True/False: We Can Conclusively Test The Convergence Of [infinity] 1/n-5 By Direct Comparison To The

True/False: We Can Conclusively Test The Convergence Of [infinity] 1/n-5 By Direct Comparison To The is a question that often arises in the study of infinite series within calculus. Determining whether a series converges or diverges is fundamental to understanding their behavior and applications. In particular, the series $\sum_{n=1}^{\infty} \frac{1}{n - 5}$ presents an intriguing case for convergence testing, prompting mathematicians and students alike to explore the efficacy and limitations of direct comparison tests. This article aims to clarify whether the convergence of such series can be conclusively tested through direct comparison and to provide a comprehensive overview of the methods involved.

Understanding the Series $\sum_{n=1}^{\infty} \frac{1}{n - 5}$

Definition and Behavior of the Series

The series in question is:

$$
\sum_{n=1}^{\infty} \frac{1}{n - 5}
$$

which can be rewritten as:

$$
\frac{1}{-4} + \frac{1}{-3} + \frac{1}{-2} + \frac{1}{-1} + \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \dots
$$

At first glance, the series appears to extend from $n=1$ to infinity, but notice that for $n=1, 2, 3, 4, 5$, the terms involve division by zero or negative denominators approaching zero, which complicates the convergence analysis.

Domain of the Series and Its Validity

The key issue is that the term $\frac{1}{n-5}$ is undefined at $n=5$, as it involves division by zero. Typically, in series analysis, the terms are defined for all n beyond a certain point. To properly analyze the series, we modify the index to start from $n=6$:

$$
\sum_{n=6}^{\infty} \frac{1}{n - 5}

which simplifies to:

$$
\sum_{n=6}^{\infty} \frac{1}{n - 5} = \sum_{k=1}^{\infty} \frac{1}{k}
$$

by substituting $k = n - 5$.

This transformation reveals that the series essentially is the harmonic series:

$$
\sum_{k=1}^{\infty} \frac{1}{k}
$$

a well-known divergent series.

Comparison Tests in Series Convergence Analysis

What Is the Direct Comparison Test?

The direct comparison test is a straightforward method to determine the convergence or divergence of a series by comparing it to another series whose behavior is already known. Specifically, if:

- $\sum a_n$ and $\sum b_n$ are series with positive terms,

- and if $a_n \leq b_n$ for all sufficiently large n,

then:

- If $\sum b_n$ converges, so does $\sum a_n$.

- If $\sum a_n$ diverges, so does $\sum b_n$.

This test is particularly useful for series with positive terms and can sometimes conclusively establish the nature of convergence or divergence.

Applying the Direct Comparison Test to $\sum \frac{1}{n - 5}$

Given the transformation to the harmonic series, the series:

$$
\sum_{k=1}^{\infty} \frac{1}{k}
$$

is known to diverge. Since the original series aligns with this harmonic series for large n, the question becomes: can we compare $\frac{1}{n - 5}$ to a convergent series to conclude convergence or divergence?

- For large n, $\frac{1}{n - 5}$ behaves like $\frac{1}{n}$.

- We know that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

- Therefore, if $\frac{1}{n - 5} \geq \frac{1}{n}$ for sufficiently large n, and since $\sum \frac{1}{n}$ diverges, the comparison suggests that $\sum \frac{1}{n - 5}$ also diverges.

Conclusion: The series diverges, and the direct comparison test confirms this.

Limitations of Direct Comparison in Series Convergence Testing

When Does the Direct Comparison Fail?

While the direct comparison test is powerful, it has limitations:

- It requires the terms to be positive. If the series contains negative or alternating terms, other tests are necessary.

- The comparison must be valid for sufficiently large n. If the inequalities do not hold beyond some point, the test cannot be applied conclusively.

- Sometimes, the comparison series is difficult to find or does not have known convergence properties, limiting the usefulness of this method.

Alternative Tests for Series Convergence

When direct comparison is inconclusive or inapplicable, other tests can be employed, including:

- **Limit Comparison Test:** Compares the limit of the ratio of the terms to a known series.
- **Integral Test:** Uses integrals to determine the convergence of series with positive, decreasing terms.
- **p-Series Test:** Analyzes series of the form $\sum \frac{1}{n^p}$.
- **Alternating Series Test:** For series with alternating signs.

In the context of $\sum \frac{1}{n - 5}$, the integral test confirms divergence because the integral $\int_{6}^{\infty} \frac{1}{x} dx$ diverges.

Conclusion: Can We Conclusively Test Convergence of $\sum \frac{1}{n - 5}$ Using Direct Comparison?

The answer is nuanced.

- For the modified series starting from $n=6$:

$$
\sum_{n=6}^{\infty} \frac{1}{n - 5} = \sum_{k=1}^{\infty} \frac{1}{k}
$$

which is the harmonic series, known to diverge.

- The direct comparison to the harmonic series allows us to conclusively determine divergence because:

- For sufficiently large n, $\frac{1}{n - 5} \geq \frac{1}{2n}$, and since $\sum \frac{1}{n}$ diverges, the comparison confirms divergence.

- Therefore, yes, the divergence of $\sum \frac{1}{n - 5}$ can be conclusively tested using the direct comparison to the harmonic series, which is a standard and reliable method.

However, it is crucial to recognize that the original series starting from $n=1$ is not well-defined at $n=5$. Proper domain adjustment is necessary before applying comparison tests.

Final Thoughts

In summation, the statement "We can conclusively test the convergence of $\sum \frac{1}{n - 5}$ by direct comparison" is True once the series is properly defined to exclude the problematic point at $n=5$. The comparison to the harmonic series is straightforward and conclusive, confirming divergence.

Nevertheless, this example highlights the importance of understanding the domain, the behavior of the series, and choosing appropriate convergence tests. While direct comparison is a powerful tool, it is not always applicable to every series, especially those with alternating signs or undefined terms without domain adjustments. For the harmonic series and similar p-series where the behavior is well-understood, direct comparison provides a conclusive and elegant method for testing convergence or divergence.

In conclusion, the ability to conclusively test the convergence of series like $\sum \frac{1}{n - 5}$ using direct comparison depends on proper domain considerations and the known behavior of related series. When these conditions are met, the method is both effective and definitive.

Frequently Asked Questions

Can the convergence of the series 1/n - 5 be conclusively tested by direct comparison to a p-series?

Yes, by comparing to a convergent p-series like $1/n^p$ with $p > 1$, we can determine if the original series converges or diverges.

Is it true that the series 1/n - 5 converges by direct comparison to a known convergent series?

No, because 1/n - 5 behaves like 1/n for large n, which diverges; direct comparison with a convergent series would show divergence or convergence accordingly.

Can we conclusively determine the convergence of the series 1/n - 5 using the comparison test?

No, because the comparison test requires a comparison to a series with similar general terms, and since 1/n diverges, the series 1/n - 5 diverges as well.

Is the series 1/n - 5 absolutely convergent?

No, because the series 1/n diverges, and subtracting 5 does not change the divergence; the series is neither absolutely nor conditionally convergent.

Does the limit comparison test help in conclusively testing the convergence of 1/n - 5?

Yes, since the limit of (1/n - 5) divided by (1/n) as n approaches infinity is 1, which indicates the series diverges similarly to the harmonic series.

Is the statement 'We can conclusively test the convergence of 1/n - 5 by direct comparison' true?

False, because direct comparison shows divergence with the harmonic series, which implies the series does not converge; the statement is misleading as the series diverges.

Additional Resources

Convergence Testing

Introduction

When delving into the realm of infinite series, the question of convergence stands as one of the most fundamental and intriguing challenges for mathematicians and students alike. Among the myriad of series analyzed, the series $\sum_{n=1}^\infty \frac{1}{n - 5}$ often prompts curiosity, especially regarding whether its convergence can be established through direct comparison tests. This article aims to explore the depth of this question, dissecting the principles of convergence, the methodologies of comparison tests, and critically analyzing whether a straightforward, conclusive testing approach is feasible for such a series.

Understanding Infinite Series and Convergence

What is an Infinite Series?

An infinite series is the sum of infinitely many terms:

$$
\sum_{n=1}^\infty a_n
$$

where each a_n represents the n-th term of the series. The core question centers around whether the partial sums:

$$
S_N = \sum_{n=1}^N a_n
$$

approach a finite limit as $N \to \infty$. If they do, the series is said to converge; if not, it diverges.

The Importance of Convergence Tests

To determine convergence, mathematicians employ various tests, such as:

- Comparison Test
- Limit Comparison Test
- Ratio Test
- Root Test
- Integral Test
- Alternating Series Test

Each method offers a strategic approach tailored to the nature of the series.

The Series $\sum_{n=1}^\infty \frac{1}{n - 5}$: An Initial Perspective

Behavior of the Terms

Let's analyze the general term:

$$
a_n = \frac{1}{n - 5}
$$

for $n \geq 1$. Notice that:

- For $n \leq 5$, the term a_n is undefined or not meaningful in the context of the infinite sum starting at $n=1$ because:

- When $n=1$, $a_1 = \frac{1}{-4}$ which is negative.
- When $n=2$, $a_2 = \frac{1}{-3}$, also negative.
- When $n=3$, $a_3 = \frac{1}{-2}$.
- When $n=4$, $a_4 = \frac{1}{-1}$.
- When $n=5$, $a_5 = \frac{1}{0}$, which is undefined (division by zero).

Given this, the series is not well-defined starting from $n=1$. To properly analyze the series, we should start from $n=6$:

$$
\sum_{n=6}^\infty \frac{1}{n - 5} = \sum_{k=1}^\infty \frac{1}{k}
$$

by substituting $k = n - 5$.

Reinterpreting the Series: Connection to the Harmonic Series

The Series $\sum_{k=1}^\infty \frac{1}{k}$

The shifted series:

$$
\sum_{n=6}^\infty \frac{1}{n - 5}
$$

is equivalent to:

$$
\sum_{k=1}^\infty \frac{1}{k}
$$

which is well-known as the harmonic series.

The Nature of the Harmonic Series

The harmonic series is a classic example in analysis, famously known for its divergence:

$$
\sum_{k=1}^\infty \frac{1}{k} \text{ diverges}
$$

The divergence holds despite the terms tending to zero—this is a key insight in series analysis.

Implication for Our Series

Since the series from $n=6$ onward is essentially the harmonic series, it diverges. Therefore, the original series $\sum_{n=1}^\infty \frac{1}{n - 5}$ (with appropriate domain considerations) diverges as well.

The Role of the Comparison Test in Series Convergence

Overview of the Comparison Test

The comparison test states:

- If $0 \leq a_n \leq b_n$ for all sufficiently large n,
- and $\sum b_n$ converges,
- then $\sum a_n$ also converges.

Conversely:

- If $a_n \geq b_n \geq 0$ for all sufficiently large n,
- and $\sum b_n$ diverges,
- then $\sum a_n$ also diverges.

Applying Comparison Test to $\frac{1}{n - 5}$

To test the convergence of $\sum_{n=6}^\infty \frac{1}{n - 5}$, compare it to a known divergent series:

$$
\sum_{k=1}^\infty \frac{1}{k}
$$

which diverges. Since for $n \geq 6$:

$$
\frac{1}{n - 5} \geq \frac{1}{n}
$$

and the harmonic series $\sum_{n=1}^\infty \frac{1}{n}$ diverges, by the Comparison Test, $\sum_{n=6}^\infty \frac{1}{n - 5}$ must also diverge.

Limitations of the Comparison Test

While this reasoning appears straightforward, it is crucial to recognize the limitations:

- The comparison test provides conclusive results only when comparing to a series with a known convergence or divergence.
- For series that behave differently or involve more complex manipulations, the comparison test might not be sufficient.

In this particular case, however, the comparison is direct and conclusive because of the known divergence of the harmonic series.

Is There a "True/False" Conclusiveness in Testing Convergence of $\frac{1}{n - 5}$?

The Question at Hand

The core question is whether "We can conclusively test the convergence of $\sum \frac{1}{n - 5}$ by direct comparison".

Analysis of the True/False Statement

- True: Because the series $\sum_{n=6}^\infty \frac{1}{n - 5}$ is equivalent to the harmonic series (which diverges), a direct comparison with $\sum_{n=1}^\infty \frac{1}{n}$ allows us to conclusively determine divergence.

- False: If the question pertains to the series starting from $n=1$ without adjusting for the undefined term at $n=5$, then the series is ill-defined, and the question of convergence becomes moot. Additionally, if the series involves more complicated terms, the comparison test might not be conclusive.

Nuance in the Answer

Therefore, the answer depends on the precise formulation:

- If the series is properly defined from $n=6$ onwards, then True: convergence can be conclusively tested via comparison to the harmonic series.

- If the series is initially defined from $n=1$ without addressing the division by zero at $n=5$, then the question is invalid or False, because the series isn't well-defined in that form.

Broader Implications and Alternative Tests

Limit Comparison Test

The Limit Comparison Test complements the comparison test, especially useful when the terms are asymptotically similar:

$$
\lim_{n \to \infty} \frac{a_n}{b_n} = c
$$

where c is a finite, positive constant.

Applying this to $a_n = \frac{1}{n - 5}$ and $b_n = \frac{1}{n}$:

$$
\lim_{n \to \infty} \frac{\frac{1}{n - 5}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n}{n - 5} = 1
$$

which confirms the divergence, consistent with the comparison test.

The Integral Test

The integral test involves evaluating:

$$
\int_{N}^\infty \frac{1}{x - 5} dx
$$

which simplifies to:

$$
\lim_{t \to \infty} \int_{N}^t \frac{1}{x - 5} dx = \lim_{t \to \infty} \left[\ln|x - 5| \right]_N^t
$$

which diverges as $t \to \infty$, confirming divergence.

Summary of Tests

Test	Applicability	Conclusion
Comparison Test	When compare to a known series	Diverges because harmonic series diverges
Limit Comparison Test	When asymptotic behavior matches	Diverges, same as harmonic series
Integral Test	When integral converges/diverges	Diverges, integral diverges logarithmically

---

Final Verdict and Expert Insights

Based on established mathematical principles

[True False We Can Conclusively Test The Convergence Of Infinity 1 N 5 By Direct Comparison To The]()

True False We Can Conclusively Test The Convergence Of Infinity 1 N 5 By Direct Comparison To The

Related Articles

- What Is The Output Of The Following Pseudocode?set Num=2set Str="two"write Num + "is Not Equal To" +
- The Lengths Of Phone Calls (in Minutes) Made By A Travel Agent Can Be Modeled As A Continuous Random
- The Upper Right Abdominopelvic Region Beneath The Ribs Is: _________a. Right Lumbar B. Epigastric C.