Two Tickets Are Drawn Without Replacement From The Following Box Of Tickets: 1, 1, 1, 2, 2, 3, 4, 5.

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This problem involves understanding probability, combinatorics, and the implications of drawing tickets without replacement. The set of tickets contains duplicates, which introduces interesting nuances in calculating probabilities and outcomes. In this article, we will explore the various probabilities associated with drawing two tickets, analyze the possible outcomes, and examine related concepts such as expected value and conditional probability.

Understanding the Setup

The Composition of the Ticket Box

The box contains a total of 8 tickets:
- Three tickets labeled 1
- Two tickets labeled 2
- One ticket labeled 3
- One ticket labeled 4
- One ticket labeled 5

This can be summarized as:

- Tickets labeled 1: 3
- Tickets labeled 2: 2
- Tickets labeled 3: 1
- Tickets labeled 4: 1
- Tickets labeled 5: 1

Total tickets: 8

Drawing Without Replacement

Drawing without replacement means once a ticket is drawn, it is not put back into the box. Therefore, the probabilities for the second draw depend on the first draw's outcome.

Possible Outcomes When Drawing Two Tickets

Number of Total Possible Outcomes

The total number of ways to draw two tickets without replacement from 8 tickets is:

1. Number of combinations: C(8, 2) = 8×7/2 = 28

Each of these 28 pairs represents a potential outcome.

Distinct Outcomes by Ticket Labels

Because tickets are labeled, outcomes can be categorized based on the labels of the drawn tickets.

- Same label outcomes:
- Both tickets labeled 1
- Both tickets labeled 2

- Different label outcomes:
- One ticket labeled 1 and the other labeled 2
- One ticket labeled 1 and another labeled 3
- One ticket labeled 1 and another labeled 4
- One ticket labeled 1 and another labeled 5
- One ticket labeled 2 and another labeled 3
- One ticket labeled 2 and another labeled 4
- One ticket labeled 2 and another labeled 5
- Ticket labeled 3 and 4
- Ticket labeled 3 and 5
- Ticket labeled 4 and 5

Calculating Probabilities of Specific Events

Probability of Drawing Two Tickets with the Same Label

Both tickets labeled 1

- Number of ways to select 2 tickets from the 3 labeled 1:
- C(3, 2) = 3
- Total outcomes: 28
- Probability:
\[
P(\text{both 1}) = \frac{3}{28}
\]

Both tickets labeled 2

- Number of ways to select 2 tickets from the 2 labeled 2:
- C(2, 2) = 1
- Probability:

$$
P(\text{both 2}) = \frac{1}{28}
$$

Both tickets with the same label (either 1 or 2)

- Sum of probabilities:

$$
P(\text{both same}) = \frac{3 + 1}{28} = \frac{4}{28} = \frac{1}{7}
$$

Probability of Drawing Tickets with Different Labels

- Since total outcomes are 28,
- Probability of different labels:

$$
P(\text{different labels}) = 1 - P(\text{both same}) = 1 - \frac{1}{7} = \frac{6}{7}
$$

Calculating Probabilities for Specific Label Pairs

Probability of Drawing a Ticket labeled 1 and a Ticket labeled 2

- Number of ways:
- Number of tickets labeled 1: 3
- Number of tickets labeled 2: 2
- Possible pairs:
- For the first ticket labeled 1, second labeled 2: 3×2 = 6
- For the first ticket labeled 2, second labeled 1: 2×3 = 6
- But since order doesn't matter in combinations, total unique pairs:
- 6 (from above calculations)
- Total pairs: 28
- Probability:

$$
P(\text{1 and 2}) = \frac{6 + 6}{28} = \frac{12}{28} = \frac{3}{7}
$$

Similarly, for other different label pairs, calculations follow the same pattern.

Expected Values and Probabilities of Specific Labels

Expected Value of the Label of the First Draw

The expected value (mean label) of the first draw is:

$$
E(\text{first label}) = \sum_{i} (\text{label value} \times P(\text{drawing that label}))
$$

Calculating the probability of drawing each label first:

- For label 1:
$$
P(\text{first ticket is 1}) = \frac{3}{8}
$$
- For label 2:
$$
P(\text{first ticket is 2}) = \frac{2}{8} = \frac{1}{4}
$$
- For label 3:
$$
P(\text{first ticket is 3}) = \frac{1}{8}
$$
- For label 4:
$$
P(\text{first ticket is 4}) = \frac{1}{8}
$$
- For label 5:
$$
P(\text{first ticket is 5}) = \frac{1}{8}
$$

Thus,

$$
E(\text{first label}) = 1 \times \frac{3}{8} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{8} + 5 \times \frac{1}{8}
$$
$$
= \frac{3}{8} + \frac{2}{4} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}
$$
$$
= \frac{3}{8} + \frac{2}{4} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}
$$
Converting all to eighths:
$$
= \frac{3}{8} + \frac{4}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} = \frac{3 + 4 + 3 + 4 + 5}{8} = \frac{19}{8} = 2.375
$$

Similarly, the expected value of the second draw can be computed, considering the updated probabilities after the first draw.

Conditional Probabilities and Expected Values

Impact of the First Draw on the Second Draw

Because tickets are drawn without replacement, the outcome of the first draw influences the probabilities of the second.

For example:
- If the first ticket drawn is labeled 1, then only 2 tickets labeled 1 remain, and total remaining tickets decrease to 7.
- The probabilities for the second draw depend on which ticket was drawn first.

Conditional Probability Example

Suppose the first ticket drawn was labeled 1:
- Remaining tickets:
- 2 tickets labeled 1
- 2 tickets labeled 2
- 1 ticket labeled 3
- 1 ticket labeled 4
- 1 ticket labeled 5

- Total remaining: 7
- Probability that the second ticket is labeled 1:

$$
P(\text{second label 1} | \text{first label 1}) = \frac{2}{7}
$$

Similarly, other conditional probabilities can be computed based on the initial outcome.

Summary and Conclusions

This problem highlights the importance of combinatorics and probability theory in understanding outcomes involving repeated elements and sampling without replacement. The presence of duplicate tickets affects the calculations of probabilities, especially for events involving identical labels.

Key takeaways include:
- Total possible outcomes are based on combinations, with 28 unique pairs.
- Probabilities for events are calculated by counting favorable outcomes over total outcomes.
- The presence of duplicates influences the likelihoods of drawing certain labels.
- Conditional probabilities are essential when considering the impact of previous draws.
- Expected values provide insight into typical outcomes and can be calculated by weighting labels according to their initial probabilities.

Analyzing such problems deepens understanding of probability distributions, combinatorial reasoning

Frequently Asked Questions

What is the probability that both tickets drawn are the number 1?

The probability is 1/7. There are 3 tickets labeled 1 out of 8 total tickets. Drawing two without replacement, the probability is (3/8) (2/7) = 6/56 = 3/28.

What is the probability that at least one ticket drawn is a 2?

First, find the probability that no 2s are drawn: tickets with no 2 are 1, 1, 1, 3, 4, 5 (6 tickets). The probability both are non-2: (6/8) (5/7) = 30/56 = 15/28. Therefore, the probability that at least one is a 2 is 1 - 15/28 = 13/28.

What is the probability that the sum of the two tickets drawn is greater than 5?

Possible pairs with sum > 5 are: (2,4), (2,5), (3,4), (3,5), (4,5), and (1,5). Calculating their probabilities yields a total of 7/28 or 1/4.

What is the probability that both tickets drawn are the same number?

Same number pairs are: (1,1), (2,2). Probability for (1,1): (3/8) (2/7) = 6/56 = 3/28. For (2,2): (2/8) (1/7) = 2/56 = 1/28. Total probability: 3/28 + 1/28 = 4/28 = 1/7.

What is the probability that the first ticket drawn is a 1 and the second is a 3?

Probability: (3/8) for first being 1, then (1/7) for second being 3 (remaining 1 ticket 3 out of 7 remaining). So, (3/8) (1/7) = 3/56.

If one ticket drawn is a 4, what is the probability that the other ticket is a 5?

Given that one ticket is 4, the remaining tickets are 1, 1, 1, 2, 2, 3, 5. The probability the other is 5 is 1/7, since only one 5 remains out of 7 remaining tickets.

Additional Resources

Two tickets are drawn without replacement from the following box of tickets: 1, 1, 1, 2, 2, 3, 4, 5. This seemingly simple scenario opens the door to a rich exploration of probability, combinatorics, and the nuances of drawing items without replacement. Whether you're a student delving into basic probability concepts or a professional analyzing complex sampling methods, understanding the intricacies of this problem provides foundational insights into how chance and combinatorial logic interplay.

Introduction to the Problem

Imagine a box filled with tickets labeled as follows: 1, 1, 1, 2, 2, 3, 4, 5. You draw two tickets without putting the first back, and you're interested in various probabilities associated with the outcomes—such as the likelihood that both tickets are the same, or that they are different, or the probability specific numbers appear.

This problem might seem straightforward at first glance, but the presence of duplicate tickets introduces interesting layers of complexity. It requires careful counting and understanding of how probabilities are affected by duplicate elements and the order of selection.

Understanding the Composition of the Ticket Box

Before diving into probabilities, it's essential to understand the composition of the ticket box:

Ticket Number	Quantity	Total Tickets
1	3	
2	2	8
3	1	
4	1	
5	1	

Total tickets in the box: 8

This composition impacts the calculation of probabilities because the likelihood of drawing a particular number depends on how many tickets of that number exist.

Fundamental Principles: Drawing Without Replacement

Drawing without replacement means once a ticket is drawn, it isn't put back into the box for the next draw. This affects probabilities because the total number of tickets decreases with each draw, and the composition of remaining tickets shifts accordingly.

Key points:
- Probabilities are conditional; the outcome of the first draw influences the second.
- The total number of possible outcomes for the two draws is combinations of 8 tickets taken 2 at a time, without regard to order—or, when considering order, permutations.

Total Possible Outcomes

When considering order (which ticket is drawn first):

- Number of ordered outcomes: $8 \times 7 = 56$

When considering unordered pairs:

- Number of combinations:

$$
\text{Number of combinations} = \binom{8}{2} = \frac{8 \times 7}{2} = 28
$$

In most probability discussions involving "drawing two tickets," the focus is on unordered pairs unless specified otherwise.

Calculating Specific Probabilities

Let's explore some common probability questions related to this setup.

1. Probability that both tickets are the same number

This is often a key question because duplicates influence the likelihood of matching outcomes.

Possible matching pairs:

- Both are 1s
- Both are 2s

Note: The tickets labeled 3, 4, 5 only appear once, so they can't form pairs with themselves.

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2. Probability that the two tickets are different

Complementary to the above, this is often equally interesting.

Step-by-Step Probability Calculations

A. Probability of Drawing Two Tickets with the Same Number

Step 1: Count the total number of unordered pairs

Total pairs: 28

Step 2: Count the favorable pairs

- Pair of 1s:

Number of ways to choose 2 tickets labeled 1:

\[

\binom{3}{2} = 3
\]

- Pair of 2s:

\[
\binom{2}{2} = 1
\]

Total favorable pairs:

\[
3 + 1 = 4
\]

Step 3: Compute the probability

\[
P(\text{both same}) = \frac{\text{favorable pairs}}{\text{total pairs}} = \frac{4}{28} = \frac{1}{7}
\]

Answer:

> The probability that both tickets drawn are the same number is 1/7.

B. Probability that both tickets are different

Since the total pairs are 28, and 4 are favorable for "both same," the remaining are for "both different."

\[
P(\text{both different}) = 1 - P(\text{both same}) = 1 - \frac{1}{7} = \frac{6}{7}
\]

Answer:

> The probability that the two tickets are different numbers is 6/7.

C. Probability that the first ticket is a specific number, say, 1

Since there are 3 tickets labeled 1 out of 8:

\[
P(\text{first ticket is 1}) = \frac{3}{8}
\]

D. Probability that the second ticket is a specific number, given the first ticket

This depends on what the first ticket was.

Case 1: First ticket is 1

- Remaining tickets: 2 ones left

- Total remaining tickets: 7

$$
P(\text{second ticket is 1} \mid \text{first was 1}) = \frac{2}{7}
$$

Case 2: First ticket is not 1 (say, 2, 3, 4, or 5)

- For example, if the first ticket is 3, remaining tickets:

- 1, 1, 2, 2, 4, 5 (total 7)

- Number of 1s remaining: 3 (unchanged, since first was 3, not 1)

- Number of 2s remaining: 2

- For 1:

$$
P(\text{second ticket is 1} \mid \text{first was 3}) = \frac{3}{7}
$$

Similarly, probabilities can be calculated based on which ticket was drawn first.

Exploring Conditional Probabilities and Expected Values

Understanding the likelihood of various outcomes involves delving into conditional probabilities, which specify the probability of an event given that another event has occurred.

1. Probability that both tickets are 1, given that at least one is 1

This involves calculating the probability that both are 1s conditioned on the event that at least one ticket is labeled 1. This is a classic application of conditional probability.

Advanced Topics: Expected Value and Variance

Beyond simple probabilities, one may be interested in the expected value (mean) of the ticket labels drawn, or the variance.

Expected value of a single draw:

$$
E(\text{single draw}) = \sum_{i} P(\text{drawing } i) \times i
$$

Calculations:

- $P(1) = \frac{3}{8}$

- $P(2) = \frac{2}{8} = \frac{1}{4}$

- $P(3) = \frac{1}{8}$

- $P(4) = \frac{1}{8}$

- $P(5) = \frac{1}{8}$

Expected value:

$$
E = 1 \times \frac{3}{8} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{8} + 5 \times \frac{1}{8}
$$

$$
E = \frac{3}{8} + \frac{2}{4} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}
$$

Simplify:

$$
E = \frac{3}{8} + \frac{2}{4} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}
$$

Convert all to eighths:

$$
E = \frac{3}{8} + \frac{4}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} = \frac{3 + 4 + 3 + 4 + 5}{8} = \frac{19}{8} = 2.375
$$

Real-World Applications and Implications

This problem isn't just a theoretical exercise; it has numerous applications:

- Quality control: Sampling items from a batch with duplicates.
- Card games: Drawing cards with duplicate values.
- Survey sampling: Selecting individuals with certain repeated characteristics.
- Inventory management: Handling items with multiple identical units.

Understanding how to compute probabilities with duplicates and without replacement enhances decision-making under uncertainty and informs strategies in various fields.

Summary of Key Takeaways

- The total number of possible unordered pairs from 8 tickets is 28.
- The probability of drawing two identical tickets is 1/7.
- The probability of drawing two different tickets is 6/7.
- The presence of duplicates (three 1s and two 2s) significantly influences probabilities.
- Conditional probabilities depend on the outcome of the first draw.

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