The Simultaneous Conditions X Y < 6, X Y < 6, And X > 0 Define A Region R. How Many Lattice

The Simultaneous Conditions X Y < 6, X Y < 6, And X > 0 Define A Region R. How Many Lattice points lie within this region? This question is fundamental in the study of coordinate geometry, linear inequalities, and integer lattice points. Understanding how to determine the number of lattice points—points with integer coordinates—within a given region defined by multiple inequalities is crucial in various mathematical fields, including combinatorics, optimization, and computational geometry. In this article, we will explore the problem systematically, analyze the constraints, and develop methods to count the lattice points within the specified region R.

Understanding the Problem and Its Constraints

Before diving into calculations, it is essential to interpret the given inequalities accurately and understand their geometric implications.

Given Conditions

The problem states:

- X Y < 6
- X Y < 6 (appears to be a repetition, possibly a typo; assuming it is central to the problem)
- X > 0

Considering the common forms of inequalities, it seems likely that the intended constraints are:

1. X > 0: The region is strictly to the right of the Y-axis.
2. X Y < 6: The product of X and Y is less than 6.

There may be a typographical error or omission, but for clarity, let's examine the primary inequalities:

- $X > 0$
- $X \times Y < 6$

The mention of "X Y < 6" twice might be emphasizing the same constraint or could be referring to an additional boundary condition. For the purpose of this analysis, we'll consider the main constraints as:

- $X > 0$
- $X \times Y < 6$

Geometric Interpretation of the Region R

Understanding the inequalities geometrically allows us to visualize the region and count lattice points effectively.

Region Defined by $X > 0$ and $X \times Y < 6$

- The inequality $X > 0$ restricts the region to the right half-plane, excluding the Y-axis.
- The inequality $X \times Y < 6$ describes the area below the hyperbola $Y = \frac{6}{X}$, assuming $X > 0$.

Key features:

- For each fixed $X > 0$, the inequality $Y < \frac{6}{X}$ bounds the Y-values from above.
- The region R is the set of all points (X, Y) with $X > 0$ and $Y < \frac{6}{X}$.

Defining the Lattice Points and Counting Strategies

Lattice points are points (x, y) where both x and y are integers.

Approach to Counting Lattice Points in R

To determine how many lattice points lie within R, we follow a systematic approach:

1. Identify the bounds for X and Y.
2. Enumerate all integer X values within the bounds.
3. For each fixed X, find all integer Y values satisfying $Y < \frac{6}{X}$.
4. Count the number of such Y points for each X.

Step-by-Step Calculation

Let's formalize the process with concrete steps.

Step 1: Determine the range of X

Since $X > 0$, and Y must satisfy $Y < \frac{6}{X}$, for the counting to be meaningful, we need to determine the maximum X value where lattice points exist.

- As $X \to 0^+$, Y can be arbitrarily large negative (since $Y < \frac{6}{X}$ becomes very large).
- As X increases, $\frac{6}{X}$ decreases.

But to have lattice points with finite Y, we consider positive integers X up to some maximum.

Note: For practical purposes, since $\frac{6}{X}$ becomes less than or equal to 1 when $X \geq 6$, the number of Y satisfying $Y < \frac{6}{X}$ becomes limited.

Step 2: Enumerate possible integer values for X

The positive integers X such that $Y < \frac{6}{X}$ can be considered for $X = 1, 2, 3, 4, 5, 6, ...$

- For $X=1$, $Y < 6$
- For $X=2$, $Y < 3$
- For $X=3$, $Y < 2$
- For $X=4$, $Y < 1.5$
- For $X=5$, $Y < 1.2$
- For $X=6$, $Y < 1$
- For $X=7$, $Y < \frac{6}{7} \approx 0.857$

Since Y must be an integer less than these values, the maximum Y for each X is $\lfloor \frac{6}{X} \rfloor$.

Step 3: Count the lattice points for each X

For each X, the possible Y values are integers satisfying:

$$Y < \frac{6}{X}$$

which implies:

$$Y \leq \lfloor \frac{6}{X} \rfloor - 1$$ (since Y must be strictly less)

But because Y can be negative as well, and the inequality is strict:

$$Y < \frac{6}{X}$$

- For positive Y, the maximum integer Y is $\lfloor \frac{6}{X} \rfloor - 1$ when $\frac{6}{X}$ is not an integer.
- For negative Y, all integers less than $\frac{6}{X}$ satisfy the inequality.

Therefore, for each X, the Y values are:

$$Y \in \left(-\infty, \lfloor \frac{6}{X} \rfloor \right)$$

but in practice, since lattice points are bounded in the plane, and the region extends infinitely downward, we need to set some lower bounds to count finitely many points.

Assumption: The problem likely considers only lattice points with Y in a reasonable range (e.g., non-negative or bounded). If not specified, the region extends infinitely downward, and the number of lattice points is infinite. So, for a finite count, we can restrict Y to, say, integers satisfying $Y \leq M$ for some large M.

Alternatively, if the problem aims to count all lattice points with Y in the range where the inequalities hold, including negative values, then the region is unbounded downward, leading to infinite lattice points.

Counting Lattice Points in a Bounded Region

Given the above, it's typical to consider bounded regions for practical counting. Suppose we restrict to $Y \geq 0$. Then, for each X, we count:

$$Y \in \{0, 1, 2, \dots, \lfloor \frac{6}{X} \rfloor - 1\}$$

provided that $\lfloor \frac{6}{X} \rfloor - 1 \geq 0$.

Let's perform this counting explicitly for $X = 1$ to 6.

Calculations for $X=1$ to 6

X	$\frac{6}{X}$	$\lfloor \frac{6}{X} \rfloor$	Number of integer $Y \ge 0$ satisfying $Y < \frac{6}{X}$	Counts of Y
1	6	6	$Y < 6$ => $Y = 0, 1, 2, 3, 4, 5$	6
2	3	3	$Y < 3$ => $Y = 0, 1, 2$	3
3	2	2	$Y < 2$ => $Y = 0, 1$	2
4	1.5	1	$Y < 1.5$ => $Y = 0, 1$	2
5	1			

Frequently Asked Questions

What is the geometric interpretation of the simultaneous conditions X Y < 6 and X > 0?

These conditions define a region in the XY-plane where the product of X and Y is less than 6, and X is positive. Geometrically, it corresponds to the area to the right of the Y-axis (since X > 0) and below the hyperbola XY = 6.

How do the inequalities X Y < 6 and X > 0 affect the shape of the region R?

They restrict the region to the right half of the plane (X > 0) and below the hyperbola XY = 6, resulting in a bounded or unbounded region depending on the specific limits, often forming a curved region approaching the axes.

How many lattice points are contained within the region R defined by X Y < 6 and X > 0?

The number of lattice points depends on the bounds set for X and Y. If X and Y are positive integers, then for each X, Y must satisfy Y < 6/X. Counting all integer pairs (X,Y) with X > 0 and Y < 6/X gives the total lattice points within R.

What is the method to count lattice points in the region R defined by the inequalities?

The common approach is to fix X as a positive integer and then count the number of integer Y values satisfying Y < 6/X, considering Y > 0 if the region is limited to positive Y. Summing over all valid X yields the total lattice points.

Are there any boundary points on the hyperbola XY = 6 within the region R?

Yes, points on the hyperbola XY = 6 with X > 0 and Y > 0 lie on the boundary of the region R. These points satisfy the inequalities as equalities and are typically included or excluded based on the problem's context.

How does the condition X > 0 influence the set of lattice points inside the region R?

It restricts the lattice points to those with positive X-coordinate, excluding points where X ≤ 0. This simplifies counting by focusing only on the right half-plane.

If X and Y are positive integers, how can we explicitly compute the number of lattice points in R?

For each positive integer X, count the number of Y = 1, 2, ..., floor(6/X) - 1, provided that Y < 6/X. Summing these counts over all X where 6/X > 1 will give the total number of lattice points within R.

Additional Resources

Understanding the Geometric and Mathematical Significance of the Conditions: X + Y < 6, XY < 6, and X > 0: An In-Depth Analysis

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Introduction

In the realm of coordinate geometry and algebraic analysis, defining regions in the XY-plane through systems of inequalities is fundamental to understanding the geometric behavior of functions, feasible solutions in optimization problems, and the structure of algebraic sets. One particularly interesting scenario involves the simultaneous conditions:

- X + Y < 6
- XY < 6
- X > 0

These inequalities collectively define a region \(R \) in the first quadrant (since \(X > 0 \)) constrained further by the sum and product of \(X \) and \(Y \). The question of how many lattice points—points with integer coordinates—lie within this region is a classical problem bridging discrete mathematics, geometry, and number theory.

This comprehensive review delves into the geometric intuition, algebraic underpinnings, implications, and computational aspects of this problem, providing clarity and depth for students, researchers, and enthusiasts alike.

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1. Geometric Interpretation of the Inequalities

1.1 The Inequality $X + Y < 6$

- Line Representation: The boundary line $X + Y = 6$ is a straight line with intercepts at $(6, 0)$ and $(0, 6)$.
- Region: The inequality $X + Y < 6$ describes the interior of the region below (or to the left of) this line in the XY-plane, excluding the boundary itself.
- Implication: Since $X > 0$, the relevant part of this region is in the first quadrant, specifically bounded by $X > 0$, $Y > 0$, and below the line.

1.2 The Inequality $XY < 6$

- Hyperbola Representation: The equality $XY = 6$ defines a rectangular hyperbola centered at the origin, with branches in the first and third quadrants.
- Region: $XY < 6$ indicates the interior of the hyperbola in the first quadrant, meaning points with positive coordinates whose product is less than 6.
- Visualization: For fixed X, the boundary $Y = \frac{6}{X}$, which decreases as X increases.

1.3 The Inequality $X > 0$

- First Quadrant: Restricts the region to the right of the Y-axis, emphasizing positive X.
- Significance: Ensures consideration of only the right half-plane, which simplifies the analysis since the hyperbola $XY = 6$ and the line $X + Y = 6$ are symmetric with respect to quadrants.

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2. Combined Geometric Region R

The region R is the intersection of all three inequalities:

$$
R = \left\{ (X, Y) \in \mathbb{R}^2\, |\, X > 0,\, X + Y < 6,\, XY < 6 \right\}
$$

Visualizing this region involves understanding how the hyperbola, the line, and the positive X-axis interact.

2.1 Boundary Curves and Lines

- Line: $X + Y = 6$
- Hyperbola: $XY = 6$

The region R lies below the line and inside the hyperbola in the first quadrant.

2.2 Intersection Points

- Find points where the hyperbola and line intersect:

$$
XY = 6 \quad \text{and} \quad X + Y = 6
$$

Substituting $Y = 6 - X$, we get:

$$
X(6 - X) = 6 \Rightarrow 6X - X^2 = 6 \Rightarrow X^2 - 6X + 6 = 0
$$

Solving for X:

$$
X = \frac{6 \pm \sqrt{36 - 24}}{2} = \frac{6 \pm \sqrt{12}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}
$$

Corresponding Y values:

$$
Y = 6 - X
$$

So the intersection points are:

$$
(X, Y) = (3 + \sqrt{3}, 3 - \sqrt{3}) \quad \text{and} \quad (3 - \sqrt{3}, 3 + \sqrt{3})
$$

Both are in the first quadrant since $3 - \sqrt{3} > 0$ and $3 + \sqrt{3} > 0$.

- These points form the vertices of the feasible region's boundary in the first quadrant.

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3. Analyzing the Feasible Region R

3.1 Shape and Boundaries

- The feasible region is bounded above by the line $X + Y = 6$,
- Inside the hyperbola $XY = 6$,
- And in the first quadrant, where $X > 0$, $Y > 0$.

The region is closed by the hyperbola and the line, and open since inequalities are strict.

3.2 Graphical Sketch

- The hyperbola $XY = 6$ in the first quadrant is decreasing, with asymptotes along axes.
- The line $X + Y = 6$ is decreasing, crossing the axes at $(6, 0)$ and $(0, 6)$.
- The intersection points mark the limits where the hyperbola and line meet.

The feasible region R:

- Lies below the line $X + Y = 6$,
- Inside the hyperbola $XY = 6$,
- In the positive X and Y axes.

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4. Counting Lattice Points within R

The core problem involves how many points with integer coordinates (x, y) satisfy the inequalities:

$$
x > 0,\quad y > 0,\quad x + y < 6,\quad xy < 6
$$

This is a classic lattice point enumeration problem in a constrained region.

4.1 Approach to Counting

To determine the number of lattice points:

1. Set bounds for x: Since $x > 0$ and $x + y < 6$, the maximum x occurs at $x \geq 1$ but must satisfy the inequalities.

2. For each fixed integer x, find corresponding y values:

- y must satisfy:
$$
y > 0,\quad y < 6 - x,\quad xy < 6
$$
- From $xy < 6$, for each x:
\[

y < \frac{6}{x}
\]
- Combining:
\[
y \in \left(0,\, \min\left(6 - x,\, \frac{6}{x}\right)\right)
\]
- Since \(y \) must be an integer, \(y \) can take all positive integers less than the minimum of these two bounds.

3. Iterate over possible integer \(x \) values:

- Because \(x + y < 6 \) and \(y > 0 \), \(x \) must be less than 6.

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5. Systematic Enumeration of Lattice Points

Let's proceed step-by-step:

For \(x = 1 \):

- \(y \in (0,\, \min(6 - 1,\, 6/1)) = (0,\, \min(5,\, 6)) = (0,\, 5) \)
- Possible integer \(y \): 1, 2, 3, 4

Check \(xy < 6 \):

- \((1,1): 1 < 6 \) — OK
- \((1,2): 2 < 6 \) — OK
- \((1,3): 3 < 6 \) — OK
- \((1,4): 4 < 6 \) — OK

Total points for \(x=1 \): 4 points

For \(x = 2 \):

- \(y \in (0,\, \min(6 - 2,\, 6/2)) = (0,\, \min(4,\, 3)) = (0,\, 3) \)
- Possible \(y \): 1, 2

Check \(xy < 6 \):

- \((2,1): 2 < 6 \) — OK
- \((2,2): 4 < 6 \) — OK

Total points

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