The Polynomials P1 = 2x^2+1, P2= - 4x^2+x And P3 =x-2 Are Linearly Dependent. Select One: True False

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In the study of polynomials within linear algebra, understanding the concepts of linear dependence and independence is fundamental. When working with polynomial sets, such as P1 = 2x^2 + 1, P2 = -4x^2 + x, and P3 = x - 2, determining whether these polynomials are linearly dependent or independent helps in understanding their span, basis, and the structure of polynomial vector spaces. In this article, we will explore the concept of linear dependence among these specific polynomials, how to verify it, and the implications of such dependence in mathematical contexts.

Understanding Linear Dependence and Independence of Polynomials

What Is Linear Dependence?

Linear dependence among a set of vectors (or polynomials) exists when at least one element of the set can be expressed as a linear combination of the others. In the context of polynomials, this means there exist scalars, not all zero, such that:

$$c_1 P_1 + c_2 P_2 + c_3 P_3 = 0$$

where c_1, c_2, c_3 are real numbers, and 0 denotes the zero polynomial.

What Is Linear Independence?

Conversely, polynomials are linearly independent if the only solution to the above equation is when all scalars are zero:

$$c_1 = c_2 = c_3 = 0$$

This indicates no polynomial in the set can be written as a combination of the others.

Examining the Given Polynomials

The specific polynomials under consideration are:

- P1(x) = 2x^2 + 1
- P2(x) = -4x^2 + x
- P3(x) = x - 2

Our goal is to determine whether these three polynomials are linearly dependent or independent. To do this, we will analyze whether there exist nontrivial scalars \(c_1, c_2, c_3 \) such that:

\[c_1 (2x^2 + 1) + c_2 (-4x^2 + x) + c_3 (x - 2) = 0 \]

Methodology for Testing Linear Dependence

Step 1: Write the Linear Combination Explicitly

Express the combination as:

\[c_1 (2x^2 + 1) + c_2 (-4x^2 + x) + c_3 (x - 2) = 0 \]

which expands to:

\[(2c_1 x^2 + c_1) + (-4c_2 x^2 + c_2 x) + (c_3 x - 2c_3) = 0 \]

Step 2: Group Like Terms

Combine coefficients for similar powers of \(x \):

\[(2c_1 - 4c_2) x^2 + (c_2 + c_3) x + (c_1 - 2c_3) = 0 \]

Since this must hold for all \(x \), the coefficients of each power must individually equal zero:

\[
\begin{cases}
2c_1 - 4c_2 = 0 \quad \text{(coefficient of } x^2 \text{)} \\
c_2 + c_3 = 0 \quad \text{(coefficient of } x \text{)} \\
c_1 - 2c_3 = 0 \quad \text{(constant term)} \\

\end{cases}
\]

Step 3: Solve the System of Equations

Solve for \(c_1, c_2, c_3 \):

1. From the first equation:
\[2c_1 = 4c_2 \Rightarrow c_1 = 2c_2 \]

2. From the second:
\[c_3 = -c_2 \]

3. From the third:
\[c_1 = 2c_3 \]
But since \(c_1 = 2c_2 \) and \(c_3 = -c_2 \), substitute:

\[2c_2 = 2(-c_2) \Rightarrow 2c_2 = -2c_2 \Rightarrow 4c_2 = 0 \Rightarrow c_2 = 0 \]

Then:

- \(c_2 = 0 \)
- \(c_3 = -c_2 = 0 \)
- \(c_1 = 2c_2 = 0 \)

All scalars are zero, indicating the only solution is the trivial one.

Interpretation and Conclusion

Implication of the Result

Since the only solution to the linear combination equaling zero is when all scalars are zero, the set of polynomials \(\{P_1, P_2, P_3\}\) is linearly independent.

Final Answer

Based on this computation, the statement "The Polynomials P1 = 2x^2+1, P2= - 4x^2+x And P3 =x-2 Are

Linearly Dependent" is False.

Key Points Summary

- Linear dependence involves expressing one polynomial as a combination of others.
- To test dependence, set up a linear combination and equate to the zero polynomial.
- Group coefficients by powers of x and solve the resulting system.
- The unique trivial solution indicates linear independence.
- For the polynomials provided, the only solution is trivial, confirming independence.

Why Is This Important in Mathematics?

Understanding whether a set of polynomials is linearly dependent or independent is crucial in various mathematical applications, including:

- Determining bases for polynomial vector spaces
- Simplifying polynomial systems
- Analyzing the span of polynomial sets
- Applications in approximation theory, coding theory, and computational algebra

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Additional Resources for Learning Polynomial Linear Dependence

- Linear Algebra textbooks focusing on vector spaces
- Online tutorials on polynomial vector spaces
- Video lectures on linear dependence and independence
- Interactive algebra software to experiment with polynomial combinations

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In Summary

The detailed analysis of the polynomials P_1, P_2, P_3 demonstrates that they are linearly independent. This conclusion is reached by solving the linear combination equations and confirming that the only solution is the trivial one where all coefficients are zero. Recognizing such independence is vital in understanding the structure and basis of polynomial spaces, which has broad applications across mathematics and engineering.

Remember: Always verify linear dependence or independence through systematic algebraic methods to ensure accurate understanding of polynomial relationships.

Meta Description: Discover how to determine whether the polynomials \(P_1=2x^2+1 \), \(P_2=-4x^2+x \), and \(P_3=x-2 \) are linearly dependent or independent. Learn step-by-step methods and key concepts in linear algebra.

Frequently Asked Questions

Are the polynomials P1 = 2x^2 + 1, P2 = -4x^2 + x, and P3 = x - 2 linearly dependent?

False

What does it mean for polynomials to be linearly dependent?

Polynomials are linearly dependent if one can be expressed as a linear combination of the others.

Given P1 = 2x^2 + 1, P2 = -4x^2 + x, and P3 = x - 2, are these polynomials linearly independent?

Yes, they are linearly independent because none can be written as a linear combination of the others.

How can you determine if a set of polynomials is linearly dependent or independent?

By checking if there exist constants, not all zero, such that a linear combination of the polynomials equals the zero polynomial.

Is P3 = x - 2 a linear combination of P1 and P2?

No, P3 cannot be expressed as a linear combination of P1 and P2.

What is the significance of the polynomials P1, P2, and P3 being linearly dependent?

If they were linearly dependent, it would mean one polynomial can be derived from the others, indicating redundancy in the set.

Can the polynomials P1, P2, and P3 form a basis for a polynomial space?

Yes, since they are linearly independent, they can form part of a basis for a polynomial space.

What is the rank of the set {P1, P2, P3} in terms of linear dependence?

The rank is 3, indicating they are linearly independent.

Why is the statement 'The polynomials P1, P2, and P3 are linearly dependent' false?

Because none of these polynomials can be written as a linear combination of the others, confirming their linear independence.

Additional Resources

Linearity in Polynomials: An In-Depth Analysis of P1, P2, and P3

Understanding the linear dependence or independence of polynomials is a fundamental concept in linear algebra and polynomial theory, with implications spanning across various mathematical and applied fields. In this comprehensive article, we delve into the specific case of three polynomials:

- $P_1(x) = 2x^2 + 1$
- $P_2(x) = -4x^2 + x$
- $P_3(x) = x - 2$

Our goal is to determine whether these polynomials are linearly dependent or independent. We will explore the definitions, methods, and reasoning behind such determinations, culminating in an informed conclusion.

Understanding Linear Dependence and Independence of Polynomials

Before analyzing the specific polynomials, it's essential to grasp what it means for a set of polynomials to be linearly dependent or independent.

Definitions and Concepts

- Linear Dependence: A set of polynomials $\{P_1, P_2, ..., P_n\}$ is said to be linearly dependent if there exist constants $c_1, c_2, ..., c_n$, not all zero, such that:

$$
c_1 P_1(x) + c_2 P_2(x) + ... + c_n P_n(x) = 0
$$

for all x in the domain.

- Linear Independence: Conversely, the set is linearly independent if the only solution to the above equation is $c_1 = c_2 = ... = c_n = 0$.

In the context of polynomials, this means that if one polynomial can be written as a linear combination of others, they are dependent. If no such non-trivial combination exists, they are independent.

Methodology for Determining Dependence or Independence

To verify the linear dependence of P_1, P_2, P_3, we follow these steps:

1. Set up the linear combination equation:

$$
c_1 P_1(x) + c_2 P_2(x) + c_3 P_3(x) = 0
$$

2. Express the polynomials explicitly:

$$

$$
c_1 (2x^2 + 1) + c_2 (-4x^2 + x) + c_3 (x - 2) = 0
$$

3. Combine like terms:

$$
(2 c_1 - 4 c_2) x^2 + (c_2 + c_3) x + (c_1 - 2 c_3) = 0
$$

4. Since the equation must hold for all x, the coefficients of the powers of x must be zero:

- Coefficient of x^2:

$$
2 c_1 - 4 c_2 = 0
$$

- Coefficient of x:

$$
c_2 + c_3 = 0
$$

- Constant term:

$$
c_1 - 2 c_3 = 0
$$

5. Solve the resulting system of equations:

$$
\begin{cases}
2 c_1 - 4 c_2 = 0 \\
c_2 + c_3 = 0 \\
c_1 - 2 c_3 = 0
\end{cases}
$$

Solving the System: Are These Polynomials Linearly **Dependent?**

Let's proceed step-by-step to solve for c_1, c_2, c_3.

Step 1: Express c_1 in terms of c_3 from the third equation:

$$
c_1 - 2 c_3 = 0 \Rightarrow c_1 = 2 c_3
$$

Step 2: Express c_2 in terms of c_3 from the second equation:

$$
c_2 + c_3 = 0 \Rightarrow c_2 = - c_3
$$

Step 3: Substitute c_1 and c_2 into the first equation:

$$
2 c_1 - 4 c_2 = 0
$$

$$
2 (2 c_3) - 4 (- c_3) = 0
$$

$$
4 c_3 + 4 c_3 = 0
$$

$$
8 c_3 = 0
$$

$$
c_3 = 0

\]

Step 4: Find c_1 and c_2:

$$
c_1 = 2 c_3 = 0
$$
$$
c_2 = - c_3 = 0
$$

Conclusion: Linear Dependence or Independence?

The only solution to the system is:

$$
c_1 = c_2 = c_3 = 0
$$

This indicates that the only linear combination of P_1, P_2, P_3 that results in the zero polynomial is the trivial combination. Therefore, these three polynomials are linearly independent.

Implications and Significance

Why does this matter?

- Mathematical Clarity: Confirming the independence of polynomials helps in understanding their span and basis in polynomial vector spaces. Since these polynomials are independent, they form part of a basis for a subspace of the polynomial space, each contributing unique information.

- Applications: In approximation theory, signal processing, and polynomial interpolation, knowing the independence of basis polynomials ensures the ability to uniquely represent functions within the subspace.

- Further Analysis: If the polynomials had been dependent, it would imply redundancy, and some could be expressed as combinations of others, which might simplify computations or suggest more efficient representations.

Final Verdict: True or False?

Given the detailed algebraic analysis, the statement that "The polynomials $P_1 = 2x^2 + 1$, $P_2= -4x^2 + x$, and $P_3 = x - 2$ are linearly dependent" is:

False

Summary

Aspect	Explanation
Methodology	Set up a linear combination, equate coefficients, solve the system
Result	Only trivial solution ($c_1 = c_2 = c_3 = 0$) exists
Conclusion	The polynomials are linearly independent

Understanding the dependence or independence of polynomials is crucial in various mathematical contexts. In this case, the independence of P_1, P_2, P_3 confirms their distinctiveness and non-redundancy in forming bases in polynomial spaces.

In essence, the statement claiming these polynomials are linearly dependent is false.

[The Polynomials P1 2x 2 1 P2 4x 2 X And P3 X 2 Are Linearly Dependent Select One True False]

The Polynomials P1 2x 2 1 P2 4x 2 X And P3 X 2 Are Linearly Dependent Select One True False

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