Use The Laplace Transform To Solve The Following System Of DE Dr Dy (a) =2+Y, = 2r, R(0) = 0, Y(0) =

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The Laplace Transform is a powerful mathematical tool used to solve a wide range of differential equations, especially systems involving linear differential equations with initial conditions. When dealing with coupled systems of differential equations, the Laplace Transform simplifies the process by converting differential equations into algebraic equations in the complex frequency domain. This article provides an in-depth guide on how to use the Laplace Transform to solve a specific system of differential equations, illustrating each step to enhance understanding and application skills.

Understanding the System of Differential Equations

Before diving into the solution process, it is essential to understand the problem statement and its components.

The Given System

The system of differential equations is presented as:

1. $\frac{dR}{dt} = 2 + Y(t)$
2. $\frac{dY}{dt} = 2R(t)$

with initial conditions:

- $R(0) = 0$
- $Y(0) = \text{[value to be specified]}$

(Note: The original problem statement seems to have some missing parts, especially the initial value for $Y(0)$. For the purpose of this tutorial, we will assume $Y(0) = 0$, a common initial condition, to proceed with the solution. If a different initial condition is provided, it can be substituted accordingly.)

Significance of the System

This coupled system models various physical and engineering phenomena, such as electrical circuits, mechanical systems, and population dynamics, where the rate of change of one variable depends on another.

Step-by-Step Solution Using the Laplace Transform

Applying the Laplace Transform involves transforming the differential equations from the time domain into the algebraic domain, solving for the transformed variables, and then applying the inverse transform to find the solution in the time domain.

Step 1: Take the Laplace Transform of Each Equation

Recall that the Laplace Transform of a derivative $\frac{dX}{dt}$ is:

$$
\mathcal{L}\left\{\frac{dX}{dt}\right\} = sX(s) - X(0)
$$

where $X(s)$ is the Laplace Transform of $X(t)$.

Applying this to the system:

- For $R(t)$:

$$
\mathcal{L}\left\{\frac{dR}{dt}\right\} = s R(s) - R(0) = s R(s) - 0 = s R(s)
$$

- For $Y(t)$:

$$
\mathcal{L}\left\{\frac{dY}{dt}\right\} = s Y(s) - Y(0) = s Y(s) - 0 = s Y(s)
$$

Transforming the equations:

1. $s R(s) = \frac{2}{s} + Y(s)$ (since $\mathcal{L}\{2\} = \frac{2}{s}$)
2. $s Y(s) = 2 R(s)$

Solving the Algebraic System in the Laplace Domain

Now, we have a system of algebraic equations:

$$
\begin{cases}
s R(s) = \frac{2}{s} + Y(s) \\

s Y(s) = 2 R(s)

\end{cases}

\]

Step 2: Express one variable in terms of the other

From the second equation:

$$
Y(s) = \frac{2 R(s)}{s}
$$

Substitute this into the first equation:

$$
s R(s) = \frac{2}{s} + \frac{2 R(s)}{s}
$$

Multiply through by s to clear denominators:

$$
s^2 R(s) = 2 + 2 R(s)
$$

Rearranged:

$$
s^2 R(s) - 2 R(s) = 2
$$

Factor $R(s)$:

$$
R(s)(s^2 - 2) = 2
$$

Solve for $R(s)$:

$$
R(s) = \frac{2}{s^2 - 2}
$$

Step 3: Find $Y(s)$

Recall:

$$
Y(s) = \frac{2 R(s)}{s} = \frac{2}{s} \times \frac{2}{s^2 - 2} = \frac{4}{s(s^2 - 2)}
$$

Inverse Laplace Transform to Find $R(t)$ and $Y(t)$

Having expressions for $R(s)$ and $Y(s)$, the next step is to find their inverse Laplace Transforms to obtain solutions in the time domain.

Step 4: Inverse Laplace Transform of $R(s)$

$$
R(s) = \frac{2}{s^2 - 2}
$$

Notice that:

$$
s^2 - 2 = (s)^2 - (\sqrt{2})^2
$$

Expressed as a difference of squares, which resembles the standard Laplace Transform of sine functions:

$$
\mathcal{L}\left\{\frac{\sin(a t)}{a}\right\} = \frac{1}{s^2 + a^2}
$$

But we have $s^2 - 2$, which suggests a hyperbolic sine if written as $s^2 + (-\sqrt{2})^2$, but since it's negative, the standard approach is to write:

$$
R(s) = \frac{2}{s^2 - (\sqrt{2})^2}
$$

which can be rewritten as:

$$
R(s) = \frac{2}{(s - \sqrt{2})(s + \sqrt{2})}
$$

Alternatively, express it in terms of partial fractions for easier inversion.

Partial Fraction Decomposition

Express:

$$
\frac{2}{s^2 - 2} = \frac{A}{s - \sqrt{2}} + \frac{B}{s + \sqrt{2}}
$$

Multiply both sides by $s^2 - 2$:

$$
2 = A(s + \sqrt{2}) + B(s - \sqrt{2})
$$

Set $s = \sqrt{2}$:

$$
2 = A(\sqrt{2} + \sqrt{2}) + B(0) \Rightarrow 2 = A (2 \sqrt{2}) \Rightarrow A = \frac{2}{2 \sqrt{2}} = \frac{1}{\sqrt{2}}
$$

Set $s = - \sqrt{2}$:

$$
2 = A(0) + B(- \sqrt{2} - \sqrt{2}) \Rightarrow 2 = B (- 2 \sqrt{2}) \Rightarrow B = - \frac{2}{2 \sqrt{2}} = - \frac{1}{\sqrt{2}}
$$

Thus,

$$
R(s) = \frac{1}{\sqrt{2}} \times \frac{1}{s - \sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{s + \sqrt{2}}
$$

Inverse Laplace Transform:

$$
R(t) = \frac{1}{\sqrt{2}} e^{\sqrt{2} t} - \frac{1}{\sqrt{2}} e^{ - \sqrt{2} t } = \frac{1}{\sqrt{2}} \left(e^{\sqrt{2} t} - e^{ - \sqrt{2} t } \right)
$$

Recognize the hyperbolic sine:

$$
R(t) = \frac{1}{\sqrt{2}} \times 2 \sinh(\sqrt{2} t) = \sqrt{2} \sinh(\sqrt{2} t)
$$

Step 5: Find $Y(t)$

Recall:

$$
Y(s) = \frac{4}{s(s^2 - 2)}
$$

Partial fractions are used again:

$$
\frac{4}{s(s^2 - 2)} = \frac{A}{s} + \frac{B}{s - \sqrt{2}} + \frac{C}{s + \sqrt{2}}
$$

Multiplying both sides by $s(s^2 - 2)$:

$$
4 = A (s^2 - 2) + B s (s + \sqrt{2}) + C s (s - \sqrt{2})
$$

Set $s = 0$:

$$
4 = A (0 - 2) + 0 + 0 \Rightarrow 4 = -2A \Rightarrow A = -2
$$

Next, set $s = \sqrt{2}$:

$$
4 = -2 (\sqrt{2})^2 + B
$$

Frequently Asked Questions

How do you apply the Laplace Transform to solve the differential equation Dr Dy (a) = 2 + Y with initial conditions R(O) = O and Y(O) = O?

First, take the Laplace Transform of both sides of the differential equation, converting derivatives into algebraic expressions involving the Laplace variable s. Then, substitute initial conditions to solve for the Laplace transforms R(s) and Y(s). Finally, perform the inverse Laplace Transform to find R(t) and Y(t).

What are the steps involved in transforming the given system of

differential equations using Laplace Transforms?

The steps include: 1) Taking Laplace Transform of each differential equation; 2) Applying initial conditions; 3) Solving the resulting algebraic equations for R(s) and Y(s); 4) Using inverse Laplace Transform to find the solutions R(t) and Y(t).

How does the initial condition R(0) = 0 and Y(0) = 0 simplify the process of using Laplace Transforms in this system?

The initial conditions eliminate additional terms from the transformed equations, making it easier to solve for the Laplace transforms R(s) and Y(s) directly, without needing to account for initial value contributions.

What challenges might arise when solving this system of differential equations with Laplace Transforms, and how can they be addressed?

Challenges include solving coupled algebraic equations for R(s) and Y(s). These can be addressed by algebraic manipulation, substitution, or using partial fraction decomposition to facilitate inverse transforms.

Can you outline the physical or practical context where solving such a system of differential equations using Laplace Transforms is beneficial?

This approach is beneficial in engineering and physics for analyzing systems such as electrical circuits, mechanical vibrations, or control systems where the system's response over time is governed by coupled differential equations, making Laplace Transforms a powerful tool for solution.

Additional Resources

Use The Laplace Transform To Solve The Following System Of DE Dr Dy (a) =2+Y, = 2r, R(0) = 0, Y(0) =

Introduction

In the realm of differential equations, systems involving multiple variables and derivatives are fundamental to modeling complex phenomena in engineering, physics, economics, and beyond. Among the array of tools available to mathematicians and engineers, the Laplace transform stands out as a powerful method for converting differential equations into algebraic equations, simplifying the process of finding solutions. This article explores how to utilize the Laplace transform to solve a specific system of differential equations, providing both a technical and accessible guide to the procedure.

Understanding the System of Differential Equations

Before delving into the solution process, it's important to clarify the system at hand. The equations are given as:

- \(R'(t) = 2 + Y(t) \)
- \(Y'(t) = 2r(t) \)

with initial conditions:

- \(R(0) = 0 \)
- \(Y(0) = 0 \)

Note: The notation appears to have some ambiguities; for the purpose of this explanation, we interpret \(r(t) \) as a function related to \(R(t) \), possibly \(R(t) \) itself, and proceed accordingly. If the original problem has a typo, the approach remains similar once the correct relations are clarified.

The goal is to find explicit expressions for $R(t)$ and $Y(t)$ that satisfy these equations and initial conditions.

The Laplace Transform: An Overview

The Laplace transform is an integral transform that converts a time-domain function into a complex frequency-domain function. It is defined for a function $f(t)$ (with $t \geq 0$) as:

$$
\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st}f(t) \, dt
$$

where:

- s is a complex variable $s = \sigma + i\omega$,
- $F(s)$ is the Laplace transform of $f(t)$.

This transformation simplifies differentiation because it turns derivatives into algebraic terms:

$$
\mathcal{L}\{f'(t)\} = sF(s) - f(0)
$$

which makes solving linear differential equations more straightforward.

Applying the Laplace Transform to the System

Step 1: Transform both equations

Starting with the two equations:

1. $R'(t) = 2 + Y(t)$
2. $Y'(t) = 2r(t)$

and given initial conditions $R(0) = 0$, $Y(0) = 0$, we apply the Laplace transform to both.

- For the first equation:

$$
\mathcal{L}\{R'(t)\} = s R(s) - R(0) = s R(s)
$$

$$
\mathcal{L}\{2 + Y(t)\} = \frac{2}{s} + Y(s)
$$

Thus, the transformed first equation becomes:

$$
s R(s) = \frac{2}{s} + Y(s)
$$

- For the second equation:

$$
\mathcal{L}\{Y'(t)\} = s Y(s) - Y(0) = s Y(s)
$$

$$
\mathcal{L}\{2 r(t)\} = 2 R(s)
$$

\]

(Note: If $r(t) = R(t)$, then the second equation is consistent with that assumption. Otherwise, adjustments are needed based on the actual definitions.)

So the second transformed equation becomes:

$$
s Y(s) = 2 R(s)
$$

Step 2: Solve algebraic equations

From the second equation:

$$
s Y(s) = 2 R(s) \Rightarrow Y(s) = \frac{2 R(s)}{s}
$$

Substitute $Y(s)$ into the first equation:

$$
s R(s) = \frac{2}{s} + \frac{2 R(s)}{s}
$$

Multiply both sides by s to clear denominators:

$$
s^2 R(s) = 2 + 2 R(s)
$$

Rearranged:

$$
s^2 R(s) - 2 R(s) = 2
$$

$$
R(s) (s^2 - 2) = 2
$$

$$
R(s) = \frac{2}{s^2 - 2}
$$

Now, recalling that $Y(s) = \frac{2 R(s)}{s}$:

$$
Y(s) = \frac{2}{s} \times \frac{2}{s^2 - 2} = \frac{4}{s (s^2 - 2)}
$$

Inverse Laplace Transform: Finding the Solutions

Having expressed $R(s)$ and $Y(s)$ explicitly, the next step is to find their inverse Laplace transforms to obtain $R(t)$ and $Y(t)$.

Calculating $R(t)$

$$
R(s) = \frac{2}{s^2 - 2}
$$

\]

Note that \(s^2 - 2 \) can be factored as:

\[

s^2 - 2 = (s - \sqrt{2})(s + \sqrt{2})

\]

This suggests partial fraction decomposition:

\[

R(s) = \frac{A}{s - \sqrt{2}} + \frac{B}{s + \sqrt{2}}

\]

Multiplying through:

\[

2 = A(s + \sqrt{2}) + B(s - \sqrt{2})

\]

Set \(s = \sqrt{2} \):

\[

2 = A(\sqrt{2} + \sqrt{2}) + B(\sqrt{2} - \sqrt{2}) \Rightarrow 2 = 2 \sqrt{2} A + 0 \Rightarrow A = \frac{2}{2 \sqrt{2}} = \frac{1}{\sqrt{2}}

\]

Set \(s = - \sqrt{2} \):

\[

2 = A(-\sqrt{2} + \sqrt{2}) + B(-\sqrt{2} - \sqrt{2}) \Rightarrow 2 = 0 + B(-2 \sqrt{2}) \Rightarrow B = -

\frac{2}{2 \sqrt{2}} = - \frac{1}{\sqrt{2}}

\]

Thus,

\[

R(s) = \frac{1}{\sqrt{2}} \times \frac{1}{s - \sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{s + \sqrt{2}}

\]

Inverse Laplace transforms are well-known:

\[

\mathcal{L}^{-1}\left\{\frac{1}{s - a}\right\} = e^{a t}

\]

Therefore,

\[

R(t) = \frac{1}{\sqrt{2}}\left(e^{\sqrt{2} t} - e^{-\sqrt{2} t} \right)

\]

This simplifies to:

\[

R(t) = \frac{1}{\sqrt{2}} \sinh(\sqrt{2} t)

\]

Calculating \(Y(t) \)

Recall:

$$
Y(s) = \frac{4}{s(s^2 - 2)}
$$

Express as partial fractions:

$$
\frac{4}{s(s - \sqrt{2})(s + \sqrt{2})} = \frac{A}{s} + \frac{B}{s - \sqrt{2}} + \frac{C}{s + \sqrt{2}}
$$

Multiply both sides by the denominator:

$$
4 = A(s - \sqrt{2})(s + \sqrt{2}) + B s (s + \sqrt{2}) + C s (s - \sqrt{2})
$$

Note:

$$
(s - \sqrt{2})(s + \sqrt{2}) = s^2 - 2
$$

Set $s = 0$:

$$
4 = A (0^2 - 2) + B \times 0 \times (\sqrt{2}) + C \times 0 \times (-\sqrt{2}) \Rightarrow 4 = -2A \Rightarrow A = -2
$$

Set \(s = \sqrt{2} \):

\[

4 = A (\sqrt{2}^2 - 2) + B \sqrt{2} (\sqrt{2} + \sqrt{2}) + C \sqrt{2} (\sqrt{2} - \sqrt{2})

\]

Calculate:

\[

\sqrt{2}^2 = 2

\]

So:

\[

4 = A (2 - 2) + B \sqrt{2} \times 2 \sqrt{2} + C \sqrt{2} \times 0

\]

\[

4 = 0 + B \sqrt{2} \times 2 \sqrt{2}

\]

Since:

\[

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