Overall leading coefficient: 1 1 4 = 4.

- End behavior:
- As $x \to \pm\infty$, the polynomial behaves like $4x^3$, tending toward positive or negative infinity depending on the sign of x.

Understanding these properties is foundational to accurate graphing.

The Common Error in Graphing

Many students and even educators have fallen into a trap when graphing this polynomial, often leading to an incorrect visual representation. The most prevalent mistake involves misinterpreting the roots and their multiplicities, especially concerning the behavior at those roots.

The specific error:

- Incorrectly assuming all roots are simple and symmetric in behavior.
- Misplacing the roots on the x-axis.
- Miscalculating the multiplicity and the impact on the polynomial's behavior at each root.
- Neglecting the influence of the leading coefficient and degree on the end behavior.

This mistake often manifests as a graph that:

- Does not intersect the x-axis at the correct points.
- Shows incorrect multiplicity behavior, such as not flattening or touching the x-axis where appropriate.
- Has incorrect end behavior, appearing inconsistent with the polynomial's degree and leading coefficient.

Why does this happen?

- Confusing the roots with their multiplicities.
- Overlooking the effect of the factor $(4x + 3)$ on the roots and the degree.
- Incorrectly plotting the roots without considering the sign and slope behavior at those roots.

Illustrative Example of the Error

Suppose a grapher identifies the roots at $x=5$, $x=2$, and $x=-3/4$. They then plot these points on the x-axis but assume:

- All roots are simple roots with multiplicity 1.
- The graph crosses the x-axis at all these points.
- The end behavior is symmetric and matches a cubic polynomial with a positive leading

coefficient.

In doing so, they may produce a graph that:

- Crosses the x-axis at x=5 and x=2, but perhaps incorrectly at x=-3/4.
- Fails to reflect the correct behavior at x=-3/4, which is an important root due to the factor 4x + 3.
- Mismatches the end behavior, especially if they neglect the leading coefficient's sign or degree.

This kind of misrepresentation leads to a flawed understanding of the polynomial's behavior, potentially affecting problem-solving and analytical skills.

Identifying the Actual Error in the Graph

The core of the error lies in misinterpreting the factor (4x + 3) and its contribution to the roots and overall graph.

Critical points:

- The root is at x = -3/4, not at x = 0 or any other point.
- The factor (4x + 3) has a multiplicity of 1 because it is linear.
- The behavior at the root x = -3/4 is that the graph crosses the x-axis at this point, not merely touches or stays tangent.

Common misinterpretation:

- Some graphers treat (4x + 3) as if it were (x + c), ignoring the coefficient 4.
- They may assume the root at x = -3/4 is a double root or higher, leading to the graph touching the x-axis rather than crossing it.

The correct interpretation:

- The root at x = -3/4 is a simple root with multiplicity 1.
- The graph should cross the x-axis at x = -3/4.
- The slope of the graph at that point is determined by the derivative, which for a simple root is non-zero.

How to Correctly Graph the Polynomial

Achieving an accurate graph involves a systematic approach:

Step 1: Factor and Identify Roots

- Roots:
- x = 5 (from x-5)
- x = 2 (from x-2)
- x = -3/4 (from 4x+3)
- Multiplicities:
- All are linear factors with multiplicity 1.

Step 2: Determine the Leading Coefficient and Degree

- Degree: 3 (product of three factors)
- Leading coefficient:
- Since (x - 5) and (x - 2) are degree 1 with leading coefficient 1, and (4x + 3) has leading coefficient 4,
- Overall leading coefficient: 1 1 4 = 4.

Step 3: Analyze End Behavior

- As x → ∞, y → ∞ (since degree is odd and leading coefficient is positive).
- As x → -∞, y → -∞.
- The graph will rise to +∞ on the right and fall to -∞ on the left.

Step 4: Determine Behavior at Roots

- At x=5 and x=2:
- The polynomial crosses the x-axis because these are simple roots.
- The graph will go from positive to negative or vice versa, depending on the sign of the polynomial around these points.
- At x=-3/4:
- The polynomial crosses the x-axis (since multiplicity is 1).
- The graph passes through the x-axis at this point.

Step 5: Plot Key Points and Sign of the Polynomial in Intervals

- Calculate the sign of Y in intervals:
- For x < -3/4
- Between -3/4 and 2
- Between 2 and 5
- For x > 5
- Use test points in each interval to determine whether the polynomial is positive or

negative.

Step 6: Sketch the Graph

- Begin at the bottom left (large negative x), where y → -∞.
- Cross the x-axis at x = -3/4, going upward.
- Pass through x=2 and x=5, crossing the x-axis.
- End at y → ∞ as x → ∞.
- Ensure the graph's shape reflects the cubic nature, with appropriate inflection points and slopes.

Additional Considerations and Common Pitfalls

Misconception 1: Ignoring the coefficient of the linear factor

- The factor 4x + 3 is often mistaken as x + 3, leading to an incorrect root at x = -3, which is not accurate.
- Always solve for the root: 4x + 3 = 0 → x = -3/4.

Misconception 2: Assuming multiplicity greater than 1

- For linear factors, multiplicity is 1 unless explicitly stated.
- Multiplicity affects whether the graph crosses or touches the x-axis:
- Odd multiplicity: the graph crosses the x-axis.
- Even multiplicity: the graph touches and turns around.

Misconception 3: Not considering end behavior

- The polynomial's degree and leading coefficient determine the end behavior.
- Failing to incorporate this leads to a graph that does not match the polynomial's true asymptotic tendencies.

Conclusion: Ensuring Accurate Polynomial Graphing

The case of Y = (x - 5)(x - 2)(4x + 3) illustrates how a small misinterpretation—particularly of the linear factor's root—can lead to significant inaccuracies in graphing. To avoid such errors:

- Always carefully factor and identify roots, including coefficients.

- Determine the multiplicity of each root.
- Analyze the degree and leading coefficient to understand end behavior.
- Use

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