Thuy Rolls A Number Cube 7 Times. Which Expression Represents The Probability Of Rolling A 4 Exactly

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When engaging in probability problems involving multiple dice rolls, understanding how to calculate the likelihood of specific outcomes becomes essential. In this article, we will explore the scenario where Thuy rolls a standard six-sided number cube (die) seven times and analyze how to determine the probability that she rolls a 4 exactly a certain number of times. We will break down the problem step-by-step, introduce relevant probability formulas, and provide clear examples to help you grasp the concepts involved.

Understanding the Scenario: Thuy Rolls a Number Cube 7 Times

Imagine Thuy has a fair, six-sided die numbered from 1 to 6. She rolls this die seven times in succession, recording the result of each roll. Each roll is independent, meaning the outcome of one roll does not influence the others.

The key question here is: What is the probability that Thuy rolls a 4 exactly a specific number of times during these seven rolls?

This type of problem falls under the umbrella of binomial probability, which deals with the number of successes in a fixed number of independent Bernoulli trials.

Defining the Key Components of the Problem

Before we construct the probability expression, let's clarify the essential elements:

1. Total number of trials (n):

- Thuy rolls the die 7 times, so $n = 7$.

2. Number of desired successes (k):

- The number of times she rolls a 4, which can range from 0 up to 7.

3. Probability of success on a single trial (p):

- Since there are six faces, and only one of them is a 4, the probability of rolling a 4 on any given roll is:

$$
p = \frac{1}{6}
$$

4. Probability of failure on a single trial (q):

- The probability that the roll is not a 4:

$$
q = 1 - p = \frac{5}{6}
$$

Formulating the Probability Expression

Given the above components, the probability that Thuy rolls exactly k 4s in n rolls follows the binomial probability formula:

$$
P(\text{exactly } k \text{ successes}) = \binom{n}{k} p^{k} q^{n - k}
$$

Where:
- $\binom{n}{k}$ is the binomial coefficient, representing the number of ways to choose k successful outcomes out of n trials.
- p^k is the probability of getting k successes.
- $q^{n - k}$ is the probability of getting n - k failures.

Applying the Formula to Our Scenario

Suppose Thuy wants to find the probability that she rolls exactly 3 fours in her 7 rolls. The expression would be:

$$
P(\text{exactly 3 fours}) = \binom{7}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{4}
$$

Similarly, if she wants to find the probability of rolling exactly 0 fours, the expression becomes:

$$
P(\text{exactly 0 fours}) = \binom{7}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^7
$$

And for exactly 7 fours:

$$
P(\text{exactly 7 fours}) = \binom{7}{7} \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0
$$

General Expression for the Probability of Exactly k Fours in 7 Rolls

The most comprehensive expression, applicable to any value of k (where $0 \leq k \leq 7$), is:

$$
\boxed{
P(\text{exactly } k \text{ fours}) = \binom{7}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7 - k}
}
$$

This formula allows us to compute the probability for any specific number of fours.

Understanding the Binomial Coefficient

The binomial coefficient $\binom{n}{k}$ (read as "n choose k") calculates the number of ways to select k successes from n trials and is calculated as:

$$
\binom{n}{k} = \frac{n!}{k! (n - k)!}
$$

where $n!$ (n factorial) is the product of all positive integers up to n.

For example:
- $\binom{7}{3} = \frac{7!}{3! \times 4!} = \frac{5040}{6 \times 24} = 35$

Examples of Calculating Probabilities

Let's explore some specific cases to deepen understanding:

Example 1: Probability of rolling exactly 2 fours in 7 rolls

$$
P = \binom{7}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5
$$

Calculate step-by-step:
- $\binom{7}{2} = 21$
- $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$
- $\left(\frac{5}{6}\right)^5 \approx 0.401$

Thus:
$$
P \approx 21 \times \frac{1}{36} \times 0.401 \approx 21 \times 0.0278 \approx 0.5838
$$

So, there is approximately a 58.38% chance of rolling exactly 2 fours in 7 rolls.

Example 2: Probability of rolling no fours at all

$$
P = \binom{7}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^7 = 1 \times 1 \times \left(\frac{5}{6}\right)^7
$$

Calculating:
- $\left(\frac{5}{6}\right)^7 \approx 0.279$

Therefore, the probability of not rolling a 4 in any of the 7 rolls is approximately 27.9%.

Applications and Importance of the Probability Formula

Understanding this probability model has several practical applications:

- **Game Strategy:** Players can analyze their chances of winning or achieving specific outcomes in dice games.

- **Educational Purposes:** Helps students grasp concepts of probability, binomial distributions, and combinatorics.

- **Simulation and Modeling:** Used in computer simulations to predict outcomes in probabilistic systems.

Additional Considerations and Variations

While the above example considers a fair die, variations can include:

- Biased dice: Adjusting p to reflect unequal probabilities for each face.
- Multiple outcomes: Calculating the probability of rolling multiple different numbers, not just 4.
- Different number of rolls: Extending the model to more or fewer than 7 rolls.
- Conditional probability: Calculating the probability given some prior information.

Summary and Key Takeaways

- The probability that Thuy rolls exactly k fours in 7 rolls of a fair six-sided die is given by the binomial probability formula:

$$
\boxed{
P(\text{exactly } k \text{ fours}) = \binom{7}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7 - k}
}
$$

- The key components involve the binomial coefficient, success probability $p = \frac{1}{6}$, and failure probability $q = \frac{5}{6}$.

- This model provides a powerful way to analyze probabilistic outcomes in repeated independent trials.

Conclusion

Calculating the probability of a specific number of successes—in this case, rolling exactly a certain number of fours in 7 die rolls—is fundamental in understanding binomial distributions. By applying the binomial probability formula, you can determine the likelihood of various outcomes and make informed predictions about random processes involving repeated independent events.

Whether for academic purposes, game strategies, or statistical analysis, mastering these concepts enhances your understanding of probability and combinatorics, equipping you to tackle more complex problems with confidence.

Frequently Asked Questions

What is the probability of rolling a 4 exactly once in 7 rolls of a number cube?

The probability is given by the binomial formula: C(7,1) (1/6)^1 (5/6)^6.

How do you calculate the probability of rolling exactly two 4s in 7 rolls?

Use the binomial probability formula: C(7,2) (1/6)^2 (5/6)^5.

What expression represents the probability of rolling a 4 exactly three times in 7 rolls?

C(7,3) (1/6)^3 (5/6)^4.

How can I find the probability of rolling no 4s in 7 rolls?

Calculate (5/6)^7, since no 4s mean all rolls are not 4s.

What is the general formula for the probability of rolling exactly k 4s in n rolls?

C(n, k) (1/6)^k (5/6)^{n - k}.

If Thuy rolls a number cube 7 times, what expression gives the probability of rolling at least one 4?

1 - (5/6)^7, which is the probability of rolling zero 4s subtracted from 1.

Which binomial coefficient should be used to calculate the probability of rolling exactly four 4s in 7 rolls?

C(7,4).

How would you write an expression to find the probability of rolling exactly five 4s in 7 rolls?

C(7,5) (1/6)^5 (5/6)^2.

What does the term C(7, k) represent in the probability expression?

It represents the number of ways to choose k successful outcomes (4s) out of 7 rolls.

Why do we use the binomial probability formula for this problem?

Because each roll is independent, and the probability of success (rolling a 4) remains constant, fitting the binomial distribution model.

Additional Resources

Thuy Rolls A Number Cube 7 Times. Which Expression Represents The Probability Of Rolling A 4 Exactly

In the realm of probability and statistics, understanding how to calculate the likelihood of specific outcomes in repeated trials is fundamental. Recently, a student named Thuy undertook a simple yet insightful experiment: rolling a standard six-sided die (or number cube) seven times. The question that naturally arises from this scenario is: What is the probability that Thuy will roll a 4 exactly once, exactly twice, or any specified number of times within those seven rolls? This problem exemplifies core concepts in probability theory, including binomial probability, the use of expressions to quantify likelihoods, and the importance of precise calculations in understanding randomness.

This article explores the mathematical framework behind Thuy's experiment, focusing on which expression accurately models the probability of rolling a 4 exactly a certain number of times in seven rolls. We will break down the problem step-by-step, introduce relevant concepts, and clarify how to determine the correct probability expression, making the topic accessible yet detailed enough for readers interested in the mechanics of probability calculations.

Understanding the Basics: The Setup of the Problem

Before delving into the specific probability expressions, it's essential to grasp the fundamental setup of Thuy's experiment:

- Number Cube (Die): A standard six-sided die, with faces numbered 1 through 6.
- Number of Rolls: 7
- Outcome of Interest: Rolling a 4 on a single roll
- Other outcomes: Rolling a number other than 4 (i.e., 1, 2, 3, 5, or 6)

The key question posed is: What is the probability that exactly k of these seven rolls result in a 4? For example, if k=2, what's the probability that Thuy rolls exactly two 4s in seven attempts?

Probability of a Single Roll Resulting in a 4

The first step is to determine the probability that a single roll results in a 4:

- Since the die is fair and numbered from 1 to 6, each side has an equal chance.
- Probability of rolling a 4: P(rolling a 4) = 1/6

Conversely, the probability of not rolling a 4:

- P(not 4) = 1 - P(4) = 1 - (1/6) = 5/6

This simple probability forms the foundation for calculating the likelihood of various outcomes across multiple rolls.

Modeling Multiple Rolls: The Binomial Framework

When Thuy rolls the die 7 times, each roll can be considered an independent Bernoulli trial—a random experiment with two possible outcomes: success (rolling a 4) or failure (not rolling a 4).

Key assumptions:

- Independence: Each roll does not affect the outcomes of others.
- Constant probability: The probability of success remains 1/6 for each roll.

Given these, the problem aligns perfectly with the binomial probability model, which calculates the probability of having exactly k successes in n independent Bernoulli trials.

Binomial probability formula:

$$
P(k\ \text{successes}) = \binom{n}{k} p^k (1 - p)^{n - k}
$$

Where:

- n = total number of trials (here, 7)
- k = number of successes (number of times 4 is rolled)
- p = probability of success on each trial (here, 1/6)
- $\binom{n}{k}$ = binomial coefficient, representing the number of ways to choose which k rolls out of n are successful

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Applying the Binomial Formula: Specific Expression for Thuy’s Problem

Suppose Thuy wants to find the probability of rolling a 4 exactly k times in 7 rolls. Substituting into the binomial formula:

$$
P(\text{exactly }k\text{ 4s}) = \binom{7}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7 - k}
$$

This expression succinctly captures the probability of exactly k successes (4s) in 7 independent rolls, each with success probability 1/6.

Important notes:

- The binomial coefficient $\binom{7}{k}$ counts the number of different sequences where exactly k rolls are 4.
- The term $\left(\frac{1}{6}\right)^k$ accounts for the probability of obtaining 4 exactly k times.
- The term $\left(\frac{5}{6}\right)^{7 - k}$ accounts for the remaining rolls being any of the other five numbers.

Examples: Calculating Specific Probabilities

Let’s see how this formula works with some concrete examples:

Example 1: Probability of rolling exactly one 4 in 7 rolls (k=1):

$$
P(1\ \text{four}) = \binom{7}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^6
$$

Calculating:

- $\binom{7}{1} = 7$
- $\left(\frac{1}{6}\right)^1 = \frac{1}{6}$
- $\left(\frac{5}{6}\right)^6 \approx 0.3349$

Multiplying:

$$
7 \times \frac{1}{6} \times 0.3349 \approx 7 \times 0.1667 \times 0.3349 \approx 0.392
$$

Example 2: Probability of rolling exactly three 4s (k=3):

$$
P(3\ \text{fours}) = \binom{7}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4
$$

Calculating:

- $\binom{7}{3} = 35$
- $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$
- $\left(\frac{5}{6}\right)^4 \approx 0.4823$

Multiplying:

$$
35 \times \frac{1}{216} \times 0.4823 \approx 35 \times 0.00463 \times 0.4823 \approx 0.078
$$

These examples illustrate how the probability varies depending on the exact number of successes.

Interpreting the Expression and Its Components

The key to understanding and correctly applying the probability expression lies in recognizing its components:

- Binomial coefficient $\binom{7}{k}$: Counts the number of different sequences where k specific positions are 4s.
- Success probability $p = 1/6$: The chance of rolling a 4 on one trial.
- Failure probability $1 - p = 5/6$: The chance of rolling any number other than 4.

This structure makes the binomial probability an elegant, powerful tool for modeling scenarios like Thuy's, where multiple independent, identical trials are performed, and the goal is to find the likelihood of a specific number of successes.

Choosing the Correct Expression: Which Formula Represents the Probability?

Given the background and the derivations, the correct expression for the probability that Thuy rolls a 4 exactly k times in 7 rolls is:

$$
\boxed{
\binom{7}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7 - k}
}
$$

Any alternative expression that does not include the binomial coefficient or misrepresents the success and failure probabilities would not accurately model the scenario.

Broader Implications and Applications

Understanding this probability expression isn't just an academic exercise—it has practical relevance in various fields:

- Gaming: Calculating odds in board games involving dice.
- Quality Control: Estimating the probability of a certain number of defective items in a batch.
- Statistics: Designing experiments and understanding sampling distributions.
- Education: Teaching foundational concepts in probability and combinatorics.

Thuy's simple act of rolling a die multiple times exemplifies these broader concepts and underscores the importance of precise mathematical language in describing randomness.

Conclusion

In summary, when Thuy rolls a standard six-sided die 7 times and wants to determine the probability of rolling a 4 exactly k times, the core mathematical model is the binomial distribution. The precise expression for this probability is:

$$
\boxed{
\binom{7}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7 - k}
}
$$

This formula accounts for all possible arrangements of successes and failures, providing a robust tool to quantify the likelihood of specific outcomes in repeated, independent trials. Whether calculating the probability of a single success, multiple successes, or no successes at

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