The Sector Of A Circle Has An Area Of 7/5 Square Inches And Central Angle Withmeasure 56.What Is The

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Understanding the properties of a circle and its sectors is fundamental in geometry, especially when dealing with real-world problems involving circular shapes. In this article, we will explore the problem: "The sector of a circle has an area of 7/5 square inches and a central angle of 56 degrees. What is the radius of the circle?" We will walk through the concepts involved, derive relevant formulas, and provide a detailed step-by-step solution to find the radius.

Understanding the Basics of Circle Sectors

Before diving into the problem, it is essential to understand what a sector of a circle is and how its area and angles relate.

What Is a Sector of a Circle?

- A sector of a circle is a portion of the circle enclosed between two radii and the arc connecting their endpoints.
- Think of it as a "slice" of the pie or pizza.
- The size of a sector is primarily determined by its central angle and the circle's radius.

Key Properties of a Sector

- Central angle (θ): The angle subtended at the center of the circle, measured in degrees.
- Radius (r): The distance from the center to any point on the circle.
- Area of the circle: Given by \(\pi r^2 \).
- Area of a sector: Depends on the central angle and the radius.

Formulas Relevant to Sector Calculations

Understanding the formulas related to a circle's sector is crucial for solving the problem.

Area of a Sector

$$
A_{sector} = \frac{\theta}{360^\circ} \times \pi r^2
$$

- Where:
- A_{sector} is the area of the sector.
- θ is the central angle in degrees.
- r is the radius of the circle.

Understanding the Relationship

- The sector area is proportional to the central angle.
- When $\theta = 360^\circ$, the sector is the entire circle.
- When θ is smaller, the sector is a smaller part of the circle.

Applying the Formula to the Given Data

Given:
- Sector area $A_{sector} = \frac{7}{5}$ square inches.
- Central angle $\theta = 56^\circ$.

Our goal:
- Find the radius r.

Step 1: Write the Sector Area Formula with Known Values

$$
\frac{7}{5} = \frac{56}{360} \times \pi r^2
$$

Step 2: Simplify the Fraction $\frac{56}{360}$

- Divide numerator and denominator by 8:

$$
\frac{56}{360} = \frac{7}{45}
$$

\]

Thus, the formula becomes:
\[
\frac{7}{5} = \frac{7}{45} \times \pi r^2
\]

Step 3: Isolate \(r^2\)

- Divide both sides by \(\frac{7}{45} \times \pi\):

\[
r^2 = \frac{\frac{7}{5}}{\frac{7}{45} \times \pi}
\]

- Simplify numerator and denominator:
\[
r^2 = \frac{\frac{7}{5}}{\frac{7}{45} \times \pi}
\]

- Multiply numerator and denominator by 1 to keep equality:
\[
r^2 = \frac{\frac{7}{5}}{\frac{7}{45} \times \pi}
\]

Step 4: Simplify the Expression

- The numerator is \(\frac{7}{5}\).
- The denominator is \(\frac{7}{45} \times \pi\).

Note that:
\[
r^2 = \frac{\frac{7}{5}}{\frac{7}{45} \times \pi} =
\frac{\frac{7}{5}}{\frac{7 \pi}{45}}
\]

- When dividing fractions, multiply numerator by reciprocal of denominator:
\[
r^2 = \frac{7}{5} \times \frac{45}{7 \pi}
\]

- Cancel the 7s:
\[
r^2 = \frac{1}{5} \times \frac{45}{\pi} = \frac{45}{5 \pi} = \frac{9}{\pi}
\]

Step 5: Find r

$$
r = \sqrt{\frac{9}{\pi}} = \frac{\sqrt{9}}{\sqrt{\pi}} = \frac{3}{\sqrt{\pi}}
$$

Final Calculation and Approximate Radius

To provide a numerical value, substitute $\pi \approx 3.1416$:

$$
r \approx \frac{3}{\sqrt{3.1416}} \approx \frac{3}{1.772} \approx 1.693 \text{ inches}
$$

Therefore, the radius of the circle is approximately 1.69 inches.

Summary of the Solution Process

1. Identify known values: Sector area $= \frac{7}{5}$, angle $\theta = 56^\circ$.
2. Write the sector area formula: $A_{sector} = \frac{\theta}{360^\circ} \times \pi r^2$.
3. Substitute the known values: $\frac{7}{5} = \frac{56}{360} \times \pi r^2$.
4. Simplify fractions: $\frac{7}{5} = \frac{7}{45} \times \pi r^2$.
5. Isolate r^2: $r^2 = \frac{7/5}{(7/45) \times \pi}$.
6. Simplify further: $r^2 = \frac{45}{5 \pi} = \frac{9}{\pi}$.
7. Calculate r: $r = \sqrt{\frac{9}{\pi}} \approx 1.69$ inches.

Additional Insights and Applications

Understanding how to compute the radius from a sector's area and central angle has practical applications across various fields:

Engineering and Design

- Calculating the dimensions of circular components like gears or pipes.
- Designing sectors in architectural structures.

Navigation and Geography

- Determining distances covered along curved paths.
- Calculating areas of sectors in map projections.

Education and Learning

- Enhancing spatial understanding and problem-solving skills.
- Developing a strong foundation in geometric formulas.

Common Mistakes to Avoid

When solving similar problems, be mindful of the following pitfalls:

1. Mixing degrees and radians: Always confirm the central angle is in degrees when using the formula that involves degrees.

2. Incorrect fraction simplification: Carefully simplify fractions to avoid errors.

3. Forgetting to square root: Remember that the radius is the positive square root of the calculated value.

4. Approximate calculations: Use consistent precision, especially when dealing with \(\pi\).

Conclusion

The problem of determining the radius of a circle when given the area of a sector and the central angle is a classic application of circle geometry formulas. By systematically applying the sector area formula, simplifying the algebra, and making accurate approximations, we found the radius to be approximately 1.69 inches. This process underscores the importance of understanding fundamental geometric relationships and their practical utility.

Whether you're a student working through coursework or a professional applying geometry in the field, mastering these calculations enhances your problem-solving toolkit and broadens your understanding of circular measurements.

Frequently Asked Questions

What is the radius of a sector with an area of 7/5 square inches and a central angle of 56 degrees?

Using the sector area formula $A = (\theta/360)\ \pi\ r^2$, we can solve for r: r = sqrt[(A 360) / (θ π)] that is r = sqrt[((7/5) 360) / (56 π)] inches.

How do you find the length of the arc of a sector with a given area and central angle?

The arc length $L = (\theta/360)\ 2\pi r$. First, find the radius using the area, then substitute into the arc length formula.

What is the formula for the area of a sector of a circle?

The area of a sector is given by $A = (\theta/360)\ \pi\ r^2$, where θ is the central angle in degrees and r is the radius.

Given the area and central angle, how can I determine the radius of the circle's sector?

Rearranged, radius r = sqrt[(A 360) / (θ π)]. Plug in the area and angle to find r.

If the area of a sector is 7/5 square inches and the central angle is 56 degrees, what is the approximate radius?

Calculating r: r ≈ sqrt[((7/5) 360) / (56 π)] ≈ 1.31 inches.

How does the central angle affect the size of a sector's area?

The area of a sector is directly proportional to the central angle; larger angles result in larger sectors, assuming the radius stays the same.

Can the radius be found if only the area and central angle are known? How?

Yes, by rearranging the sector area formula to solve for r: r = sqrt[(A 360) / (θ π)].

What are common units used in calculating the area and dimensions of a circle sector?

Typically, inches, centimeters, or meters are used for lengths and square inches, square centimeters, or square meters for areas.

Why is understanding the sector's area important in real-world applications?

It helps in fields like engineering, architecture, and design where precise measurements of circular segments are necessary for construction, manufacturing, or analysis.

Additional Resources

The Sector Of A Circle Has An Area Of 7/5 Square Inches And Central Angle With Measure 56°: An In-Depth Analysis

Understanding the properties of a circle and its sectors is fundamental in geometry, offering insights into how angles, areas, and arcs interrelate. The problem involving a sector with an area of 7/5 square inches and a central angle of 56° provides a practical example of these concepts. This article aims to dissect this problem thoroughly, exploring the mathematical principles involved, solving the problem step-by-step, and highlighting the key features, potential applications, and common pitfalls.

Understanding the Sector of a Circle

A sector of a circle is a portion of the circle enclosed by two radii and the arc connecting their endpoints. Think of it as a "slice" of a pie, where the size of the slice depends on the central angle. The sector's area and arc length are two primary characteristics derived from the circle's radius and the central angle.

Key Features of a Sector:

- Area: The region enclosed within the two radii and the arc.
- Arc Length: The length of the curved boundary of the sector.
- Central Angle (θ): The angle subtended at the circle's center by the sector.

Understanding how these features relate to the circle's radius is essential in solving related problems.

Mathematical Foundations and Formulas

The properties of a sector are governed by specific formulas:

1. Area of a Sector

The area A of a sector with radius r and central angle θ (in degrees) is given by:

$$
A = \frac{\theta}{360} \times \pi r^2
$$

This formula indicates that the sector's area is proportional to the fraction of the circle's total area determined by $\theta/360$.

2. Arc Length of a Sector

The arc length L of the sector is:

$$
L = \frac{\theta}{360} \times 2 \pi r
$$

which represents the length of the curved boundary.

Analyzing the Given Data

The problem states:

- The area of the sector is $\frac{7}{5}$ square inches.
- The central angle θ is 56°.

Our goal is to determine the radius r of the circle, and potentially other properties like the arc length.

Step-by-Step Solution

1. Apply the Sector Area Formula

Given:

$$
A = \frac{7}{5} \text{ sq inches}
$$

$$
\theta = 56^\circ
$$

Plug into the area formula:

$$
\frac{7}{5} = \frac{56}{360} \times \pi r^2
$$

Simplify the fraction:

$$
\frac{56}{360} = \frac{14}{90} = \frac{7}{45}
$$

So:

$$
\frac{7}{5} = \frac{7}{45} \times \pi r^2
$$

Divide both sides by 7:

$$
\frac{1}{5} = \frac{1}{45} \times \pi r^2
$$

Multiply both sides by 45:

$$
45 \times \frac{1}{5} = \pi r^2
$$

$$
9 = \pi r^2
$$

Solve for r^2:

$$
r^2 = \frac{9}{\pi}
$$

Calculate r:

$$
r = \sqrt{\frac{9}{\pi}} = \frac{3}{\sqrt{\pi}}
$$

For a numerical approximation:

$$
\pi \approx 3.1416
$$

$$
r \approx \frac{3}{\sqrt{3.1416}} \approx \frac{3}{1.772} \approx 1.693 \text{ inches}
$$

Result: The radius of the circle is approximately 1.693 inches.

---

2. Calculate the Arc Length (Optional)

Using the arc length formula:

$$
L = \frac{\theta}{360} \times 2 \pi r
$$

Plug in the known values:

$$
L = \frac{56}{360} \times 2 \pi \times 1.693
$$

Simplify:

$$
\frac{56}{360} = \frac{14}{90} = \frac{7}{45}
$$

$$
L = \frac{7}{45} \times 2 \pi \times 1.693
$$

Calculate:

$$
L \approx \frac{7}{45} \times 2 \times 3.1416 \times 1.693
$$

$$
L \approx 0.1556 \times 6.2832 \times 1.693
$$

$$
L \approx 0.1556 \times 10.648
$$

$$
L \approx 1.658 \text{ inches}
$$

Approximate arc length: 1.658 inches.

Key Takeaways and Features

- The problem demonstrates how sector area and central angle are interconnected.
- The formulas used are versatile tools in geometry, applicable to various problems involving circles.
- Calculating the radius from the sector's area and angle involves algebraic manipulation and understanding proportional relationships.

Features of the Problem:

- Clear relationship between area, radius, and angle.
- Application of basic formulas with straightforward algebra.
- Supports understanding of sectors beyond theoretical exercises, including practical applications.

Potential Applications of Sector Calculations

Understanding sectors is crucial in fields like engineering, architecture, and design. For example:

- Design of pie charts and data visualization.
- Mechanical components like gears and rotors.
- Land division and plot planning involving circular sectors.
- Optics and light distribution systems.

This problem reinforces foundational skills that can be extended to these complex real-world scenarios.

Common Pitfalls and Errors to Avoid

- Incorrect unit conversions: Always ensure angles are in degrees when using the formulas or convert to radians if needed.
- Misapplication of formulas: Remember that the formulas are specific to degrees; for radians, the formulas differ.
- Arithmetic mistakes: Simplify fractions carefully and double-check calculations, especially when dealing with multiple steps.
- Assuming the radius without proper calculation: Always derive the radius from the given data rather than guessing.

Conclusion

The problem involving a sector with an area of \(\frac{7}{5}\) square inches and a central angle of 56° offers a rich illustration of geometric principles in action. By applying the sector area formula, we derived the radius to be approximately 1.693 inches, showcasing how algebra and geometry complement each other. Calculating the arc length further demonstrates the interconnectedness of a sector's properties. Such problems not only sharpen mathematical skills but also prepare one for analyzing practical scenarios involving circles and sectors. Mastery of these concepts opens doors to more advanced topics in geometry, trigonometry, and their applications across science and engineering.

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