# Two Conducting Spheres Of Different Radii, As Shown In The Figure, Each Have Charge -Q. Which Of The

## Two Conducting Spheres Of Different Radii, As Shown In The Figure, Each Have Charge -Q. Which Of The

When analyzing electrostatics problems involving conducting spheres, one common scenario involves two spheres of different radii, each initially carrying an equal charge. Understanding how charges distribute themselves on conductors of varying sizes is fundamental in electrostatics, with applications in capacitor design, shielding, and electrostatic induction. This article explores the principles governing the charge distribution on two conducting spheres of different radii, both initially charged with -Q, and investigates the resulting equilibrium state when they are brought into proximity or connected, either directly or through a conducting wire.

Introduction to Conducting Spheres and Electrostatics

Conducting spheres serve as ideal models in electrostatics because their free electrons can move freely, allowing the charge to distribute uniformly over their surfaces in equilibrium. When two such conductors are placed near each other or connected, they influence each other's charge distribution through electrostatic induction and Coulomb forces.

Basic Principles

- Charge Distribution on Conductors: In electrostatics, the charge on a conductor resides on its surface. For a sphere, the charge distributes uniformly if isolated; however, when multiple conductors interact, the distribution becomes uneven and depends on their sizes and positions.
- Electrostatic Equilibrium: Charges rearrange themselves until the electric potential on each conductor's surface is constant. This is the condition for electrostatic equilibrium.
- Conservation of Charge: The total charge on a conductor or a system remains constant unless there's an external charge transfer.

Physical Setup and Assumptions

Consider two conducting spheres:

- Sphere 1 with radius $R_1$
- Sphere 2 with radius $R_2$

Each initially carries a charge of $-Q$. The spheres are either:

- Brought into contact and then separated, or
- Connected via a conducting wire, allowing charge redistribution to reach equilibrium.

The goal is to determine:

1. How the charges distribute on each sphere after equilibrium.
2. The potential on each sphere.

3. How the difference in radii influences the final charge distribution.

Potential and Charge Distribution on a Single Conducting Sphere

For a solitary conducting sphere of radius $R$ carrying a charge $q$:

$$
V = \frac{1}{4 \pi \varepsilon_0} \frac{q}{R}
$$

where:

- $V$ is the potential of the sphere,
- $\varepsilon_0$ is the permittivity of free space.

This relation indicates that larger spheres with the same charge have lower potential, and vice versa.

Interaction of Two Conducting Spheres

When two spheres are near each other, their charges influence each other through their electric fields. The key factors are:

- The initial charges ($-Q$ on both),
- The distance $d$ between their centers,
- The radii $R_1$ and $R_2$.

When Connected by a Conducting Wire

Connecting the two spheres with a conducting wire allows charge to flow until they reach a common potential. This process obeys:

$$
V_1 = V_2
$$

where $V_1$ and $V_2$ are the potentials of spheres 1 and 2, respectively.

Charge Redistribution

Since the initial charges are both $-Q$, the total charge remains:

$$
Q_{\text{total}} = -Q - Q = -2Q
$$

After redistribution, the charges on the spheres are:

$$
q_1, \quad q_2
$$

with:

$$
q_1 + q_2 = -2Q
$$

and the potentials:

$$
V_1 = \frac{1}{4 \pi \varepsilon_0} \frac{q_1}{R_1}
$$

$$
V_2 = \frac{1}{4 \pi \varepsilon_0} \frac{q_2}{R_2}
$$

The equilibrium condition:

$$
V_1 = V_2
$$

leads to:

$$
\frac{q_1}{R_1} = \frac{q_2}{R_2}
$$

which simplifies to:

$$
q_2 = q_1 \frac{R_2}{R_1}
$$

Using the total charge conservation:

$$
q_1 + q_2 = -2Q
$$

substituting $q_2$:

$$
q_1 + q_1 \frac{R_2}{R_1} = -2Q
$$

$$
q_1 \left( 1 + \frac{R_2}{R_1} \right) = -2Q
$$

$$
q_1 = -2Q \frac{1}{1 + \frac{R_2}{R_1}} = -2Q \frac{R_1}{R_1 + R_2}
$$

Similarly,

$$
q_2 = -2Q \frac{R_2}{R_1 + R_2}
$$

## Final Charge on Each Sphere

$$
\boxed{
\begin{aligned}
q_1 &= -2Q \frac{R_1}{R_1 + R_2} \\
q_2 &= -2Q \frac{R_2}{R_1 + R_2}
\end{aligned}
}
$$

This result demonstrates that larger spheres acquire a greater share of the total charge magnitude.

## Physical Interpretation and Implications

- Since both spheres initially have charge $-Q$, the total charge is $-2Q$.
- When connected, charges redistribute such that each sphere's potential is equal.
- The sphere with the larger radius ends up with a larger magnitude of charge in the negative direction.
- The charge distribution is proportional to the radii, indicating size plays a crucial role in electrostatic behavior.

## Effect of Different Initial Charges and Non-Connected Spheres

If the initial charges are different or the spheres are not connected, the analysis becomes more complex. Factors such as:

- Induced charges,
- Mutual polarization,
- Potential differences,

must be considered.

In such cases, methods like the method of images, multipole expansion, or numerical simulation are employed to analyze the charge distribution.

## Practical Applications and Examples

### Capacitor Design

Two conducting spheres of different sizes can serve as model capacitors, where the charge distribution affects capacitance and potential. Understanding how charge distributes helps optimize energy storage.

### Electrostatic Shielding

Large spheres can shield smaller objects from external fields by distributing charge in a way that cancels internal fields, a principle used in Faraday cages.

## Induction and Charge Transfer

When conducting spheres of different radii are brought into contact or proximity, charges may transfer or induce surface charges, affecting electrical and electrostatic properties in devices.

## Experimental Verification

Experimental setups verifying these principles involve:

- Conducting spheres connected with wires,
- Applying known charges,
- Measuring surface potentials and charge distributions using electrometers or field meters.

Such experiments confirm theoretical predictions and help refine models of electrostatic behavior.

## Summary of Key Points

- When two conducting spheres of different radii are connected, charges redistribute to equalize potential.
- The final charge on each sphere is proportional to its radius.
- The larger sphere acquires a greater share of the total charge magnitude.
- Initial charges influence the final distribution, but the size ratio is a dominant factor in equilibrium.
- These principles are fundamental in understanding electrostatics, capacitor behavior, and electrostatic shielding.

## Conclusion

The behavior of two conducting spheres of different radii, each initially charged with $-Q$, exemplifies core electrostatic principles such as charge conservation, potential equalization, and the influence of geometry on charge distribution. Recognizing how size affects charge sharing enables engineers and physicists to design better electrical devices, from capacitors to shielding systems, and enhances our understanding of electrostatic phenomena in both theoretical and practical contexts.

## References

- Griffiths, D. J. Introduction to Electrodynamics. 4th Edition. Pearson, 2013.
- Halliday, D., Resnick, R., and Walker, J. Fundamentals of Physics. 10th Edition. Wiley, 2014.
- Serway, R. A., and Jewett, J. W. Physics for Scientists and Engineers. 9th Edition. Cengage Learning, 2014.

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Note: For visual clarity, diagrams illustrating the setup, charge distribution, and potential lines are highly recommended in a full article but are omitted here due to format constraints.

# Frequently Asked Questions

## How does the electric field differ between two conducting spheres of different radii, both charged with the same charge -Q?

The electric field on the surface of each sphere depends on its radius; the smaller sphere experiences a higher surface charge density, leading to a stronger electric field at its surface compared to the larger sphere.

## Which sphere has a higher potential: the smaller or the larger conducting sphere, given both have charge -Q?

The larger sphere has a higher potential because, for a given charge, the potential increases with the radius of the conductor.

## Does the charge distribution on each sphere remain uniform, and how does the radius affect this?

Yes, since both are conductors and isolated, the charge distributes uniformly on each sphere's surface. The sphere's radius does not affect the uniformity but influences the surface charge density.

## If the two spheres are brought close to each other, how do their charges interact?

They will influence each other's charge distribution through induction, potentially causing a redistribution of charges to minimize the system's energy, but the total charges on each sphere remain -Q.

## What is the potential energy stored in each sphere, and how is it related to their radii?

The potential energy in each sphere is proportional to $Q^2$ divided by its radius; thus, the smaller sphere (with smaller radius) has higher potential energy for the same charge.

## How does the capacitance of each conducting sphere compare, and what is its relation to the radius?

The capacitance of a sphere is directly proportional to its radius ($C = 4\pi\varepsilon_0 r$); hence, the larger sphere has greater capacitance than the smaller one.

## Can the two spheres exchange charge, and under what

## conditions?

If the spheres are connected by a conducting path or close enough to influence each other, charge can flow until electrostatic equilibrium is reached, typically resulting in a redistribution to maintain the same potential.

## Which sphere will experience a greater electrostatic force if placed near another charged object, and why?

The smaller sphere, with a higher surface charge density and electric field, will generally experience a stronger electrostatic force when near another charged object.

# Additional Resources

Two Conducting Spheres Of Different Radii, Each Have Charge -Q: An Investigative Analysis

In the realm of electrostatics, the behavior of conductors under the influence of charge distributions provides fundamental insights into electric fields, potential, and charge redistribution. Among classic problems, the scenario involving two conducting spheres of different radii, each initially charged with an equal magnitude but opposite in sign (−Q), presents both theoretical intrigue and practical significance. This investigative review aims to analyze the electrostatic properties, charge distributions, and potential interactions in such a system, offering a comprehensive understanding rooted in classical electrostatics.

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# Introduction to Conducting Spheres in Electrostatics

Conducting spheres are idealized objects often used to elucidate principles of electrostatics due to their symmetry and well-understood behavior. When charged, free electrons within conductors redistribute themselves to minimize the system's electrostatic energy, typically resulting in a uniform potential throughout each sphere. The interaction between two such spheres depends on their charges, sizes, initial conditions, and the distance separating them.

In this specific scenario, we consider two conducting spheres of different radii, labeled as Sphere 1 with radius $R_1$ and Sphere 2 with radius $R_2$, each initially charged with charge −Q. The key questions involve understanding how their charges and potentials evolve when they are brought into proximity, how the electric fields behave, and what the resulting charge distributions look like after equilibrium is achieved.

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# Background and Theoretical Foundations

## Electrostatics of Isolated Conducting Spheres

For a single isolated conducting sphere of radius R carrying a total charge Q, the potential V at its surface is given by:

$V = Q / (4\pi\varepsilon_0 R)$

where $\varepsilon_0$ is the permittivity of free space. The charge resides on the surface, creating an electric field outside the sphere analogous to that of a point charge Q located at its center.

## Interaction of Two Conducting Spheres

When two conducting spheres are brought into proximity, their electric fields interact. If both are charged, the field lines emanate from positive charges and terminate on negative charges, influencing each other's charge distribution. The problem becomes more complex when the spheres are of different sizes, due to the asymmetric influence of their electric fields.

The key principles governing their behavior include:

- Electrostatic Equilibrium: Each conducting sphere maintains a constant potential throughout its volume.
- Charge Redistribution: Charges may move between the spheres and within each sphere to reach equilibrium.
- Induced Charges: The presence of one sphere induces a charge distribution on the other, especially when they are close.

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# System Description and Initial Conditions

The system under consideration involves:

- Sphere 1: Radius $R_1$, charge $-Q$, initially isolated.
- Sphere 2: Radius $R_2$, charge $-Q$, initially isolated.
- Separation Distance: d, the center-to-center distance between the spheres, with the assumption that $d > R_1 + R_2$ to prevent physical contact.

The initial conditions are such that each sphere carries a uniform charge distribution consistent with a total charge of $-Q$, and they are brought near each other without overlapping.

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# Charge Distribution and Potential Equilibrium Analysis

## Induced Charges and Polarization Effects

When the spheres are brought close, their mutual electric fields cause induced charge distributions:

- On Sphere 1: The negative charge tends to repel charges within the sphere, but the presence of Sphere 2 induces positive charge regions on the side facing Sphere 2.
- On Sphere 2: Similar effects occur, leading to a non-uniform charge distribution, with regions of induced positive and negative charges.

The net charges on each sphere remain $-Q$ after equilibrium, but the local distributions are altered.

## Methodology for Charge and Potential Calculation

The complex nature of the problem often necessitates advanced computational techniques or approximation methods:

- Method of Images: Suitable when the spheres are small or the separation is large, but less accurate for finite-sized spheres at close distances.
- Multipole Expansion: Expanding the potential in spherical harmonics to account for higher-order interactions.
- Numerical Simulations: Finite element methods or boundary element methods (BEM) to solve for charge densities and potentials accurately.

In most practical analyses, the key is to determine the effective charge distribution, the resulting potential difference, and the net force between the spheres.

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# Electrostatic Potential and Force Computation

## Potential on the Surface of Each Sphere

Due to mutual induction, the potential on each sphere's surface deviates from the initial uniform potential. The final equilibrium potential $V_1$ on Sphere 1 and $V_2$ on Sphere 2 are related through boundary conditions:

- Each sphere's surface is an equipotential.
- The total charge on each sphere remains $-Q$.

The potentials are affected by the induced charges and can be expressed as:

$V_1 = (Q_1 + Q_{ind,1}) / (4\pi\varepsilon_0 R_1)$

$V_2 = (Q_2 + Q_{ind,2}) / (4\pi\varepsilon_0 R_2)$

where $Q_{ind,1}$ and $Q_{ind,2}$ are the induced charges due to the other sphere.

## Force Between the Spheres

The electrostatic force can be estimated using the concept of the interaction energy. For two point charges, Coulomb’s law applies directly:

$F = (1 / (4\pi\varepsilon_0)) |Q_1 Q_2| / d^2$

However, for finite conducting spheres, especially with induced charges, this approximation is inadequate. Instead, the force is derived from the energy stored in the system or from the Maxwell stress tensor evaluated on the surfaces.

- When the spheres are far apart ($d >> R_1, R_2$), the force approaches that of point charges.
- When the spheres are close, induced charges significantly alter the force, generally leading to an attractive interaction when charges are opposite.

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# Special Cases and Limiting Behaviors

## Equal and Opposite Charges

Since each sphere carries charge $-Q$, the total system charge is $-2Q$. When they are separated, the electrostatic potential energy is simply the sum of the self-energies plus the interaction energy.

- At large distances: The interaction energy diminishes as $1/d$.
- At close proximity: Induced charges lead to a strong attractive force, potentially resulting in charge redistribution and energy minimization.

## Equal Radii vs. Different Radii

- Equal radii: Symmetric charge distributions and potentials.
- Different radii: Asymmetry introduces complexity, with larger spheres exerting a more significant influence on the smaller one, leading to asymmetric charge distributions and different potential values.

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## Implications and Practical Significance

Understanding the electrostatic interactions between two conducting spheres of different radii with charges of the same magnitude but opposite signs has multiple applications:

- Electrostatic Precursors: In designing equipment like particle accelerators or electrostatic precipitators.
- Capacitor Design: For systems involving spherical conductors with varying sizes.
- Nanotechnology: Manipulating charged nanoparticles or colloids where size disparities are common.
- Fundamental Physics: Testing theories of charge redistribution and field interactions.

Moreover, this analysis helps refine models used in computational electromagnetics, providing insights into the subtleties of charge induction, potential distribution, and force calculations in asymmetric systems.

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## Conclusion

The investigation into two conducting spheres of different radii, each charged with −Q, reveals a rich tapestry of electrostatic phenomena driven by charge induction, potential redistribution, and mutual interaction. While the initial conditions suggest symmetrical charges, the asymmetry in size induces complex charge distributions and potential differences, ultimately influencing the force and energy of the system.

Through a combination of theoretical models, approximation techniques, and numerical simulations, it becomes possible to predict the behavior of such systems with high accuracy. These insights not only deepen our fundamental understanding of electrostatics but also underpin numerous practical applications across physics, engineering, and nanotechnology.

In essence, the electrostatic interplay between differently sized conducting spheres exemplifies the nuanced balance between symmetry, size effects, and charge interactions—an area ripe for ongoing research and technological innovation.

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