The Probability Distribution Of The Discrete Random Variable X Is Given Below F(x)=(3x)(72) X(75)

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Understanding probability distributions is fundamental in the field of statistics and probability theory. They provide a mathematical framework for modeling and analyzing the likelihood of various outcomes for a random variable. In this article, we will explore the probability distribution of a specific discrete random variable \(X\) with the given distribution function \(F(x) = (3x)(72) \) for \(X(75)\). We will delve into its properties, how to interpret the distribution, and the implications for statistical analysis.

Overview of Discrete Random Variables and Probability Distributions

Before analyzing the specific distribution, it is essential to understand the basic concepts related to discrete random variables and their probability distributions.

What Is a Discrete Random Variable?

- A discrete random variable is a variable that can take on a finite or countably infinite set of distinct values.
- Examples include the number of heads in coin tosses, the number of cars passing through an intersection, or the number of defectives in a batch.
- Discrete variables are characterized by their probability mass function (PMF), which assigns probabilities to each possible value.

Probability Distribution of a Discrete Random Variable

- The probability distribution of a discrete random variable describes the likelihood of each possible outcome.
- It is represented mathematically by the PMF: $P(X = x)$, where x is a possible value of X.
- The sum of all probabilities in the distribution must equal 1: $\sum P(X = x) = 1$.

Understanding the Given Distribution Function $F(x) = (3x)(72)$ for $X(75)$

The provided distribution function appears to have some notation that may seem ambiguous. Typically, in probability theory:

- $F(x)$ denotes the cumulative distribution function (CDF), which gives the probability that $X \leq x$.
- The notation $X(75)$ could imply the value of X at point 75 or might be indicating the support of the distribution.

Given the form $F(x) = (3x)(72)$, it suggests a function involving x, but the notation is somewhat unconventional. To interpret this correctly, we will analyze the possible meaning and structure.

Interpreting the Distribution Function

- The expression $F(x) = (3x)(72)$ might imply a linear function of x, scaled by constants.
- Alternatively, the notation could be a typo or shorthand for a more complex function.

Assuming the key points:
- The distribution is discrete.
- $F(x)$ could be the cumulative distribution function (CDF).
- The mention of $X(75)$ might mean that the distribution is supported on values up to 75.

Potential interpretation:

- Suppose the distribution function is $F(x) = \frac{3x}{72}$, for x within the support.
- This would be a linear CDF, scaled between 0 and 1, as typical for probability distributions.

Clarifying assumptions:

- $F(x) = \frac{3x}{72}$ for x in the support.
- The support of X is from 0 to 75, as indicated by the mention of 75.
- The distribution is discrete, so X takes specific values within this range.

Note:
Without explicit clarification, this interpretation is based on common forms of distribution functions. If the original formula had typographical issues, the following analysis will serve as a comprehensive guide on how to interpret such functions.

Analyzing the Distribution: Properties and Calculations

Now, let's assume the distribution function is:

$$ F(x) = \frac{3x}{72} \quad \text{for} \quad x = 0, 1, 2, \dots, 75 $$

This form suggests a linear cumulative distribution function (CDF) over the support $0 \leq x \leq 75$.

Verifying the Distribution Properties

- Non-decreasing: $F(x)$ increases as x increases, satisfying the monotonicity condition.
- Limits: At $x=0$, $F(0)=0$. At $x=75$, $F(75)=\frac{3 \times 75}{72} = \frac{225}{72} \approx 3.125$.

Since $F(75)$ exceeds 1, which is invalid for a CDF, this suggests that the distribution function needs normalization or correction.

Adjusted assumption:

- To keep $F(x)$ within [0,1], perhaps the function is:

$$
F(x) = \frac{3x}{75}
$$

which normalizes the maximum at $x=75$:

$$
F(75) = \frac{3 \times 75}{75} = 3
$$

Again, exceeds 1. Therefore, an alternative is:

$$
F(x) = \frac{x}{75}
$$

which is a standard linear CDF on [0,75], increasing from 0 to 1.

Final assumption for clarity:

- The discrete distribution of X is supported on integers from 0 to 75.
- The cumulative distribution function is:

$$

$$
F(x) = \frac{x}{75}
$$

for integer x in $[0,75]$.

Deriving the Probability Mass Function (PMF)

Given the CDF $F(x)$, the PMF $p(x) = P(X=x)$ can be obtained as:

$$
p(x) = F(x) - F(x-1)
$$

for $x=1, 2, \dots, 75$, with $p(0) = F(0)$.

Calculations:

- $p(0) = F(0) = 0/75 = 0$ (or possibly $p(0) = 1/75$ if starting at 1)
- For $x=1$:

$$
p(1) = F(1) - F(0) = \frac{1}{75} - 0 = \frac{1}{75}
$$

- For $x=2$:

$$
p(2) = \frac{2}{75} - \frac{1}{75} = \frac{1}{75}
$$

- Similarly, for any x:

$$
p(x) = \frac{x}{75} - \frac{x-1}{75} = \frac{1}{75}
$$

This indicates a uniform distribution over the integers from 1 to 75, with each value having probability $1/75$.

Characteristics of the Distribution

Based on the above, the distribution of X is:

- Uniform discrete distribution over $\{1, 2, \dots, 75\}$.
- Probability mass function:

$$
p(x) = \frac{1}{75}, \quad x=1, 2, \dots, 75
$$

- Expected value (mean):

$$
E[X] = \frac{a + b}{2} = \frac{1 + 75}{2} = 38
$$

- Variance:

$$
Var(X) = \frac{(b - a + 1)^2 - 1}{12} = \frac{(75 - 1 + 1)^2 - 1}{12} = \frac{75^2 - 1}{12} = \frac{5625 - 1}{12} = \frac{5624}{12} \approx 468.67
$$

Applications and Significance of the Distribution

Understanding such a distribution has numerous applications in real-world scenarios involving discrete outcomes.

Practical Use Cases

- Quality Control: Modeling the number of defective items in a batch, assuming uniform probabilities.
- Survey Sampling: Random selection of respondents with equal likelihood.
- Gaming and Simulations: Designing random events with uniform probabilities.

Statistical Analysis and Inference

- The uniform distribution simplifies calculations of probabilities and expectations.
- It provides a baseline for comparing other distributions.
- Facilitates hypothesis testing involving discrete uniform variables.

Conclusion

In summary, the probability distribution of the discrete random variable X, as interpreted from the given function, resembles a uniform distribution over the integers from 1 to 75. Each outcome has an equal probability of \(1/75\), and key statistics such as the mean and variance are straightforward to compute. Recognizing the structure of the distribution is crucial for applying it correctly in statistical modeling, hypothesis testing, and simulations.

Note: If the original distribution function differs or contains additional parameters, the analysis should be adjusted accordingly. Always ensure the distribution functions satisfy the fundamental properties of probability, such as non-negativity, normalization (total probability equals 1), and monotonicity for the CDF.

Further Reading and Resources
- "Introduction to Probability" by Joseph K. Blitz

Frequently Asked Questions

What does the given probability distribution function F(x) = (3x)(72) for the discrete random variable X represent?

It represents the probability distribution of X, specifying the probability associated with each discrete value of X, where F(x) is calculated as (3x) multiplied by 72.

How do you interpret the formula F(x) = (3x)(72) in the context of a probability distribution?

It suggests that the probability for each value x of the variable X is proportional to 3x, scaled by 72, indicating a linear relation between x and its probability.

What are the possible values of X for which the distribution F(x) is valid?

To determine valid values, we need to ensure that the sum of all probabilities equals 1 and that each probability is between 0 and 1, which typically restricts X to certain discrete values within a specific range.

How do you verify if F(x) is a valid probability distribution?

By summing F(x) over all possible x values and checking if the total equals 1, and ensuring that each individual F(x) is between 0 and 1.

How can we find the expected value E[X] given this distribution?

Calculate E[X] by summing x multiplied by its probability F(x) over all possible values: $E[X] = \sum x F(x)$.

What is the significance of normalizing F(x) to ensure it sums to 1?

Normalization ensures that the total probability across all possible values of X equals 1, which is a fundamental property of any probability distribution.

How do you compute the variance of X from this distribution?

First compute E[X], then compute $E[X^2]$, and use the formula $Var(X) = E[X^2] - (E[X])^2$.

If F(x) = (3x)(72), what is the probability when X takes its maximum value?

You substitute the maximum value of X into the formula to find F(x), then verify if it satisfies the probability criteria (between 0 and 1).

What steps are involved in deriving the probability mass function from F(x) = (3x)(72)?

Identify the discrete values of X, compute F(x) for each, and then normalize the values so that their sum equals 1, ensuring a valid pmf.

Can F(x) = (3x)(72) be a valid probability distribution directly, or does it need adjustment?

It likely needs normalization because the sum over all x of F(x) may not equal 1 initially; dividing each F(x) by the total sum ensures it becomes a valid pmf.

Additional Resources

Probability Distribution

Understanding the probability distribution of a discrete random variable is fundamental in the realm of probability theory and statistics. It provides a comprehensive picture of how possible outcomes are distributed and the likelihood associated with each. Today, we delve deeply into a specific case: the probability distribution of the discrete random variable X, defined by the function:

$$
F(x) = \frac{3x}{72} \quad \text{for} \quad x \leq 75
$$

This exploration will examine the characteristics, interpretation, and implications of this distribution, presenting it as a detailed product review or expert analysis that unpacks its components thoroughly.

Understanding the Distribution Function $F(x)$: A Primer

At the outset, the function provided appears to be a probability function, but the notation and form suggest that it could be either a cumulative distribution function (CDF) or a probability mass function (PMF). The notation $F(x)$ typically denotes a cumulative distribution function, which gives the probability that the random variable X takes a value less than or equal to x.

However, the expression:

$$
F(x) = \frac{3x}{72}
$$

is a linear function of x, which increases with x, suggesting it could be a CDF if it satisfies certain properties.

Important Clarification:
- For a valid CDF, $F(x)$ must be non-decreasing, right-continuous, with limits $F(-\infty) = 0$ and $F(\infty) = 1$.
- For a discrete random variable, the probability mass function $p(x) = P(X = x)$ sums to 1 over its support.

Given the form, it seems more consistent to interpret this as a CDF, especially because the function depends on x directly and is scaled over the range.

Analyzing the Distribution: Key Properties

To fully understand this distribution, we need to examine several core properties:

1. Domain and Range

- Domain: The function is defined for $x \leq 75$. Usually, for a CDF, the domain extends over the support of the distribution, which in this case appears to be up to 75.
- Range: As x increases from the minimum to maximum, $F(x)$ increases from 0 to 1, assuming the function is scaled appropriately.

2. Behavior of $F(x)$

- For $x \leq 0$:

$$
F(x) = \frac{3x}{72} \leq 0
$$

Since probabilities cannot be negative, $F(x)$ should be 0 for all x less than the minimum support value.

- For $0 \leq x \leq 75$:

$$
F(x) = \frac{3x}{72}
$$

This is a linear, increasing function from 0 at $x=0$ to $\frac{3 \times 75}{72} = \frac{225}{72} \approx 3.125$. But probabilities cannot exceed 1, so this suggests that the function must be scaled or normalized over the support.

- For $x \geq 75$:

$$
F(x) = 1

\]

Assuming the cumulative distribution reaches 1 at the maximum support.

Correcting the Distribution: Normalization and Validity

The above indicates that the initial function as provided is not a proper CDF unless normalized properly. Since at $x=75$:

$$
F(75) = \frac{3 \times 75}{72} = \frac{225}{72} \approx 3.125
$$

which exceeds 1, it violates the fundamental property of a CDF. Therefore, the distribution needs to be scaled to ensure $F(75) = 1$.

Normalization Process:

Define the normalized CDF as:

$$
F(x) = \frac{\frac{3x}{72}}{\text{Max value at } x=75} = \frac{\frac{3x}{72}}{\frac{3 \times 75}{72}} = \frac{3x/72}{225/72} = \frac{3x}{72} \times \frac{72}{225} = \frac{3x}{225}
$$

Simplify:

$$
F(x) = \frac{x}{75}
$$

Interpreted as:

$$
F(x) = \frac{x}{75}
$$

for $0 \leq x \leq 75$, with:

$$
F(x) = 0 \quad \text{for} \quad x < 0
$$

$$
F(x) = 1 \quad \text{for} \quad x \geq 75
$$

This is a classic linear CDF corresponding to a uniform distribution over $[0,75]$.

Implication: The Distribution as a Uniform Model

Given the normalized form:

$$
F(x) = \frac{x}{75}
$$

we recognize this as the CDF of a discrete uniform distribution over the integers from 0 up to 75, assuming X takes integer values within that range uniformly.

Characteristics:

- Support: The set $\{0, 1, 2, \ldots, 75\}$.
- Probability Mass Function (PMF):

Since the distribution is uniform, the probability for each value x in the support is:

$$
p(x) = P(X = x) = \frac{1}{76}
$$

because there are 76 possible integer outcomes from 0 to 75 inclusive.

Key Statistical Measures

Understanding the distribution allows us to compute crucial statistics:

1. Expected Value (Mean)

$$
E[X] = \frac{a + b}{2} = \frac{0 + 75}{2} = 37.5
$$

since it's a uniform distribution over $[0,75]$.

2. Variance

\[

Var(X) = \frac{(b - a + 1)^2 - 1}{12} = \frac{(75 - 0 + 1)^2 - 1}{12} = \frac{76^2 - 1}{12} = \frac{5776 -

1}{12} = \frac{5775}{12} \approx 481.25

\]

3. Standard Deviation

\[

\sigma = \sqrt{481.25} \approx 21.92

\]

Practical Applications and Interpretation

The implications of this distribution are significant in various contexts:

- Simulation and Modeling:

The uniform distribution is often used in simulations where each outcome within the range is equally likely. For example, modeling the time to complete a task that can take anywhere uniformly between 0 and 75 minutes.

- Quality Control:

If a product's defect rate is uniformly distributed over a certain range, understanding its mean and variability helps in planning quality assurance processes.

- Decision-Making:

When outcomes are equally likely within a range, decision-makers can use the mean and variance to assess risks and expected returns.

Conclusion: The Essence of the Distribution

In conclusion, the initial form $F(x) = \frac{3x}{72}$ when correctly normalized reveals a classic uniform distribution over the integers from 0 to 75. Its properties—mean of 37.5, variance of approximately 481.25, and standard deviation around 21.92—highlight the spread and central tendency of the variable X.

This distribution embodies simplicity and symmetry, making it a foundational model in probability theory. Whether used in simulations, statistical modeling, or theoretical analysis, understanding the nuances of its probability structure ensures accurate interpretation and application in real-world scenarios.

Embracing the clarity of this distribution equips analysts and statisticians with a robust tool for modeling scenarios where outcomes are equally likely within a specified range. This detailed exploration underscores the importance of proper normalization and interpretation in probability distributions, elevating our comprehension from raw formulas to practical insights.

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