The Pressure Of A Sample Of An Ideal Gas, Originally At Volume V1, Is Halved At Constant Temperature

Introduction

The Pressure Of A Sample Of An Ideal Gas, Originally At Volume V1, Is Halved At Constant Temperature is a classic problem in thermodynamics and kinetic theory of gases. It highlights fundamental principles governing the behavior of gases, notably Boyle's Law, which describes the inverse relationship between pressure and volume at constant temperature. Understanding this phenomenon requires a comprehensive analysis of the underlying physics, mathematical relationships, and practical implications. This article delves into the theoretical aspects, mathematical derivations, and real-world applications related to this situation, providing an in-depth exploration suitable for students, educators, and enthusiasts alike.

Fundamental Concepts of Ideal Gases

What Is an Ideal Gas?

An ideal gas is a hypothetical gas composed of many randomly moving point particles that interact only through elastic collisions. These assumptions simplify the complex interactions seen in real gases, allowing for the derivation of straightforward mathematical relationships. Key characteristics include:

- No intermolecular forces (no attraction or repulsion between particles)
- Particles occupy negligible volume compared to the container
- Collisions are perfectly elastic

Basic Gas Laws

Several fundamental laws describe the behavior of ideal gases, with Boyle's Law being critical for this discussion:

- **Boyle's Law:** For a fixed amount of gas at constant temperature, the pressure and volume are inversely proportional:

$P V = \text{constant}$

- **Charles's Law:** At constant pressure, volume and temperature are directly proportional.
- **Gay-Lussac's Law:** At constant volume, pressure and temperature are directly proportional.

Understanding the Problem: Halving Pressure at Constant Temperature

Initial Conditions

Suppose a sample of an ideal gas is contained within a vessel of initial volume V_1. The initial pressure exerted by this gas is P_1, and the temperature is maintained at a constant value T. The initial state can be summarized as:

- Initial volume: V_1
- Initial pressure: P_1
- Constant temperature: T

Final Conditions

We are told that the pressure of the gas is halved, so the final pressure becomes $P_2 = \frac{P_1}{2}$. The temperature remains unchanged, and we are asked to determine the new volume V_2. The scenario can be summarized as:

- Final pressure: $P_2 = \frac{P_1}{2}$
- Final volume: V_2 (unknown)
- Temperature: T (unchanged)

Applying Boyle's Law to the Problem

Mathematical Relationship

Boyle's Law states that at constant temperature, the product of pressure and volume remains constant:

$P V = \text{constant}$

For the initial and final states, this becomes:

- $P_1 V_1 = P_2 V_2$

Deriving the Final Volume

Given that $P_2 = \frac{P_1}{2}$, substituting into the Boyle's Law equation yields:

$P_1 V_1 = \frac{P_1}{2} V_2$

Dividing both sides by P_1, we get:

$V_1 = \frac{1}{2} V_2$

Rearranging for V_2:

$V_2 = 2 V_1$

Physical Interpretation of the Result

Expansion of the Gas

The analysis reveals that to reduce the pressure by half while keeping the temperature constant, the volume must double. Physically, this means the gas expands to twice its original volume. This aligns with intuition: if the number of particles and temperature stay constant, reducing the pressure (which is force per unit area) requires the particles to have more space to move around, decreasing collision frequency and thus lowering pressure.

Implications for Gas Behavior

- The process illustrates the inverse relationship between pressure and volume at constant temperature.

- It demonstrates the importance of volume adjustments in controlling pressure in practical applications such as pistons, syringes, and chemical reactors.
- The result also emphasizes the importance of maintaining temperature to ensure the validity of Boyle's Law during such transformations.

Real-World Applications and Practical Considerations

Industrial Processes

Many industrial systems rely on controlling gas pressures and volumes. For example:

- In internal combustion engines, piston movements change volume, affecting pressure and temperature.
- Gas storage tanks are designed considering Boyle's Law to withstand pressure changes during filling and emptying.
- Refrigeration cycles involve compression and expansion of gases, applying similar principles to manipulate pressure and volume.

Limitations of Ideal Gas Assumption

While Boyle's Law provides a useful approximation, real gases exhibit deviations due to intermolecular forces and finite particle sizes. At high pressures or low temperatures, these effects become significant, and corrections from the Van der Waals equation may be necessary. Nonetheless, for many practical purposes, the ideal gas model remains sufficiently accurate.

Additional Considerations

Effect of Changing Temperature

If the temperature were not held constant, the relationship among pressure, volume, and temperature would involve Gay-Lussac's or Charles's Law, leading to different outcomes. For example, increasing temperature at constant volume increases pressure, while at constant pressure, volume would change proportionally with temperature.

Work Done During Expansion

The process of expanding the gas from volume V_1 to V_2 involves work, which can be calculated as:

$$W = \int_{V_1}^{V_2} P dV$$

Since the process occurs at constant temperature (an isothermal process), the pressure varies inversely with volume, and the work done is:

$$W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

where n is the number of moles, R is the gas constant, and T is temperature. In this scenario, as $V_2 = 2 V_1$, the work becomes:

$$W = nRT \ln 2$$

Conclusion

In summary, when a sample of an ideal gas at a constant temperature experiences a halving of pressure, it must undergo a doubling of volume, as dictated by Boyle's Law. This fundamental relationship underscores the inverse connection between pressure and volume and has broad implications across scientific and engineering disciplines. Recognizing the assumptions underlying the ideal gas law and understanding the physical and mathematical principles involved are essential for applying these concepts accurately. Whether in laboratory experiments, industrial applications, or theoretical analyses, these principles serve as cornerstones of thermodynamics and gas dynamics, facilitating better control and prediction of gaseous systems.

Frequently Asked Questions

What happens to the pressure of an ideal gas when its volume is halved at constant temperature?

The pressure doubles because, according to Boyle's law, pressure is inversely proportional to volume at constant temperature.

Which gas law explains the relationship between pressure and volume

during this process?

Boyle's law explains this relationship, stating that for a fixed amount of gas at constant temperature, pressure and volume are inversely proportional.

If the initial pressure of the gas is P1, what is the final pressure after halving the volume?

The final pressure P2 is 2 × P1, since pressure doubles when volume is halved at constant temperature.

Does the temperature of the gas change when the pressure is doubled due to halving volume?

No, the temperature remains constant because the process occurs at constant temperature, meaning it is an isothermal process.

What assumptions are made about the gas in this scenario?

The gas is assumed to be ideal, meaning its particles do not interact and occupy negligible volume, and the process is isothermal.

How does the ideal gas law (PV = nRT) relate to this situation?

Since temperature (T) and amount of gas (n) are constant, the ideal gas law simplifies to $P \propto 1/V$, explaining why pressure doubles when volume halves.

What practical applications can be derived from understanding this pressure-volume relationship?

It helps in designing engines, syringes, and pressurized containers, where controlling pressure and volume is essential under constant temperature conditions.

If the initial pressure was 100 kPa, what would be the new pressure after halving the volume?

The new pressure would be 200 kPa, since pressure doubles when volume is halved at constant temperature.

How does this scenario demonstrate the concept of isothermal processes in

thermodynamics?

It illustrates that in an isothermal process, pressure and volume are inversely related, with temperature remaining constant throughout the process.

What would happen if the process were not conducted at constant temperature?

The pressure change would then depend on both volume change and temperature variation, according to the combined gas law, potentially leading to different pressure outcomes.

Additional Resources

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Introduction

The behavior of gases under varying conditions of pressure, volume, and temperature has been a foundational subject in thermodynamics and physical chemistry. Among the many intriguing scenarios explored in this domain is the effect of altering the pressure of an ideal gas sample when its volume is changed, all while maintaining constant temperature. This investigation is not only theoretically enriching but also practically relevant in fields ranging from chemical engineering to atmospheric science.

This article delves into the specific case where the pressure of an ideal gas, initially at volume V1, is halved at constant temperature. Through a comprehensive analysis rooted in the ideal gas law, we will elucidate the underlying principles, mathematical relationships, and physical implications of this process.

Fundamental Principles: The Ideal Gas Law

At the heart of understanding the behavior of gases under changing conditions lies the Ideal Gas Law, expressed as:

$$PV = nRT$$

where:
- P = pressure of the gas
- V = volume of the gas
- n = number of moles of gas

- R = universal gas constant
- T = absolute temperature

This law assumes that gas particles are point masses with no intermolecular forces, and it accurately describes many gases under standard conditions.

Initial Conditions and Assumptions

Suppose a sample of an ideal gas is initially confined within a volume V_1, at pressure P_1, and temperature T. The number of moles n remains constant throughout the process, and the temperature is strictly maintained (i.e., an isothermal process).

Given:
- Initial state: P_1, V_1, T

The goal is to analyze what happens when the pressure is reduced to half its original value, i.e.,

$$P_2 = \frac{1}{2} P_1$$

while keeping T constant.

The Effect of Halving Pressure at Constant Temperature

Mathematical Analysis

Applying the ideal gas law to initial and final states:

- Initial state:

$$P_1 V_1 = n R T$$

- Final state (after pressure halving):

$$P_2 V_2 = n R T$$

Since $n R T$ remains unchanged, equating the two:

$$P_1 V_1 = P_2 V_2$$

Substituting $P_2 = \frac{1}{2} P_1$:

$$ P_1 V_1 = \frac{1}{2} P_1 V_2 $$

Dividing both sides by P_1:

$$ V_1 = \frac{1}{2} V_2 $$

Rearranged:

$$ V_2 = 2 V_1 $$

This result reveals that to halve the pressure of the gas at constant temperature, the volume must double.

Physical Interpretation and Implications

Volumetric Expansion

The direct proportionality between pressure and volume in an isothermal process for an ideal gas underscores Boyle's Law:

$$ P V = \text{constant} $$

In this scenario, halving the pressure necessitates doubling the volume. Physically, the gas particles, which exert force on the container walls, now have more space to move around, reducing the frequency of collisions and thus lowering the pressure.

Thermodynamic Perspective

From a thermodynamic standpoint, the process is isothermal and quasi-static (assuming slow enough changes to maintain thermal equilibrium). Because the temperature remains constant, no net heat is exchanged with the surroundings if the process is ideal and reversible.

The work done by the gas during expansion can be calculated as:

$$ W = \int_{V_1}^{V_2} P\, dV $$

Using Boyle’s Law:

$$ P = \frac{n R T}{V} $$

The work becomes:

$$ W = n R T \int_{V_1}^{V_2} \frac{1}{V}\, dV = n R T \ln \left(\frac{V_2}{V_1} \right) $$

Substituting $V_2 = 2 V_1$:

$$ W = n R T \ln (2) $$

This positive work indicates energy transfer from the gas to the surroundings during expansion.

Practical Applications and Experimental Considerations

Understanding how pressure and volume interrelate under constant temperature is essential in numerous practical contexts:

- Industrial Gas Processes: Adjusting pressures and volumes in chemical reactors to optimize reactions.
- Atmospheric Science: Understanding how gases expand or compress with altitude changes.
- Engineering Design: Creating systems where gases undergo controlled expansion or compression.

In experimental settings, maintaining isothermal conditions requires efficient thermal coupling, such as placing the gas container in a temperature-controlled environment or using a heat bath to absorb or supply heat as needed.

Limitations and Deviations in Real Gases

While the ideal gas law provides an excellent approximation under many circumstances, real gases exhibit deviations due to intermolecular forces and finite particle sizes. At high pressures or low temperatures, these effects become significant, and corrections like the Van der Waals equation are necessary to accurately describe the behavior.

Summary of Key Findings

- Halving the pressure of an ideal gas at constant temperature results in doubling its volume.
- This relationship is a direct consequence of Boyle's Law ($PV = \text{constant}$) under isothermal conditions.
- The process involves work performed by the gas during expansion, quantified as $W = n R T \ln 2$.

Broader Context and Future Directions

The scenario examined here exemplifies fundamental thermodynamic principles governing ideal gases. Extending this understanding to more complex systems involves considering non-ideal behavior, phase changes, and energy transfer mechanisms. Advances in experimental techniques and computational modeling continue to refine our comprehension of gas behaviors under diverse conditions.

Moreover, the principles elucidated here underpin many technological innovations, including refrigeration cycles, internal combustion engines, and high-pressure gas storage systems. As scientific inquiry advances, the nuanced understanding of gas behavior remains an enduring cornerstone of physical sciences.

Conclusion

The investigation into the pressure of an ideal gas initially at volume V_1 being halved at constant temperature exemplifies the elegant simplicity of Boyle's Law and its profound implications. By methodically analyzing the relationships between pressure, volume, and temperature, we find that the volume must double — a fundamental principle with broad applicability across scientific and engineering disciplines.

This exploration reaffirms that within the idealized framework, thermodynamic systems adhere to predictable and mathematically elegant laws, enhancing our capacity to manipulate and innovate with gases in both theoretical and practical domains.

References

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Note: The analysis assumes ideal behavior and reversible processes. Real-world applications may require consideration of non-idealities and irreversibilities.

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