# The Position Of A Particle As A Function Of Time Is Given By X=(6m/s)t+(2m/s2)t2.A) Find The Average

**The Position Of A Particle As A Function Of Time Is Given By X=(6m/s)t+(2m/s2)t2. A) Find The Average**

Understanding the motion of particles is fundamental in physics, especially when analyzing how objects move over time. The given function, $X = (6 \text{ m/s})\, t + (2 \text{ m/s}^2)\, t^2$, describes the position of a particle as a function of time. In this article, we will delve deep into interpreting this function, calculating the average velocity over a time interval, and exploring related concepts in kinematics. Whether you're a student preparing for exams or an enthusiast seeking to strengthen your understanding, this comprehensive guide aims to clarify the intricacies of particle motion.

## Understanding the Position Function

## The Equation: $X(t) = (6 \text{ m/s})\, t + (2 \text{ m/s}^2)\, t^2$

The given position function:

$$ X(t) = (6\, \text{m/s})\, t + (2\, \text{m/s}^2)\, t^2 $$

can be interpreted as a quadratic function of time. Here:

- The term $(6\, \text{m/s})\, t$ represents the initial velocity component contributing linearly to position.
- The term $(2\, \text{m/s}^2)\, t^2$ accounts for acceleration effects on the particle's displacement.

This form resembles the standard kinematic equation for uniformly accelerated motion:

$$ X(t) = X_0 + v_0 t + \frac{1}{2} a t^2 $$

Comparing, we observe:

- Initial position $X_0$ is zero (assuming starting at origin).
- Initial velocity $v_0 = 6\, \text{m/s}$.
- Acceleration $a = 4\, \text{m/s}^2$ because:

\[

\frac{1}{2} a = 2\, \text{m/s}^2 \Rightarrow a = 4\, \text{m/s}^2
\]

Thus, the particle starts from the origin with an initial velocity of 6 m/s and accelerates at $4\ \text{m/s}^2$.

# Calculating Average Velocity

The core question is: "Find the average velocity of the particle over a specific time interval."

In physics, the average velocity \( v_{avg} \) between times \( t_1 \) and \( t_2 \) is defined as:

\[ v_{avg} = \frac{X(t_2) - X(t_1)}{t_2 - t_1} \]

This formula calculates the net displacement divided by the total time elapsed.

## Selecting a Time Interval

The problem as stated does not specify the time interval. Common choices are:

- From \( t=0 \) seconds to some final time \( t=T \).
- Or over a specific interval, such as from \( t=1\, \text{s} \) to \( t=3\, \text{s} \).

For clarity, we'll demonstrate the calculation over the interval \( t = 0 \) to \( t = T \), where \( T \) is a positive real number.

## Calculating Position at Specific Times

Using the position function:

\[ X(t) = 6 t + 2 t^2 \]

- At \( t=0 \):

\[ X(0) = 6 \times 0 + 2 \times 0^2 = 0 \]

- At \( t=T \):

\[ X(T) = 6 T + 2 T^2 \]

## Average Velocity Over the Interval [0, T]

Applying the formula:

$$ v_{avg} = \frac{X(T) - X(0)}{T - 0} = \frac{6 T + 2 T^2 - 0}{T} $$

Simplify:

$$ v_{avg} = \frac{6 T + 2 T^2}{T} = 6 + 2 T $$

Result:

$$
\boxed{
v_{avg} = 6 + 2 T \quad \text{(meters per second)}
}
$$

This expression indicates that the average velocity depends linearly on the chosen time $T$.

# Understanding the Results

The average velocity formula:

$$ v_{avg} = 6 + 2 T $$

has several implications:

- At $T=0$: $v_{avg} = 6\, \text{m/s}$, which is the initial velocity.
- As $T$ increases: $v_{avg}$ increases linearly, reflecting the particle's acceleration.

This behavior aligns with the physics of uniformly accelerated motion, where the average velocity over an interval is the mean of the initial and final velocities.

## Connecting Average and Instantaneous Velocities

- Instantaneous velocity at any time $t$:

$$ v(t) = \frac{dX}{dt} = 6 + 4 t $$

- Average velocity over [0, T]:

$$ v_{avg} = \frac{X(T) - X(0)}{T} $$

which simplifies to:

$$ v_{avg} = \frac{\int_0^T v(t) dt}{T} $$

- Average velocity as the mean of initial and final velocities:

$$
v_{avg} = \frac{v(0) + v(T)}{2} = \frac{6 + (6 + 4 T)}{2} = \frac{12 + 4 T}{2} = 6 + 2 T
$$

confirming our earlier calculation.

# Practical Applications of These Calculations

Understanding how to compute average velocity from position functions is crucial in various real-world scenarios:

- **Vehicle Motion Analysis:** Determining average speed over a trip.
- **Particle Physics:** Analyzing particle trajectories in accelerators.
- **Engineering:** Designing systems with precise motion parameters.
- **Sports Science:** Measuring average speeds during athletic activities.

Accurate calculations enable better predictions, optimizations, and understanding of physical behaviors.

# Additional Concepts Related to Particle Motion

## Instantaneous Velocity and Acceleration

- Instantaneous velocity $v(t)$:

$$ v(t) = \frac{dX}{dt} = 6 + 4 t $$

- Acceleration $a(t)$:

$$ a(t) = \frac{dv}{dt} = 4\, \text{m/s}^2 $$

which remains constant, confirming uniform acceleration.

## Displacement and Total Distance

- Displacement over time interval [0, T]:

$$ \Delta X = X(T) - X(0) = 6 T + 2 T^2 $$

- Total distance traveled could differ if the particle changes direction, but in this case, since velocity remains positive for $t \ge 0$, displacement equals total distance.

## Summary and Key Takeaways

- The position function $X(t) = 6 t + 2 t^2$ describes a particle starting at the origin with an initial velocity of 6 m/s and acceleration of 4 $m/s^2$.
- The average velocity over the interval [0, T] is:

$$ v_{avg} = 6 + 2 T $$

- This derivation confirms that average velocity is the mean of initial and final velocities for uniformly accelerated motion.
- Instantaneous velocity increases linearly with time:

$$ v(t) = 6 + 4 t $$

- Understanding these relationships equips students and professionals to analyze real-world motion scenarios effectively.

## Conclusion

Analyzing particle motion through functions like $X = 6 t + 2 t^2$ offers valuable insights into dynamic systems. Calculating the average velocity over a time interval involves understanding the position function, instantaneous velocity, and the fundamental principles of kinematics. These calculations are not only academically significant but also practically applicable across various fields such as engineering, physics, and sports science. Mastery of these concepts enables accurate modeling of motion, prediction of future states, and optimization of systems involving moving particles.

Whether you're solving academic problems or applying these principles in real-world contexts, understanding the relationship between position, velocity, and acceleration is essential. Remember, the key steps involve deriving the position function, calculating the change in position over the interval, and then dividing by the elapsed time to find the average velocity.

# Frequently Asked Questions

## What is the given position function of the particle?

The position function is $X(t) = (6 \text{ m/s})t + (2 \text{ m/s}^2)t^2$.

## How do you interpret the terms in the position function?

The first term, $(6 \text{ m/s})t$, represents the initial velocity component, while the second term, $(2 \text{ m/s}^2)t^2$, accounts for the acceleration's effect over time.

## What is the formula to find the average velocity over a time interval?

Average velocity over an interval from $t_1$ to $t_2$ is given by $(X(t_2) - X(t_1)) / (t_2 - t_1)$.

## How can you calculate the average velocity from the given position function?

By evaluating X(t) at the initial and final times and applying the average velocity formula: $(X(t_2) - X(t_1)) / (t_2 - t_1)$.

## What is the average velocity between t = 0 and t = T?

Between t = 0 and t = T, the average velocity is [X(T) - X(0)] / T. Since X(0) = 0, it simplifies to X(T) / T.

## Calculate the average velocity at t = T for the given position function.

The average velocity over [0, T] is $(6 T + 2 T^2) / T = 6 + 2 T$, representing the mean rate of change of position over the interval.

# Additional Resources

### Understanding the Motion of a Particle: Analyzing the Position Function and Calculating the Average Velocity

In the realm of classical mechanics, the motion of particles and objects is fundamental to understanding the physical universe. From the simple movement of a rolling ball to the complex trajectories of celestial bodies, the principles governing motion are both universal and intricate. One of the most insightful approaches to studying such motion involves examining the position of a particle as a function of time. This allows us to derive quantities such as velocity, acceleration, and averages that describe the particle’s behavior over specific time intervals. In this article, we focus on a specific problem: analyzing a particle whose position is given by the function \( x(t) = (6\, \text{m/s})\, t +

$(2\, \text{m/s}^2)\, t^2$, and specifically, calculating its average velocity over a certain period. We will explore the mathematical foundations, physical interpretations, and implications of this problem in a detailed and structured manner.

---

## Mathematical Representation of Particle Motion

### The Position Function $x(t)$

The position of a particle as a function of time, $x(t)$, encapsulates how the particle moves through space over time. In the given problem, the position function is:

$$
x(t) = (6\, \text{m/s})\, t + (2\, \text{m/s}^2)\, t^2
$$

This quadratic function indicates that the particle's position depends both linearly and quadratically on time, suggesting the presence of both constant velocity components and acceleration.

- The term $(6\, \text{m/s})\, t$ signifies a uniform velocity component, indicating that, at least initially, the particle is moving forward with a speed of 6 meters per second.
- The term $(2\, \text{m/s}^2)\, t^2$ indicates an acceleration component, which causes the velocity to change over time.

Understanding the form of this function is fundamental because it allows us to derive other key quantities such as velocity and acceleration, which are essential for describing the particle's motion comprehensively.

---

## Deriving Velocity and Acceleration

### Instantaneous Velocity $v(t)$

The instantaneous velocity of the particle at any given time is the first derivative of the position function with respect to time:

$$
v(t) = \frac{dx(t)}{dt}
$$

Applying differentiation to our specific $x(t)$:

$$
v(t) = \frac{d}{dt} \left[ 6t + 2t^2 \right] = 6 + 4t
$$

This linear function of time indicates that the velocity increases uniformly, starting at 6 m/s when $t = 0$, and increasing by 4 m/s every second.

- At $t = 0$, $v(0) = 6\, \text{m/s}$
- At $t = 1\, \text{s}$, $v(1) = 6 + 4(1) = 10\, \text{m/s}$

This reveals an acceleration $a(t)$:

$$
a(t) = \frac{dv(t)}{dt} = 4\, \text{m/s}^2
$$

which is constant, confirming uniform acceleration.

## Implications of the Velocity and Acceleration

The velocity function indicates that the particle accelerates at a constant rate of 4 $m/s^2$. This insight is critical because it allows us to predict the particle's behavior at any point in time, and it simplifies calculations of average quantities over specified intervals.

---

# Calculating the Average Velocity

## Definition of Average Velocity

Average velocity over a time interval $[t_1, t_2]$ is defined as:

$$
v_{avg} = \frac{\text{Displacement}}{\text{Time Interval}} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}
$$

This measure provides a broad overview of the particle’s overall motion during the interval, contrasting with the instantaneous velocity which reflects the particle’s speed at a specific moment.

## Choosing the Time Interval

In the absence of a specified interval in the problem, typical choices include:

- From $t = 0$ to $t = T$, where $T$ is a specific time, such as 5 seconds.
- Or, for illustrative purposes, a standard interval like $t = 0$ to $t = 4\, \text{s}$.

Let’s proceed with calculating the average velocity over the interval from $t = 0$ to $t = 5\, \text{s}$.

---

# Step-by-Step Calculation of the Average Velocity

## Calculate $x(0)$ and $x(5)$

Using the position function:

$$
x(t) = 6t + 2t^2
$$

- At $t = 0$:

$$
x(0) = 6 \times 0 + 2 \times 0^2 = 0\, \text{m}
$$

- At $t = 5\, \text{s}$:

$$
x(5) = 6 \times 5 + 2 \times 5^2 = 30 + 2 \times 25 = 30 + 50 = 80\, \text{m}
$$

## Compute the average velocity over $[0, 5]$ seconds

$$
v_{avg} = \frac{x(5) - x(0)}{5 - 0} = \frac{80\, \text{m} - 0\, \text{m}}{5\, \text{s}} = \frac{80\, \text{m}}{5\, \text{s}} = 16\, \text{m/s}
$$

Interpretation: The particle’s average velocity over 5 seconds is 16 m/s, which aligns with the increasing velocity due to acceleration. It’s important to note that this average velocity is greater than the initial velocity (6 m/s) because the particle accelerates during the interval.

---

# Analyzing the Result: Physical Significance and Insights

## Understanding the Implications of the Average Velocity

The average velocity calculated (16 m/s over 5 seconds) reflects the net displacement divided by total time, considering the acceleration's effect. Since the particle accelerates at a constant rate, its velocity increases linearly over time, and the average velocity over an interval lies midway between the initial and final velocities.

- Initial velocity at $t=0$: 6 m/s
- Final velocity at $t=5\, \text{s}$: $v(5) = 6 + 4 \times 5 = 26\, \text{m/s}$

The average of these:

$$
\frac{6 + 26}{2} = 16\, \text{m/s}
$$

which matches our calculated average velocity, confirming the consistency of the results.

Physical Insight: For uniformly accelerated motion starting with initial velocity $u$, the average velocity over a time interval is simply the arithmetic mean of the initial and final velocities:

$$
v_{avg} = \frac{u + v_f}{2}
$$

This is a key principle in kinematics and holds true for constant acceleration scenarios.

## Broader Context and Applications

Understanding how to compute and interpret average velocity is crucial in various real-world contexts, such as:

- Planning travel times in transportation engineering
- Analyzing the motion of vehicles under acceleration
- Interpreting experimental data in physics labs
- Designing control systems where motion profiles are critical

The ability to relate the position function to average quantities offers foundational insights that are essential for more advanced topics like differential calculus applications, velocity profiles, and energy considerations.

---

# Extensions and Additional Considerations

## Calculating Average Acceleration

While the problem focused on average velocity, a natural extension involves calculating the average acceleration over the interval:

$$
a_{avg} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}
$$

Using the velocity function $v(t) = 6 + 4t$:

$$
a_{avg} = \frac{v(5) - v(0)}{5 - 0} = \frac{26 - 6}{5} = \frac{20}{5} = 4\, \text{m/s}^2
$$

which confirms the constant acceleration derived earlier.

## Implications in Real-World Scenarios

In real-world applications, such idealized models are instrumental in:

- Designing acceleration and deceleration profiles for vehicles
- Understanding the motion of projectiles
- Planning trajectories in robotics and automation
- Analyzing sports dynamics

The quadratic form of the position function exemplifies how acceleration influences motion, transitioning from initial velocities to higher speeds over time.

---

# Conclusion: The Significance of Kinematic

## Analysis

By analyzing the position function \( x

## The Position Of A Particle As A Function Of Time Is Given By X 6m S T 2m S2 T2 A Find The Average

The Position Of A Particle As A Function Of Time Is Given By X 6m S T 2m S2 T2 A Find The Average

## Related Articles


- The U.S. Bureau Of The Census Prediction For The Percentage Of The Population 65 Years And Older Can
- The Proof For The Product Property Of Logarithms Requires Simplifying The Expression Logb(bx Y) To X
- The Diagram Shows A Square ABCD With Side Length K Cm. MDE Is A Sector Of A Circle, Centre D. E Lies

Back to Home