The Random Variable X Is Known To Be Uniformly Distributed Between 2 And 12. What Is The Probability

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Understanding probability distributions is fundamental in statistics and probability theory. When a random variable is uniformly distributed over a certain interval, it implies that all outcomes within that interval are equally likely. Specifically, if the variable X is known to be uniformly distributed between 2 and 12, this indicates that the probability density function (PDF) of X is constant over this interval. This article explores the concept of the uniform distribution, how to compute probabilities associated with such a variable, and addresses specific questions like "What is the probability that X falls within a particular range?" in detail.

Understanding the Uniform Distribution

Definition of Uniform Distribution

A uniform distribution is a type of probability distribution where every outcome in a specified interval has an equal chance of occurring. It is characterized by two parameters: the minimum value (a) and the maximum value (b). When a random variable X follows a uniform distribution between a and b, denoted as $X \sim U(a, b)$, its probability density function is constant over the interval $[a, b]$.

Mathematically, the PDF of a uniform distribution is expressed as:

$$
f(x) = \begin{cases}
\frac{1}{b - a} & \text{for } a \leq x \leq b \\
0 & \text{otherwise}
\end{cases}
$$

This indicates that the probability density is uniform across the interval, and the total probability over the entire interval sums to 1.

Parameters of the Distribution: a and b

In the context of the problem, the parameters are:

- $a = 2$
- $b = 12$

This means that the random variable X can take any value between 2 and 12 with equal likelihood, and the probability density function is:

$$
f(x) = \frac{1}{12 - 2} = \frac{1}{10}
$$

for all x in $[2, 12]$.

Calculating Probabilities for a Uniform Distribution

Basic Probability Calculation

The probability that a uniformly distributed random variable falls within a specific subinterval $[c, d]$ within its support $[a, b]$ is proportional to the length of that subinterval relative to the total interval.

$$
P(c \leq X \leq d) = \int_{c}^{d} f(x) dx = (d - c) \times \frac{1}{b - a}
$$

Given the uniform density, this simplifies to:

$$
P(c \leq X \leq d) = \frac{d - c}{b - a}
$$

provided $a \leq c \leq d \leq b$.

Key points:

- The probability depends solely on the length of the interval $[c, d]$.
- The total probability over the entire support is 1.

Examples of Probability Calculations

Suppose we want to find the probability that X falls between 4 and 10:

$$
P(4 \leq X \leq 10) = \frac{10 - 4}{12 - 2} = \frac{6}{10} = 0.6
$$

Similarly, the probability that X is less than 5:

$$
P(X \leq 5) = \frac{5 - 2}{12 - 2} = \frac{3}{10} = 0.3
$$

These calculations are straightforward due to the constant density of the uniform distribution.

Specific Problem: Calculating the Probability for X Between Certain Values

Question Restatement

Given that $X \sim U(2, 12)$, what is the probability that X is at least a certain value, or within a particular range? For example, what is $P(X \geq k)$ for some k between 2 and 12? Or more generally, what is $P(c \leq X \leq d)$?

Calculating $P(X \geq k)$

To compute the probability that X is greater than or equal to a certain value k within the support:

$$
P(X \geq k) = \int_{k}^{12} f(x) dx = \frac{12 - k}{12 - 2} = \frac{12 - k}{10}

\]

where:

- \(k\) must satisfy \(2 \leq k \leq 12\) to be relevant.
- If \(k < 2\), then \(P(X \geq k) = 1\) because X is always ≥ 2.
- If \(k > 12\), then \(P(X \geq k) = 0\) because X cannot exceed 12.

Example:

Calculate the probability that \(X \geq 8\):

\[
P(X \geq 8) = \frac{12 - 8}{10} = \frac{4}{10} = 0.4
\]

Calculating \(P(c \leq X \leq d)\)

Similarly, for a range \([c, d]\) within \([2, 12]\):

\[
P(c \leq X \leq d) = \frac{d - c}{10}
\]

Example:

Calculate the probability that \(X\) is between 3 and 9:

\[
P(3 \leq X \leq 9) = \frac{9 - 3}{10} = \frac{6}{10} = 0.6
\]

Applications and Practical Implications

Real-World Situations Using Uniform Distribution

Uniform distributions are commonly used in various practical scenarios, including:

- Random sampling: When each outcome between two bounds is equally likely, such as selecting a random number or point.
- Simulations: For generating random variables when no prior knowledge biases the outcomes.
- Modeling uncertainties: Where the exact probability distribution is unknown but outcomes are believed to be equally likely.

Significance of Knowing the Distribution

Understanding that X is uniformly distributed between 2 and 12 enables:

- Precise calculation of probabilities for any subinterval.
- Simplified modeling in simulations or decision-making processes.
- Clear interpretation of likelihoods associated with specific outcomes.

Summary and Key Takeaways

- The uniform distribution is characterized by equal likelihood across its support interval.
- For $X \sim U(a, b)$, the probability that X falls within $[c, d]$ is $\frac{d - c}{b - a}$.
- To find probabilities involving inequalities (like $X \geq k$), adjust the bounds accordingly.
- The total probability over the entire support is always 1.
- Practical applications span sampling, simulations, and modeling uncertainties in various fields.

Conclusion: Calculating the Probability for the Given Distribution

Given that X is uniformly distributed between 2 and 12, the probability that X takes on a value within any subinterval $[c, d]$ within this range is straightforward to compute:

$$
P(c \leq X \leq d) = \frac{d - c}{10}
$$

\]

where the numerator is the length of the subinterval, and 10 is the length of the total interval \([2, 12]\).

For example, if asked, "What is the probability that \(X\) is at least 4?" the answer is:

\[
P(X \geq 4) = \frac{12 - 4}{10} = 0.8
\]

In summary, understanding the uniform distribution's properties allows for quick and effective probability calculations, aiding in statistical analysis, decision-making, and modeling in diverse scientific and engineering disciplines.

Frequently Asked Questions

What is the probability density function (PDF) of the uniform random variable X between 2 and 12?

The PDF of X is constant between 2 and 12 and zero elsewhere, given by $f(x) = 1 / (12 - 2) = 1/10$ for $2 \leq x \leq 12$.

How do you calculate the probability that X falls between two values a and b within its range?

The probability is the area under the uniform distribution between a and b, calculated as $(b - a) / (12 - 2)$, provided $2 \leq a < b \leq 12$.

What is the probability that X is greater than 8?

$P(X > 8) = (12 - 8) / (12 - 2) = 4 / 10 = 0.4$.

What is the probability that X is less than 5?

$P(X < 5) = (5 - 2) / (12 - 2) = 3 / 10 = 0.3$.

Calculate the probability that X is between 4 and 10.

$P(4 \leq X \leq 10) = (10 - 4) / (12 - 2) = 6 / 10 = 0.6$.

What is the expected value (mean) of the uniformly distributed variable X between 2 and 12?

The mean is (2 + 12) / 2 = 7.

What is the variance of X for a uniform distribution between 2 and 12?

Variance = [(b - a)^2] / 12 = (10)^2 / 12 = 100 / 12 ≈ 8.33.

How do you find the probability that X is exactly equal to a specific value, say 5?

For a continuous uniform distribution, P(X = 5) = 0, since the probability at a single point is zero.

Additional Resources

The Random Variable X Is Known To Be Uniformly Distributed Between 2 And 12. What Is The Probability?

In the world of probability and statistics, understanding the behavior of random variables is fundamental. When a variable's distribution is known, it unlocks a wealth of insights into predicting outcomes and assessing risks. Today, we explore a specific case: a random variable \(X \) that is uniformly distributed between 2 and 12. The central question is: What is the probability of \(X \) falling within a specific interval? This seemingly simple query opens the door to a deeper understanding of uniform distributions and their properties.

Understanding the Uniform Distribution

What Is a Uniform Distribution?

A uniform distribution is one of the most straightforward probability distributions. It describes a situation where every outcome within a specified interval is equally likely. Imagine rolling a fair die: each face from 1 to 6 has an equal chance of appearing. Similarly, if a variable \(X \) is uniformly distributed over an interval \([a, b]\), then each value between \(a \) and \(b \) has an equal probability density.

Formally, the uniform distribution over \([a, b]\) (denoted as \(U(a, b) \)) has the following probability density function (pdf):

\[

f(x) = \begin{cases}
\frac{1}{b - a} & \text{for } a \leq x \leq b \\
0 & \text{otherwise}
\end{cases}
\]

This implies that the probability of \(X \) falling within any sub-interval \([c, d]\) within \([a, b]\) is proportional to the length of that sub-interval:

\[
P(c \leq X \leq d) = \frac{d - c}{b - a}
\]

provided \(a \leq c \leq d \leq b \).

Key Properties

- Constant probability density within the interval: \(f(x) = \frac{1}{b - a} \).
- Equal likelihood of all outcomes in the interval.
- Symmetry: The distribution is symmetric about the midpoint \(\frac{a + b}{2} \).

The Scenario: X Uniform from 2 to 12

Given that \(X \sim U(2, 12) \), the key parameters are:

- Lower bound \(a = 2 \)
- Upper bound \(b = 12 \)

Thus, the total range (or support) of \(X \) is 10 units (from 2 to 12).

The probability density function (pdf) for \(X \) is:

\[
f(x) = \frac{1}{12 - 2} = \frac{1}{10}
\]

for \(2 \leq x \leq 12 \).

This means that for any \(x \) in \([2, 12]\), the probability density is constant at 0.1.

Calculating Probabilities for Specific Intervals

Understanding the probability that X takes a value within a certain interval involves integrating the pdf over that interval. Since the pdf is constant, the calculation simplifies to:

$$
P(c \leq X \leq d) = \frac{d - c}{b - a}
$$

where c and d are within the bounds of the distribution.

Example 1: Probability that X is between 4 and 8

- $c = 4$
- $d = 8$

Applying the formula:

$$
P(4 \leq X \leq 8) = \frac{8 - 4}{12 - 2} = \frac{4}{10} = 0.4
$$

This indicates a 40% chance that X falls between 4 and 8.

Example 2: Probability that X is less than 5

- $c = 2$
- $d = 5$

Calculating:

$$
P(2 \leq X \leq 5) = \frac{5 - 2}{10} = \frac{3}{10} = 0.3
$$

So, there's a 30% probability that X is less than 5.

Example 3: Probability that X exceeds 11

- $c = 11$
- $d = 12$

Calculating:

$$
P(11 \leq X \leq 12) = \frac{12 - 11}{10} = \frac{1}{10} = 0.1
$$

A 10% chance that X exceeds 11.

General Approach to Finding Probabilities

The key steps to evaluate any probability involving $X \sim U(2, 12)$ are:

1. Identify the interval $[c, d]$ for which the probability is sought.
2. Ensure the interval is within the bounds: If the interval exceeds $[2,12]$, truncate it accordingly.
3. Apply the formula:

$$
P(c \leq X \leq d) = \frac{\max(0, \min(d, 12) - \max(c, 2))}{10}
$$

This accounts for cases when the interval overlaps only partially, or not at all, with the distribution support.

Applications and Implications

Understanding the uniform distribution and its probability calculations is crucial in various fields:

- Quality control: Estimating the likelihood of measurements falling within certain tolerances.
- Simulation and modeling: Generating random samples uniformly across a specified range.
- Risk assessment: Calculating probabilities of outcomes within specific bounds.

Moreover, because the uniform distribution is so intuitive—every outcome is equally likely—it often serves as a baseline or null model against which more complex distributions are compared.

Visualizing the Distribution

A helpful way to grasp the uniform distribution is through its graphical representation: a rectangle with height $\frac{1}{b - a}$ spanning from a to b. For our case:

- The rectangle spans from 2 to 12 on the x-axis.

- The height is constant at 0.1.

Any sub-interval within this rectangle represents an event with a probability proportional to its length. Visualizing this can clarify why the probability of the variable falling within a certain segment is simply the proportion of the total range.

Practical Considerations

While the theoretical calculations are straightforward, real-world data may introduce complexities:

- Measurement errors might distort the assumed uniformity.
- Boundaries: If the actual data range is not perfectly known or if the distribution is only approximately uniform, the probability estimates might need adjustment.
- Discrete vs. continuous: The uniform distribution discussed here is continuous. For discrete uniform distributions, the probability calculations differ slightly.

Summary and Key Takeaways

- The distribution of X is uniform over $[2, 12]$, meaning every value within this range is equally likely.
- The probability that X falls within any sub-interval $[c, d]$ can be calculated as:

$$
P(c \leq X \leq d) = \frac{d - c}{10}
$$

where the interval is within the bounds of $[2, 12]$.

- This simplicity makes the uniform distribution a fundamental concept in probability theory, serving as both a theoretical tool and a practical model for randomness.

In conclusion, knowing that X is uniformly distributed between 2 and 12 allows us to rapidly compute the likelihood of various events. Whether estimating the probability of a measurement falling within a certain range or simulating random outcomes, understanding the properties and calculations of the uniform distribution is essential for statisticians, engineers, and data scientists alike.

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