Use Equation 9 From Section 13.6 To Find The Surface Area Of That Part Of The Plane 10x+4y+z=10 That

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In multivariable calculus, calculating the surface area of a specific portion of a surface is a fundamental problem that illustrates the application of surface integrals. Equation 9 from Section 13.6 provides a systematic way to determine the surface area of a surface defined implicitly or explicitly. In this context, we focus on the plane described by the equation 10x + 4y + z = 10, and our goal is to find the surface area of a particular part of this plane—usually bounded by a certain region in the xy-plane or another specified boundary. This article will walk through the process step-by-step, illustrating how to utilize Equation 9 from Section 13.6, which involves setting up the appropriate surface integral, parametrization, and the necessary derivatives to evaluate the integral effectively.

Understanding the Surface and Its Boundaries

Equation of the Surface

The plane under consideration is given by:

$$ 10x + 4y + z = 10 $$

This is a linear surface in three-dimensional space, commonly referred to as a plane. To analyze its surface area, we need to understand its geometry and the region over which we want to compute the surface area.

Identifying the Part of the Plane

Typically, the problem specifies a region in the xy-plane that bounds the part of the surface we're interested in. For example, the region might be a specific polygon or a bounded domain such as a rectangle or circle. For the purposes of this discussion, assume that the part of the plane is bounded above by a region R in the xy-plane.

Common choices include:

- A rectangular region, e.g., $0 \leq x \leq a$, $0 \leq y \leq b$
- A circular region, e.g., $x^2 + y^2 \leq r^2$
- An arbitrary polygonal region

The specific boundary conditions will influence the limits of integration used in the surface area calculation.

Applying Equation 9 from Section 13.6

General Form of Equation 9

Equation 9 from Section 13.6 typically states the formula for the surface area S of a surface $z = f(x, y)$ over a region R:

$$
S = \iint_{R} \sqrt{1 + \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2} \, dA
$$

This integral accounts for the surface's slope in both x and y directions, effectively "stretching" the area element dA in the xy-plane to account for the surface's inclination.

Expressing the Surface as a Function $z = f(x, y)$

Given the plane's equation:

$$
10x + 4y + z = 10
$$

solve for z:

$$
z = 10 - 10x - 4y
$$

This explicit form allows us to compute the partial derivatives necessary for applying the formula.

Calculating Partial Derivatives

$$
\frac{\partial z}{\partial x} = -10
$$

$$
\frac{\partial z}{\partial y} = -4
$$

Notice that these derivatives are constants because the surface is a plane with constant slopes.

Setting Up the Surface Area Integral

Integral Expression

Using the derivatives, the surface area becomes:

$$
S = \iint_{R} \sqrt{1 + (-10)^2 + (-4)^2} \, dA
$$

which simplifies to:

$$
S = \iint_{R} \sqrt{1 + 100 + 16} \, dA = \iint_{R} \sqrt{117} \, dA
$$

Since $\sqrt{117}$ is a constant, the integral reduces to:

$$
S = \sqrt{117} \times \text{Area of } R
$$

Key Point: For a plane with constant slopes, the surface area over a region is simply the product of $\sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}$ and the area of the projection R in the xy-plane.

Calculating the Area of Region R

To determine the exact surface area, we need to:

1. Define the region R in the xy-plane.
2. Calculate its area, possibly via double integration.

Suppose, for example, that R is a rectangle:

$$
0 \leq x \leq a,\quad 0 \leq y \leq b
$$

then,

$$
\text{Area of } R = a \times b
$$

Alternatively, if the boundary is more complex, a double integral over the specified boundary will be necessary.

Explicit Calculation for a Specific Region

Example: Region R as a Rectangle

Suppose the part of the plane is bounded over the rectangle:

$$
0 \leq x \leq 2,\quad 0 \leq y \leq 3
$$

then, the surface area is:

$$
S = \sqrt{117} \times (2 \times 3) = 6 \sqrt{117}
$$

which numerically approximates to:

$$
S \approx 6 \times 10.8167 \approx 64.9002
$$

Example: Circular Region R with Radius r

If R is a circle of radius r:

$$
x^2 + y^2 \leq r^2
$$

then, the area of R:

$$
\text{Area} = \pi r^2
$$

and the surface area:

$$
S = \sqrt{117} \times \pi r^2
$$

This demonstrates the direct proportionality between the projection area and the surface area of the plane segment.

General Procedure for Any Region R

To compute the surface area for an arbitrary region R, follow these steps:

1. Identify the boundary of R in the xy-plane.
2. Set up the double integral over R using the formula:

   \[

$$
S = \iint_{R} \sqrt{1 + \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2} \, dA
$$

which simplifies in this case to:

$$
S = \sqrt{117} \iint_{R} dA
$$

since the integrand is constant.

3. Calculate the area of R, either directly if it's a simple shape or via integration if it's more complex.

4. Multiply the area of R by $\sqrt{117}$ to find the surface area.

Conclusion and Summary

In summary, the process of finding the surface area of a part of the plane $10x + 4y + z = 10$ involves translating the plane into a function $z = f(x, y)$, calculating the derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, and applying Equation 9 from Section 13.6. Because the derivatives are constants in this case, the integrand simplifies to a constant $\sqrt{117}$, and the surface area reduces to this constant multiplied by the area of the projection region R.

This approach exemplifies the power of surface integrals in multivariable calculus, allowing us to handle straightforward planes and more complex surfaces in a systematic way. Whether the region R is a simple rectangle or a complicated polygon, the method remains consistent—determine the projection area and multiply by the square root terms derived from the surface's slopes.

By mastering this technique, students and practitioners can efficiently compute surface areas for a variety of surfaces, which has applications across physics, engineering, and mathematics, including surface physics, material science, and geometric modeling.

Note: If the problem specifies particular boundaries for the region R, substitute those limits into the double integral, and evaluate accordingly.

Frequently Asked Questions

How is Equation 9 from Section 13.6 used to find the surface area of the plane 10x + 4y + z = 10?

Equation 9 provides a formula for the surface area by integrating the magnitude of the surface's gradient over the projection area. For the plane, it involves expressing z in terms of x and y, computing the surface element using partial derivatives, and integrating over the relevant domain.

What is the process to set up the surface integral for the plane 10x + 4y + z = 10 using Equation 9?

First, rewrite the plane as $z = 10 - 10x - 4y$. Then, compute the partial derivatives of z with respect to x and y, find the magnitude of the surface element using these derivatives, and integrate over the projection region in the xy-plane.

How do the coefficients 10 and 4 in the plane's equation influence the surface area calculation?

The coefficients determine the slopes of the plane in x and y directions, affecting the partial derivatives used in the surface element calculation. Larger coefficients increase the surface area contribution, as they increase the magnitude of the surface differential.

Can Equation 9 be applied directly to find the surface area of any plane, or are there limitations?

Equation 9 can be applied to any smooth surface, including planes, as long as the surface can be expressed as a function $z = f(x,y)$. For planes, the calculation simplifies, but the method remains valid for more complex surfaces as well.

What is the significance of choosing the correct projection domain when using Equation 9 to find the surface area of the plane?

The projection domain in the xy-plane determines the limits of integration. Accurate identification of this domain ensures the computed surface area correctly corresponds to the portion of the plane being analyzed, especially if the region is bounded or irregular.

Additional Resources

Use Equation 9 From Section 13.6 To Find The Surface Area Of That Part Of The Plane 10x+4y+z=10 That

Imagine peering down at a sloped surface, perhaps a segment of a rooftop or a section of a modern sculpture, and wondering just how much material covers it. Calculating the surface area of such a segment might seem daunting at first, especially when the surface is described by an equation in three dimensions. However, with the right tools and a systematic approach, this task becomes manageable. Today, we'll explore how to determine the surface area of a specific part of a plane, specifically the plane given by the equation $10x + 4y + z = 10$, using a vital formula—Equation 9 from Section 13.6 of a typical calculus textbook.

This method leverages calculus's power to convert a three-dimensional problem into an integral that can be evaluated systematically. Let's delve into the mathematical framework that makes this possible and walk through a step-by-step example to clarify the process.

Understanding the Problem: The Geometric Context

Before jumping into formulas and calculations, it's essential to grasp what the problem entails.

The Plane Equation

The given plane is:

$10x + 4y + z = 10$

This is a linear equation representing a flat, infinite surface. However, in typical applications—say, when dealing with a specific bounded region—we're interested only in a particular portion, such as the part lying over a certain region in the xy-plane.

The Region of Interest

Suppose we're asked to find the surface area of the part of this plane that lies over a specific domain D in the xy-plane. For example, D could be a rectangle, a triangle, or any other region. Without loss of generality, let's assume D is a rectangular region defined by:

$a \leq x \leq b$
$c \leq y \leq d$

Our task is to find the surface area of the surface portion lying directly above this domain D.

The Mathematical Foundation: Surface Area of a Surface

Calculating the surface area of a surface defined implicitly by a function $z = f(x, y)$ involves integrating the infinitesimal surface elements over the region D.

General Approach

- If a surface is described as $z = f(x, y)$, the differential surface element dS over a region D is given by:

$dS = \sqrt{1 + (\partial f/\partial x)^2 + (\partial f/\partial y)^2}\ dx\ dy$

- The total surface area S over D is then:

$S = \iint_D \sqrt{1 + (\partial f/\partial x)^2 + (\partial f/\partial y)^2} \, dx \, dy$

This formula accounts for the fact that the surface may be inclined or curved, adjusting the projected area accordingly.

Application to Our Plane

Our plane is given by:

$z = 10 - 10x - 4y$

This explicit form enables straightforward calculation of the partial derivatives needed for the surface area integral.

Using Equation 9 from Section 13.6: The Key Formula

In many calculus texts, Equation 9 from Section 13.6 provides a standard formula for computing the surface area of a graph $z = f(x, y)$:

Surface Area Formula:

$S = \iint_D \sqrt{1 + (\partial f/\partial x)^2 + (\partial f/\partial y)^2} \, dx \, dy$

This formula is fundamental because it simplifies the task to computing the partial derivatives of the function $f(x, y)$ over the domain D.

Why is this formula important?

- It reduces a three-dimensional surface area problem to a two-dimensional integral.
- It requires only the first derivatives of the function, which are often straightforward to compute.
- It allows for precise calculations of the area of inclined or curved surfaces.

In our case, since the surface is planar, the derivatives are constant, simplifying the integral significantly.

Step-by-Step Calculation

Let's proceed with a concrete example—calculating the surface area of the part of the plane $10x + 4y + z = 10$ that lies over a specified rectangular region in the xy-plane.

Step 1: Express z as a function of x and y

Rearranged:

$z = 10 - 10x - 4y$

Step 2: Compute the partial derivatives

- $\partial z/\partial x = -10$
- $\partial z/\partial y = -4$

Since these are constants, the integrand in the surface area formula simplifies greatly.

Step 3: Write the surface area integral

Using the formula:

$$S = \iint_D \sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}\, dx\, dy$$

Plugging in the derivatives:

$$S = \iint_D \sqrt{1 + (-10)^2 + (-4)^2}\, dx\, dy$$

Calculate inside the square root:

$$1 + 100 + 16 = 117$$

So:

$$S = \iint_D \sqrt{117}\, dx\, dy$$

Step 4: Determine the domain D

Suppose D is the rectangle:

$0 \leq x \leq 2$
$0 \leq y \leq 3$

Then the surface area becomes:

$$S = \sqrt{117} \times \text{Area}(D)$$

The area of D:

$$\text{Area}(D) = (b - a) \times (d - c) = (2 - 0) \times (3 - 0) = 6$$

Step 5: Final calculation

$$S = \sqrt{117} \times 6$$

Calculate $\sqrt{117}$:

$$\sqrt{117} \approx 10.8167$$

Therefore:

$$S \approx 10.8167 \times 6 \approx 64.9002$$

Result:

The surface area of the specified part of the plane over the given rectangle is approximately 64.9 square units.

Interpreting the Result and Extending the Method

This example illustrates a straightforward application of Equation 9 from Section 13.6. The key steps involve:

- Expressing the surface as a function $z = f(x, y)$
- Computing the partial derivatives
- Setting up the double integral over the domain D
- Calculating the area of D
- Multiplying by the constant $\sqrt{1 + (\partial f/\partial x)^2 + (\partial f/\partial y)^2}$

For more complex regions D, the integral becomes more involved, potentially requiring numerical methods or more advanced calculus techniques. When the surface is not planar, derivatives vary across D, and the integral must be evaluated accordingly.

This method isn't limited to planes; it applies broadly to any surface where $z = f(x, y)$ is known, providing a powerful tool for engineers, architects, and scientists needing precise surface measurements.

Practical Considerations and Applications

Understanding how to compute surface areas has several real-world applications:

- Architecture & Construction: Calculating material quantities for sloped roofs or facades.
- Manufacturing: Estimating surface coatings or paint needed for curved surfaces.
- Environmental Science: Measuring the surface area of landforms or water bodies for modeling.
- Aerospace & Mechanical Design: Determining aerodynamic surface areas for components.

In each case, the core approach remains consistent: express the surface mathematically, determine derivatives, define the domain, and evaluate the double integral.

Conclusion: The Power of Calculus in Surface Area Calculations

By leveraging Equation 9 from Section 13.6, we transform a seemingly complex three-dimensional problem into a manageable two-dimensional integral. Whether dealing with simple planes or intricate curved surfaces, this formula provides a systematic way to quantify surface areas with precision.

In our example, the planarity of the surface simplified derivatives to constants, making the calculation straightforward. For more complicated surfaces, the same principles apply, but the derivatives may vary, requiring more advanced techniques or numerical methods.

Ultimately, mastering this approach enables professionals and students alike to solve practical problems involving surface measurements with confidence and mathematical rigor. As the foundation of many engineering designs, environmental assessments, and scientific analyses, the ability to compute surface areas accurately remains a vital skill in the toolkit of modern applied mathematics.

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