Trigonometry Solve For X: A)tan^2 (x) - 1 = 0 B)2 Cos^2 (x) - 1 = 0 C)2 Sin^2(x) + 15 Sin(x) + 7 = 0

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Understanding how to solve trigonometric equations for x is a fundamental skill in mathematics, especially in trigonometry. These equations often appear in various applications, from physics to engineering, and mastering their solutions enhances your problem-solving capabilities. In this article, we'll explore how to approach and solve three common trigonometric equations:

- A) $\tan^2(x) - 1 = 0$
- B) $2 \cos^2(x) - 1 = 0$
- C) $2 \sin^2(x) + 15 \sin(x) + 7 = 0$

Let's dive into each problem, understand the methods involved, and see how to find all solutions within a specified domain.

Understanding Trigonometric Equations

Before solving specific equations, it's essential to grasp some key concepts:

Basic Trigonometric Functions

- Sine (sin x): Opposite side / Hypotenuse
- Cosine (cos x): Adjacent side / Hypotenuse
- Tangent (tan x): sin x / cos x

Important Identities

- Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$
- Expressing functions in terms of each other:
- $\tan^2 x = \sec^2 x - 1$
- $\sec^2 x = 1 / \cos^2 x$

Solving Trigonometric Equations

Most equations can be simplified into algebraic forms involving sine or cosine, then solved using algebraic techniques like factoring, quadratic formula, or substitution.

Equation A: $\tan^2(x) - 1 = 0$

This is a straightforward trigonometric equation involving tangent squared. Let's analyze and solve step-by-step.

Step 1: Isolate $\tan^2(x)$

$\tan^2(x) - 1 = 0$
=> $\tan^2(x) = 1$

Step 2: Take the square root of both sides

$\tan(x) = \pm\sqrt{1}$
=> $\tan(x) = \pm 1$

Step 3: Find x for tan x = 1 and tan x = -1

Recall the basic tangent values:
- $\tan x = 1$ at $x = \pi/4 + n\pi$
- $\tan x = -1$ at $x = -\pi/4 + n\pi$

where n is any integer, representing the periodic nature of tangent with period π.

Step 4: Write the general solutions

- For $\tan x = 1$:
$x = \pi/4 + n\pi$, where $n \in \mathbb{Z}$

- For $\tan x = -1$:
$x = -\pi/4 + n\pi$, where $n \in \mathbb{Z}$

Summary of solutions for Equation A

- $X = \pi/4 + n\pi$
- $X = -\pi/4 + n\pi$

Equation B: $2\cos^2(x) - 1 = 0$

This equation involves cosine squared. Let's analyze and solve.

Step 1: Isolate $\cos^2(x)$

$2\cos^2(x) - 1 = 0$
=> $2\cos^2(x) = 1$
=> $\cos^2(x) = 1/2$

Step 2: Take the square root of both sides

$\cos x = \pm\sqrt{(1/2)}$
=> $\cos x = \pm 1/\sqrt{2}$
=> $\cos x = \pm\sqrt{2}/2$

Step 3: Find x for $\cos x = \sqrt{2}/2$ and $\cos x = -\sqrt{2}/2$

- $\cos x = \sqrt{2}/2$ at $x = \pi/4 + 2n\pi$ and $x = 7\pi/4 + 2n\pi$
- $\cos x = -\sqrt{2}/2$ at $x = 3\pi/4 + 2n\pi$ and $x = 5\pi/4 + 2n\pi$

Alternatively, since cosine is positive in quadrants I and IV, and negative in II and III:

- $\cos x = \sqrt{2}/2$ at:
$x = \pi/4 + 2n\pi$
$x = 7\pi/4 + 2n\pi$

- $\cos x = -\sqrt{2}/2$ at:
$x = 3\pi/4 + 2n\pi$
$x = 5\pi/4 + 2n\pi$

where $n \in \mathbb{Z}$.

Step 4: Summarize solutions

- $X = \pi/4 + 2n\pi$
- $X = 3\pi/4 + 2n\pi$
- $X = 5\pi/4 + 2n\pi$
- $X = 7\pi/4 + 2n\pi$

Equation C: $2 \sin^2(x) + 15 \sin(x) + 7 = 0$

This is a quadratic in terms of sin(x). Let's approach it as an algebraic quadratic.

Step 1: Substitution

Let $y = \sin x$
Then, the equation becomes:
$2 y^2 + 15 y + 7 = 0$

Step 2: Solve the quadratic equation for y

Apply quadratic formula:
$y = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

where a=2, b=15, c=7.

Calculate discriminant:
$D = (15)^2 - 427 = 225 - 56 = 169$

Calculate square root:
$\sqrt{D} = \sqrt{169} = 13$

Find roots:
- $y = [-15 + 13] / (22) = (-2) / 4 = -1/2$
- $y = [-15 - 13] / (22) = (-28) / 4 = -7$

Step 3: Check for valid sine values

Since sin x must be between -1 and 1:

- For $y = -1/2$: Valid, as $-1 \leq -1/2 \leq 1$
- For $y = -7$: Not valid, since outside the range of sine

Therefore, only $y = -1/2$ is valid.

Step 4: Find x such that sin x = -1/2

Standard solutions:
- $\sin x = -1/2$ at $x = 3\pi/2 + 2n\pi$ (but check the specific angles)
- Actually, $\sin x = -1/2$ at:
- $x = 7\pi/6 + 2n\pi$
- $x = 11\pi/6 + 2n\pi$

since these are the angles where sine equals -1/2.

Summary of solutions for Equation C

- $X = 7\pi/6 + 2n\pi$
- $X = 11\pi/6 + 2n\pi$

Summary of Solutions and Tips for Solving Trigonometric Equations

Equation	Solutions	Domain Considerations
A) $\tan^2(x) - 1 = 0$	$x = \pi/4 + n\pi$, $x = -\pi/4 + n\pi$	All real x, periodic with π
B) $2\cos^2(x) - 1 = 0$	$x = \pi/4 + 2n\pi$, $3\pi/4 + 2n\pi$, $5\pi/4 + 2n\pi$, $7\pi/4 + 2n\pi$	All real x, period 2π
C) $2\sin^2(x) + 15\sin(x) + 7 = 0$	$x = 7\pi/6 + 2n\pi$, $11\pi/6 + 2n\pi$	All real x, period 2π

Tips for Solving Trigonometric Equations:
- Always attempt to rewrite equations into basic forms involving sine or cosine.
- Use identities to simplify complex expressions.
- When quadratic in sine or cosine appears, solve as algebraic quadratics.
- Check the validity of solutions within the domain of sin x and cos x.
- Remember the periodicity of trig functions to write the general solutions.

Conclusion

Mastering how to solve trigonometric equations is crucial for students and professionals working with mathematical models. The key steps involve algebraic manipulation, applying identities, and understanding the periodic nature of trig functions. Whether it's solving for tangent, sine, or cosine, the methods remain consistent: isolate the trig function, solve algebraically, and then interpret solutions within the context of their

Frequently Asked Questions

How do you solve the equation tan^2(x) - 1 = 0 for x?

Set $\tan^2(x) = 1$, which implies $\tan(x) = \pm 1$. Therefore, x = arctangent of ±1, leading to solutions $x = \pi/4 + n\pi$ and $x = 3\pi/4 + n\pi$, where n is any integer.

What are the solutions to 2 Cos^2(x) - 1 = 0?

Rewrite as $\cos^2(x) = 1/2$. Taking square roots, $\cos(x) = \pm\sqrt{1/2} = \pm 1/\sqrt{2} = \pm\sqrt{2}/2$. So, $x = \pi/4 + n\pi$ and $x = 3\pi/4 + n\pi$.

How do you solve the quadratic equation 2 Sin^2(x) + 15 Sin(x) + 7 = 0?

Treat Sin(x) as a variable, say y. The quadratic becomes $2y^2 + 15y + 7 = 0$. Solve for y using the quadratic formula: $y = [-15 \pm \sqrt{225 - 56}] / 4$. Then, find x from Sin(x) = y, considering the domain.

What is the general solution for tan(x) = 1?

Since $\tan(x) = 1$ at $x = \pi/4 + n\pi$, the general solutions are $x = \pi/4 + n\pi$, where n is any integer.

How can you find all solutions to Cos(x) = -√2/2?

$\cos(x) = -\sqrt{2}/2$ at $x = 3\pi/4 + 2n\pi$ and $x = 5\pi/4 + 2n\pi$, where n is any integer, covering all quadrants where cosine is negative.

What are the solutions for Sin(x) when solving 2 Sin^2(x) + 15 Sin(x) + 7 = 0?

Using the quadratic formula, the solutions for y = Sin(x) are $y = [-15 \pm \sqrt{225 - 56}] / 4$, which simplifies to $y = [-15 \pm \sqrt{169}] / 4 = [-15 \pm 13] / 4$. Therefore, $y = (-15 + 13)/4 = -0.5$ and $y = (-15 - 13)/4 = -7$. Both are valid if within the sine range [-1, 1], so only y = -0.5 is acceptable. Then, $x = \arcsin(-0.5) + 2n\pi$ and $x = \pi - \arcsin(-0.5) + 2n\pi$.

Why do we include nπ or 2nπ when expressing the solutions to these trigonometric equations?

Because trigonometric functions are periodic, their solutions repeat every π or 2π radians, so we include nπ or 2nπ to represent all possible solutions.

Additional Resources

Trigonometry Solve For X: An In-Depth Investigation into Fundamental Equations and Solutions

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Introduction

The realm of trigonometry, a branch of mathematics that deals with the relationships between the angles and sides of triangles, extends far beyond simple geometric concepts. It serves as the backbone for various scientific and engineering disciplines, including physics, astronomy, and signal processing. At the heart of many trigonometric applications lies the challenge of solving equations involving trigonometric functions for unknown angles, often denoted as x.

This comprehensive review focuses on three fundamental trigonometric equations—each presenting unique characteristics and solution strategies. These are:

- A) $\tan^2(x) - 1 = 0$
- B) $2 \cos^2(x) - 1 = 0$
- C) $2 \sin^2(x) + 15 \sin(x) + 7 = 0$

Through meticulous analysis, this article explores the methods to solve these equations, discusses their solutions in different contexts, and elucidates their significance in broader mathematical and applied fields.

The Significance of Solving Trigonometric Equations

Before delving into specific equations, it is essential to understand why solving trigonometric equations is crucial. These equations often model periodic phenomena such as sound waves, electromagnetic signals, and planetary motions. Moreover, they underpin calculus techniques, Fourier analysis, and differential equations.

Key aspects of solving trigonometric equations include:

- Determining all possible angles within a given interval, often $[0, 2\pi)$ or $[0°, 360°)$.
- Understanding the periodic nature of trigonometric functions.
- Applying identities to simplify complex expressions.
- Recognizing the domain restrictions and multiple solutions inherent in these functions.

Equation A: $\tan^2(x) - 1 = 0$

Derivation and Solution Strategy

Equation A presents a quadratic form in terms of $\tan(x)$:

$$
\tan^2(x) - 1 = 0
$$

This can be rearranged as:

$$
\tan^2(x) = 1
$$

Taking square roots gives:

$$
\tan(x) = \pm 1
$$

The solutions for $\tan(x) = 1$ and $\tan(x) = -1$ are fundamental since tangent has a period of π, and its values repeat every π radians.

Solving for x

1. For $\tan(x) = 1$:

$$
x = \arctan(1) + n\pi = \frac{\pi}{4} + n\pi
$$

2. For $\tan(x) = -1$:

$$
x = \arctan(-1) + n\pi = -\frac{\pi}{4} + n\pi
$$

To express solutions within the principal interval $[0, 2\pi)$:

- $x = \frac{\pi}{4}$
- $x = \frac{5\pi}{4}$ (adding π to $\pi/4$)
- $x = \frac{7\pi}{4}$ (adding π to $-\frac{\pi}{4}$)
- $x = \frac{3\pi}{4}$ (adding π to $\frac{\pi}{4}$, considering the negative solution)

Summary of solutions in $[0, 2\pi)$:

- $x = \frac{\pi}{4}$
- $x = \frac{3\pi}{4}$
- $x = \frac{5\pi}{4}$
- $x = \frac{7\pi}{4}$

Visual and Conceptual Understanding

Graphically, $\tan(x)$ exhibits periodicity and asymptotes at $x = \frac{\pi}{2} + n\pi$. The equation $\tan^2(x) = 1$ corresponds to points where the tangent function intersects lines $y = 1$ and $y = -1$. These intersections recur every π radians, reflecting the periodic nature of tangent.

Equation B: $2 \cos^2(x) - 1 = 0$

Recasting and Solution Approach

Equation B involves the cosine squared function:

$$
2 \cos^2(x) - 1 = 0
$$

Rearranged as:

$$
2 \cos^2(x) = 1
$$

$$
\cos^2(x) = \frac{1}{2}
$$

Taking square roots:

$$
\cos(x) = \pm \frac{\sqrt{2}}{2}
$$

Recognizing that $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$, solutions correspond to angles where cosine equals $\pm \frac{\sqrt{2}}{2}$.

Solving for x

The general solutions are:

- For $\cos(x) = \frac{\sqrt{2}}{2}$:

$$
x = \pm \frac{\pi}{4} + 2n\pi
$$

- For $\cos(x) = -\frac{\sqrt{2}}{2}$:

$$
x = \pi \pm \frac{\pi}{4} + 2n\pi
$$

Within the interval $[0, 2\pi)$, the solutions are:

- $x = \frac{\pi}{4}$
- $x = \frac{7\pi}{4}$
- $x = \frac{3\pi}{4}$
- $x = \frac{5\pi}{4}$

Summary of solutions:

Solution	Value (radians)	Degrees
$\frac{\pi}{4}$	0.785	45°
$\frac{3\pi}{4}$	2.356	135°
$\frac{5\pi}{4}$	3.927	225°
$\frac{7\pi}{4}$	6.108	315°

Geometric Interpretation

Cosine's positive and negative values correspond to points on the unit circle where the x-coordinate (projection on the x-axis) is $\pm \frac{\sqrt{2}}{2}$. These points are symmetric with respect to the y-axis, reflecting cosine's even symmetry.

Equation C: $2 \sin^2(x) + 15 \sin(x) + 7 = 0$

Recognizing Quadratic Form

Equation C is quadratic in $\sin(x)$:

$$
2 \sin^2(x) + 15 \sin(x) + 7 = 0
$$

Let $y = \sin(x)$. The equation becomes:

$$
2 y^2 + 15 y + 7 = 0
$$

Solving for y

Applying the quadratic formula:

$$
y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=2$, $b=15$, $c=7$:

$$
\Delta = b^2 - 4ac = 225 - 56 = 169
$$

$$
y = \frac{-15 \pm \sqrt{169}}{4}
$$

$$
y = \frac{-15 \pm 13}{4}
$$

Thus,

- First solution:

$$
y = \frac{-15 + 13}{4} = \frac{-2}{4} = -\frac{1}{2}
$$

- Second solution:

$$
y = \frac{-15 - 13}{4} = \frac{-28}{4} = -7
$$

Since $\sin(x)$ must be within $[-1, 1]$, the value $y = -7$ is invalid.

Valid solution:

$$
\sin(x) = -\frac{1}{2}
$$

Finding x in $[0, 2\pi)$

The sine function equals $-\frac{1}{2}$ at:

$$
x = \frac{7\pi}{6}, \quad \frac{11\pi}{6}
$$

which correspond to 210° and 330°. These are the solutions within one period.

Summary of solutions:

Solution in radians	Degrees	Description
$\frac{7\pi}{6}$	210°	Third quadrant
$\frac{11\pi}{6}$	330°	Fourth quadrant

Broader Implications and Applications

Contextualizing Solutions in Scientific Fields

The solutions derived from these equations have real-world counterparts. For example, equations involving $\tan(x)$, $\cos(x)$, and $\sin(x)$ appear in:

- Wave analysis: Understanding phase shifts and amplitudes.

- Electrical engineering: Signal processing involving sinusoidal functions.
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